Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.2-Inverse-hyperbolic-cosine/7.2.2-d-x-^m-a+b-arccosh-c-x-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [166]. This is test number [189].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (166)	% 0. (0)
Mathematica	% 100. (166)	% 0. (0)
Maple	% 67.47 (112)	% 32.53 (54)
Maxima	% 33.13 (55)	% 66.87 (111)
Fricas	% 31.33 (52)	% 68.67 (114)
Sympy	% 30.12 (50)	% 69.88 (116)
Giac	% 31.93 (53)	% 68.07 (113)

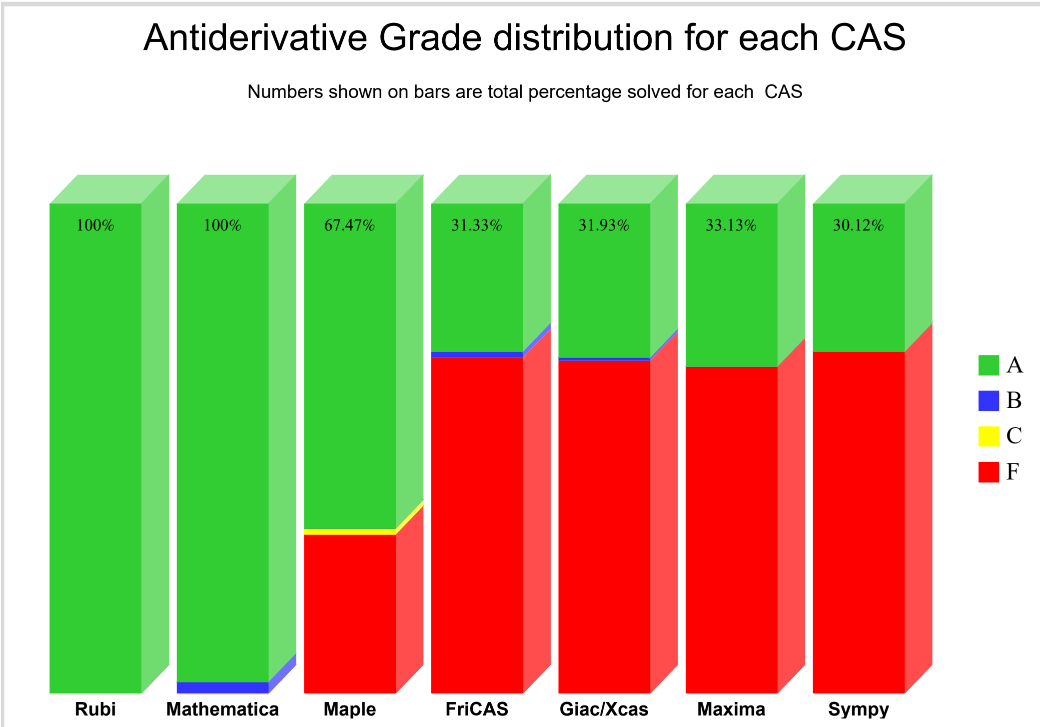
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

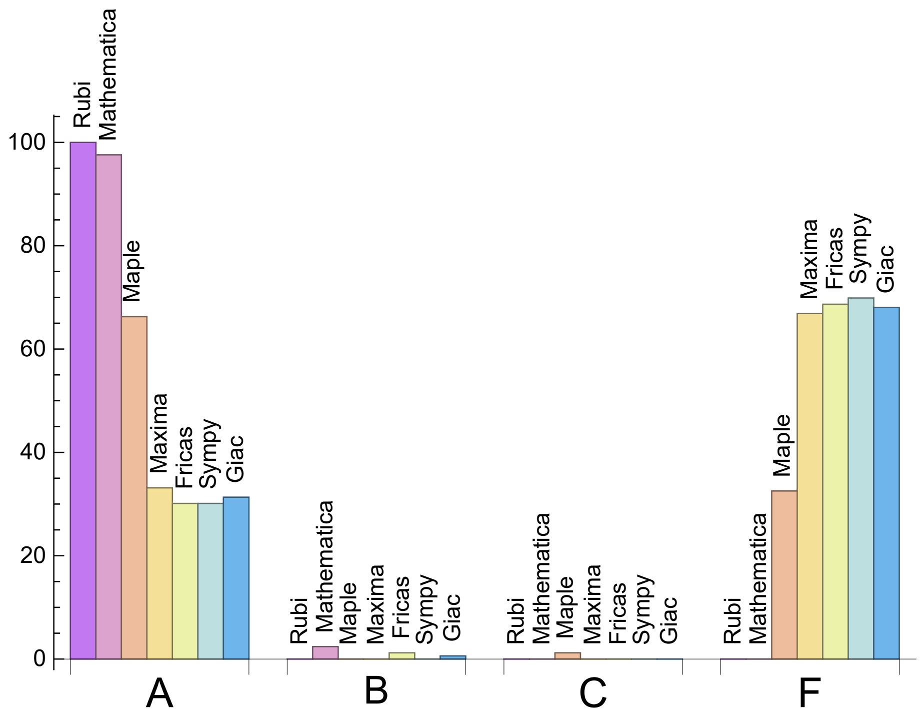
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	97.59	2.41	0.	0.
Maple	66.27	0.	1.2	32.53
Maxima	33.13	0.	0.	66.87
Fricas	30.12	1.2	0.	68.67
Sympy	30.12	0.	0.	69.88
Giac	31.33	0.6	0.	68.07

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.36	106.27	0.83	92.	1.
Mathematica	0.69	107.6	0.84	83.5	0.9
Maple	0.08	68.54	0.77	58.5	0.86
Maxima	0.73	49.29	0.6	34.	0.95
Fricas	1.69	145.48	1.67	134.5	1.85
Sympy	1.86	53.34	0.46	13.	0.41
Giac	0.67	59.32	0.73	0.	0.

1.4 list of integrals that has no closed form antiderivative

{49, 50, 56, 57, 63, 64, 70, 71, 77, 83, 89, 95, 96, 102, 108, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 132, 166}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {137}

Mathematica {2, 4, 6, 17, 18, 20, 27, 28, 29, 30, 31, 38, 39, 40, 41, 51, 52, 53, 54, 55, 66, 67, 68, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 90, 91, 92, 94, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 137, 142, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

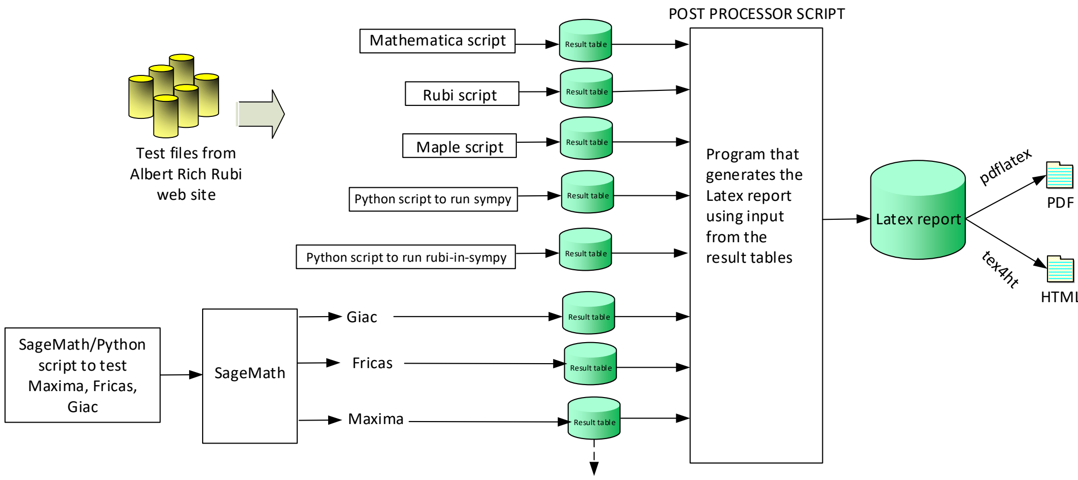
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166 }

B grade: { 39, 41, 148, 150 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 32, 33, 34, 35, 36, 37, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 75, 76, 77, 81, 82, 83, 87, 88, 89, 93, 94, 95, 96, 100, 101, 102, 106, 107, 108, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 166 }

B grade: { }

C grade: { 130, 131 }

F grade: { 28, 30, 39, 41, 72, 73, 74, 78, 79, 80, 84, 85, 86, 90, 91, 92, 97, 98, 99, 103, 104, 105, 109, 110, 111, 117, 118, 127, 128, 129, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 16, 19, 21, 22, 24, 26, 33, 35, 37, 49, 50, 56, 57, 63, 64, 70, 71, 77, 83, 89, 95, 96, 102, 108, 114, 119, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 136, 138, 139, 140, 141, 166

B grade: { }

C grade: { }

F grade: { 6, 13, 15, 17, 18, 20, 23, 25, 27, 28, 29, 30, 31, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 115, 116, 117, 118, 127, 128, 129, 130, 131, 137, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 49, 50, 56, 57, 63, 64, 70, 71, 115, 116, 119, 120, 121, 126, 132, 133, 134, 135, 136, 139, 140, 141, 166 }

B grade: { 7, 138 }

C grade: { }

F grade: { 6, 17, 18, 20, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 122, 123, 124, 125, 127, 128, 129, 130, 131, 137, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 49, 50, 56, 57, 63, 64, 70, 71, 77, 83, 95, 96, 102, 108, 115, 116, 119, 120, 121, 123, 124, 125, 126, 132, 133, 134, 135, 136, 166 }

B grade: { }

C grade: { }

F grade: { 6, 7, 8, 9, 10, 11, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 117, 118, 122, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 16, 21, 22, 24, 26, 33, 35, 37, 49, 50, 56, 57, 63, 64, 70, 71, 77, 83, 89, 95, 96, 102, 108, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 136, 166 }

B grade: { 19 }

C grade: { }

F grade: { 6, 13, 15, 17, 18, 20, 23, 25, 27, 28, 29, 30, 31, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 117, 118, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	55	52	92	135	76	90
normalized size	1	1.	0.59	0.56	0.99	1.45	0.82	0.97
time (sec)	N/A	0.037	0.033	0.014	1.092	2.505	2.482	1.23

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	71	99	116	131	68	109
normalized size	1	1.	0.92	1.29	1.51	1.7	0.88	1.42
time (sec)	N/A	0.03	0.062	0.017	1.144	2.274	1.346	1.353

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	46	43	65	113	54	70
normalized size	1	1.	0.71	0.66	1.	1.74	0.83	1.08
time (sec)	N/A	0.023	0.023	0.009	1.111	2.402	0.619	1.244

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	61	77	88	109	44	95
normalized size	1	1.	1.24	1.57	1.8	2.22	0.9	1.94
time (sec)	N/A	0.015	0.025	0.011	1.132	2.346	0.265	1.277

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	34	78	26	47
normalized size	1	1.	1.	0.97	1.13	2.6	0.87	1.57
time (sec)	N/A	0.006	0.016	0.001	1.159	2.363	0.208	1.373

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	43	43	42	66	0	0	0	0
normalized size	1	1.	0.98	1.53	0.	0.	0.	0.
time (sec)	N/A	0.059	0.034	0.034	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	57	51	32	158	0	49
normalized size	1	1.	1.78	1.59	1.	4.94	0.	1.53
time (sec)	N/A	0.017	0.025	0.013	1.781	2.403	0.	1.396

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	40	36	86	0	68
normalized size	1	1.	0.92	1.05	0.95	2.26	0.	1.79
time (sec)	N/A	0.013	0.007	0.012	1.728	2.419	0.	1.209

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	78	73	61	215	0	78
normalized size	1	1.	1.2	1.12	0.94	3.31	0.	1.2
time (sec)	N/A	0.027	0.075	0.013	1.827	2.451	0.	1.335

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	45	50	65	109	0	104
normalized size	1	1.	0.68	0.76	0.98	1.65	0.	1.58
time (sec)	N/A	0.024	0.023	0.012	1.772	2.466	0.	1.345

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	104	95	88	238	0	103
normalized size	1	1.	1.12	1.02	0.95	2.56	0.	1.11
time (sec)	N/A	0.041	0.044	0.016	1.766	2.519	0.	1.349

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	80	168	134	234	122	153
normalized size	1	1.	0.61	1.27	1.02	1.77	0.92	1.16
time (sec)	N/A	0.49	0.107	0.04	1.282	2.53	4.914	1.332

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	77	126	0	205	99	0
normalized size	1	1.	0.73	1.19	0.	1.93	0.93	0.
time (sec)	N/A	0.44	0.072	0.035	0.	2.367	2.876	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	64	100	95	188	85	120
normalized size	1	1.	0.71	1.11	1.06	2.09	0.94	1.33
time (sec)	N/A	0.311	0.095	0.031	1.181	2.257	1.295	1.323

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	58	58	0	166	60	0
normalized size	1	1.	0.91	0.91	0.	2.59	0.94	0.
time (sec)	N/A	0.251	0.05	0.029	0.	2.193	0.639	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	39	43	134	39	84
normalized size	1	1.	1.	1.	1.1	3.44	1.	2.15
time (sec)	N/A	0.127	0.02	0.026	1.137	2.325	0.244	1.353

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	63	98	0	0	0	0
normalized size	1	1.	1.02	1.58	0.	0.	0.	0.
time (sec)	N/A	0.093	0.028	0.029	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	92	137	0	0	0	0
normalized size	1	1.	1.53	2.28	0.	0.	0.	0.
time (sec)	N/A	0.209	0.251	0.061	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	73	53	158	0	149
normalized size	1	1.	1.	1.52	1.1	3.29	0.	3.1
time (sec)	N/A	0.192	0.019	0.063	1.733	2.515	0.	1.425

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	144	177	0	0	0	0
normalized size	1	1.	1.26	1.55	0.	0.	0.	0.
time (sec)	N/A	0.392	0.243	0.106	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	69	109	97	194	0	198
normalized size	1	1.	0.73	1.15	1.02	2.04	0.	2.08
time (sec)	N/A	0.365	0.078	0.107	1.555	2.545	0.	1.626

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	130	246	223	359	206	243
normalized size	1	1.	0.56	1.06	0.97	1.55	0.89	1.05
time (sec)	N/A	0.771	0.117	0.048	1.198	2.419	8.787	1.607

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	143	184	0	327	170	0
normalized size	1	1.	0.78	1.01	0.	1.79	0.93	0.
time (sec)	N/A	0.666	0.131	0.042	0.	2.528	5.034	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	103	150	157	281	138	190
normalized size	1	1.	0.66	0.97	1.01	1.81	0.89	1.23
time (sec)	N/A	0.474	0.093	0.037	1.252	2.509	2.599	1.583

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	113	88	0	261	102	0
normalized size	1	1.	1.06	0.82	0.	2.44	0.95	0.
time (sec)	N/A	0.381	0.082	0.03	0.	2.45	1.258	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	61	77	205	63	132
normalized size	1	1.	1.	0.9	1.13	3.01	0.93	1.94
time (sec)	N/A	0.183	0.025	0.031	1.21	2.476	0.57	1.362

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	82	132	0	0	0	0
normalized size	1	1.	0.94	1.52	0.	0.	0.	0.
time (sec)	N/A	0.105	0.05	0.037	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	128	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.31	0.168	0.099	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	92	113	0	0	0	0
normalized size	1	1.	0.94	1.15	0.	0.	0.	0.
time (sec)	N/A	0.317	0.794	0.082	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	201	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.577	0.546	0.169	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	220	180	0	0	0	0
normalized size	1	1.	1.26	1.03	0.	0.	0.	0.
time (sec)	N/A	0.578	0.598	0.122	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	175	344	0	497	275	0
normalized size	1	1.	0.57	1.12	0.	1.62	0.9	0.
time (sec)	N/A	2.193	0.153	0.052	0.	2.398	25.086	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	158	300	271	466	248	293
normalized size	1	1.	0.58	1.09	0.99	1.7	0.91	1.07
time (sec)	N/A	1.628	0.127	0.05	1.231	2.615	14.652	1.85

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	143	224	0	402	197	0
normalized size	1	1.	0.67	1.05	0.	1.88	0.92	0.
time (sec)	N/A	1.314	0.109	0.045	0.	2.477	8.668	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	122	180	193	358	165	230
normalized size	1	1.	0.67	0.99	1.06	1.97	0.91	1.26
time (sec)	N/A	0.881	0.112	0.04	1.205	2.358	5.032	1.779

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	104	104	0	316	110	0
normalized size	1	1.	0.87	0.87	0.	2.63	0.92	0.
time (sec)	N/A	0.603	0.07	0.033	0.	2.606	2.612	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	71	99	262	70	169
normalized size	1	1.	1.	0.92	1.29	3.4	0.91	2.19
time (sec)	N/A	0.298	0.028	0.033	1.157	2.312	1.181	1.467

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	103	165	0	0	0	0
normalized size	1	1.	1.	1.6	0.	0.	0.	0.
time (sec)	N/A	0.12	0.029	0.037	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	478	0	0	0	0	0
normalized size	1	1.	3.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.335	0.648	0.086	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	112	149	0	0	0	0
normalized size	1	1.	0.97	1.3	0.	0.	0.	0.
time (sec)	N/A	0.358	1.073	0.071	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	268	268	595	0	0	0	0	0
normalized size	1	1.	2.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.802	3.075	0.181	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	40	40	0	0	0	0
normalized size	1	1.	0.73	0.73	0.	0.	0.	0.
time (sec)	N/A	0.096	0.095	0.039	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	33	0	0	0	0
normalized size	1	1.	0.77	0.77	0.	0.	0.	0.
time (sec)	N/A	0.084	0.083	0.033	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	31	0	0	0	0
normalized size	1	1.	0.76	0.76	0.	0.	0.	0.
time (sec)	N/A	0.08	0.071	0.026	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	0	0	0
normalized size	1	1.	0.83	0.83	0.	0.	0.	0.
time (sec)	N/A	0.066	0.061	0.027	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	22	0	0	0	0
normalized size	1	1.	0.74	0.81	0.	0.	0.	0.
time (sec)	N/A	0.064	0.052	0.03	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	0	0	0
normalized size	1	1.	1.	0.93	0.	0.	0.	0.
time (sec)	N/A	0.037	0.022	0.024	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0
normalized size	1	1.	1.	1.11	0.	0.	0.	0.
time (sec)	N/A	0.017	0.022	0.021	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.194	0.065	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.421	0.089	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	101	83	0	0	0	0
normalized size	1	1.	1.38	1.14	0.	0.	0.	0.
time (sec)	N/A	0.064	0.209	0.037	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	58	54	0	0	0	0
normalized size	1	1.	0.95	0.89	0.	0.	0.	0.
time (sec)	N/A	0.05	0.231	0.033	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	58	59	0	0	0	0
normalized size	1	1.	0.98	1.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.226	0.03	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	42	42	44	28	0	0	0	0
normalized size	1	1.	1.05	0.67	0.	0.	0.	0.
time (sec)	N/A	0.025	0.241	0.026	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	39	39	60	33	0	0	0	0
normalized size	1	1.	1.54	0.85	0.	0.	0.	0.
time (sec)	N/A	0.184	0.1	0.026	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	4.35	0.05	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	7.917	0.091	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	107	123	0	0	0	0
normalized size	1	1.	1.05	1.21	0.	0.	0.	0.
time (sec)	N/A	0.645	0.135	0.042	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	75	82	0	0	0	0
normalized size	1	1.	0.86	0.94	0.	0.	0.	0.
time (sec)	N/A	0.598	0.166	0.036	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	69	84	0	0	0	0
normalized size	1	1.	0.81	0.99	0.	0.	0.	0.
time (sec)	N/A	0.504	0.146	0.029	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	67	43	0	0	0	0
normalized size	1	1.	0.99	0.63	0.	0.	0.	0.
time (sec)	N/A	0.393	0.049	0.026	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	45	0	0	0	0
normalized size	1	1.	1.	0.82	0.	0.	0.	0.
time (sec)	N/A	0.188	0.042	0.027	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.659	0.065	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	2.194	0.089	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	126	175	0	0	0	0
normalized size	1	1.	0.74	1.03	0.	0.	0.	0.
time (sec)	N/A	0.617	0.609	0.043	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	188	114	0	0	0	0
normalized size	1	1.	1.21	0.74	0.	0.	0.	0.
time (sec)	N/A	0.588	0.373	0.043	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	183	121	0	0	0	0
normalized size	1	1.	1.2	0.79	0.	0.	0.	0.
time (sec)	N/A	0.676	0.365	0.035	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	131	60	0	0	0	0
normalized size	1	1.	1.25	0.57	0.	0.	0.	0.
time (sec)	N/A	0.396	0.294	0.026	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	116	67	0	0	0	0
normalized size	1	1.	1.35	0.78	0.	0.	0.	0.
time (sec)	N/A	0.378	0.179	0.027	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	9.826	0.069	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	11.941	0.093	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	182	162	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.475	0.095	0.183	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	101	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.403	0.084	0.103	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	100	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.397	0.083	0.089	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	65	73	0	0	0	0
normalized size	1	1.	0.7	0.78	0.	0.	0.	0.
time (sec)	N/A	0.35	0.075	0.089	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	45	41	0	0	0	0
normalized size	1	1.	0.85	0.77	0.	0.	0.	0.
time (sec)	N/A	0.219	0.036	0.074	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.283	0.07	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	345	345	152	0	0	0	0	0
normalized size	1	1.	0.44	0.	0.	0.	0.	0.
time (sec)	N/A	1.115	0.112	0.187	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	101	0	0	0	0	0
normalized size	1	1.	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.874	0.085	0.097	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	100	0	0	0	0	0
normalized size	1	1.	0.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.64	0.087	0.091	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	84	105	0	0	0	0
normalized size	1	1.	0.66	0.83	0.	0.	0.	0.
time (sec)	N/A	0.438	0.125	0.112	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	45	68	0	0	0	0
normalized size	1	1.	0.52	0.79	0.	0.	0.	0.
time (sec)	N/A	0.22	0.029	0.095	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.289	0.069	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	394	394	162	0	0	0	0	0
normalized size	1	1.	0.41	0.	0.	0.	0.	0.
time (sec)	N/A	1.822	0.097	0.191	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	257	257	101	0	0	0	0	0
normalized size	1	1.	0.39	0.	0.	0.	0.	0.
time (sec)	N/A	1.375	0.083	0.109	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	100	0	0	0	0	0
normalized size	1	1.	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	1.055	0.082	0.088	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	92	139	0	0	0	0
normalized size	1	1.	0.59	0.89	0.	0.	0.	0.
time (sec)	N/A	0.709	0.165	0.115	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	99	99	45	81	0	0	0	0
normalized size	1	1.	0.45	0.82	0.	0.	0.	0.
time (sec)	N/A	0.395	0.035	0.098	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.294	0.071	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	150	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	0.11	0.203	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	101	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.086	0.113	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	100	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	0.083	0.101	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	49	37	0	0	0	0
normalized size	1	1.	0.78	0.59	0.	0.	0.	0.
time (sec)	N/A	0.077	0.036	0.053	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	43	43	45	26	0	0	0	0
normalized size	1	1.	1.05	0.6	0.	0.	0.	0.
time (sec)	N/A	0.047	0.028	0.043	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.228	0.071	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.617	0.102	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	193	193	201	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.184	0.292	0.183	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	124	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.159	0.092	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	139	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.166	0.086	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	63	83	0	0	0	0
normalized size	1	1.	0.71	0.93	0.	0.	0.	0.
time (sec)	N/A	0.066	0.1	0.111	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	76	66	0	0	0	0
normalized size	1	1.	1.12	0.97	0.	0.	0.	0.
time (sec)	N/A	0.218	0.06	0.095	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.341	0.066	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	228	228	278	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.864	1.542	0.174	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	175	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.755	0.759	0.102	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	194	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.627	0.66	0.086	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	83	122	0	0	0	0
normalized size	1	1.	0.67	0.99	0.	0.	0.	0.
time (sec)	N/A	0.483	0.255	0.113	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	121	84	0	0	0	0
normalized size	1	1.	1.36	0.94	0.	0.	0.	0.
time (sec)	N/A	0.234	0.151	0.106	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.343	0.067	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	300	300	374	0	0	0	0	0
normalized size	1	1.	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.932	1.988	0.172	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	244	244	291	0	0	0	0	0
normalized size	1	1.	1.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.757	0.556	0.098	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	286	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.848	0.796	0.086	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	91	153	0	0	0	0
normalized size	1	1.	0.58	0.97	0.	0.	0.	0.
time (sec)	N/A	0.495	0.286	0.118	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	147	111	0	0	0	0
normalized size	1	1.	1.2	0.91	0.	0.	0.	0.
time (sec)	N/A	0.444	0.181	0.101	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.345	0.066	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	58	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.277	1.809	0.646	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	58	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.298	1.735	0.569	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	167	143	0	0	0	0	0
normalized size	1	1.08	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.238	0.355	0.693	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	82	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.095	0.676	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	1.301	0.485	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	1.319	0.576	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	1.326	0.457	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	1.871	0.07	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	2.169	0.067	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	2.011	0.071	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	1.932	0.069	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	1.396	0.082	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	144	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.247	0.194	0.155	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	97	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.191	0.095	0.08	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	95	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	0.111	0.092	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	58	38	0	0	0	0
normalized size	1	1.	0.98	0.64	0.	0.	0.	0.
time (sec)	N/A	0.087	0.045	0.031	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	43	40	0	0	0	0
normalized size	1	1.	0.88	0.82	0.	0.	0.	0.
time (sec)	N/A	0.049	0.024	0.041	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.327	0.053	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	105	109	130	161	87	123
normalized size	1	1.	1.25	1.3	1.55	1.92	1.04	1.46
time (sec)	N/A	0.039	0.037	0.012	1.17	2.459	1.459	1.469

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	54	55	78	140	71	84
normalized size	1	1.	0.76	0.77	1.1	1.97	1.	1.18
time (sec)	N/A	0.03	0.044	0.004	1.172	2.429	0.677	1.453

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	76	86	101	132	61	108
normalized size	1	1.	1.38	1.56	1.84	2.4	1.11	1.96
time (sec)	N/A	0.02	0.035	0.004	1.15	2.5	0.373	1.433

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	34	41	95	31	55
normalized size	1	1.	1.	0.97	1.17	2.71	0.89	1.57
time (sec)	N/A	0.014	0.021	0.003	1.106	2.493	0.167	1.296

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	48	75	0	0	0	0
normalized size	1	1.	0.87	1.36	0.	0.	0.	0.
time (sec)	N/A	0.077	0.04	0.03	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	65	59	43	171	0	0
normalized size	1	1.	1.76	1.59	1.16	4.62	0.	0.
time (sec)	N/A	0.024	0.072	0.004	1.676	2.596	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	48	52	49	108	0	0
normalized size	1	1.	1.12	1.21	1.14	2.51	0.	0.
time (sec)	N/A	0.02	0.018	0.004	1.74	2.319	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	101	82	73	234	0	0
normalized size	1	1.	1.42	1.15	1.03	3.3	0.	0.
time (sec)	N/A	0.033	0.101	0.006	1.7	2.642	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	50	62	77	139	0	0
normalized size	1	1.	0.69	0.86	1.07	1.93	0.	0.
time (sec)	N/A	0.032	0.029	0.004	1.645	2.512	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	214	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.788	0.481	0.109	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	136	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.649	0.38	0.108	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	100	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.412	0.222	0.092	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	292	292	540	0	0	0	0	0
normalized size	1	1.	1.85	0.	0.	0.	0.	0.
time (sec)	N/A	1.251	2.255	0.099	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	184	184	165	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.816	1.008	0.106	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	269	0	0	0	0	0
normalized size	1	1.	1.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.428	0.625	0.094	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	337	337	924	0	0	0	0	0
normalized size	1	1.	2.74	0.	0.	0.	0.	0.
time (sec)	N/A	2.09	10.694	0.102	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	228	228	207	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	1.328	1.88	0.124	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	452	0	0	0	0	0
normalized size	1	1.	2.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.751	2.351	0.095	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	195	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.36	0.346	0.108	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	104	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	0.22	0.115	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	100	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.1	0.094	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	247	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.31	0.658	0.105	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	135	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.155	1.202	0.105	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	132	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.41	0.237	0.099	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	276	276	340	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	1.33	2.04	0.095	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	188	188	157	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.893	1.517	0.118	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	192	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.456	1.015	0.094	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	361	361	394	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	1.627	2.461	0.099	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	175	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.87	1.61	0.111	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	188	188	214	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.772	0.698	0.109	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	141	118	0	0	0	0	0
normalized size	1	1.1	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.285	0.397	0.29	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	194	164	0	0	0	0	0
normalized size	1	1.07	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.31	0.246	2.246	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	87	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.137	2.023	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.457	0.811	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [41] had the largest ratio of [1.]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.	8	0.5
2	A	5	5	1.	8	0.625
3	A	4	4	1.	8	0.5
4	A	3	3	1.	6	0.5
5	A	2	2	1.	4	0.5
6	A	5	5	1.	8	0.625
7	A	3	3	1.	8	0.375
8	A	2	2	1.	8	0.25
9	A	5	5	1.	8	0.625
10	A	4	4	1.	8	0.5
11	A	7	5	1.	8	0.625
12	A	7	5	1.	10	0.5
13	A	6	4	1.	10	0.4
14	A	5	5	1.	10	0.5
15	A	4	4	1.	8	0.5

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	3	3	1.	6	0.5
17	A	6	6	1.	10	0.6
18	A	7	5	1.	10	0.5
19	A	3	3	1.	10	0.3
20	A	9	7	1.	10	0.7
21	A	5	5	1.	10	0.5
22	A	16	7	1.	10	0.7
23	A	12	7	1.	10	0.7
24	A	9	7	1.	10	0.7
25	A	6	5	1.	8	0.625
26	A	4	3	1.	6	0.5
27	A	7	7	1.	10	0.7
28	A	9	6	1.	10	0.6
29	A	7	7	1.	10	0.7
30	A	13	9	1.	10	0.9
31	A	10	9	1.	10	0.9
32	A	23	4	1.	10	0.4
33	A	19	6	1.	10	0.6
34	A	14	4	1.	10	0.4
35	A	11	6	1.	10	0.6
36	A	7	4	1.	8	0.5
37	A	5	3	1.	6	0.5
38	A	8	7	1.	10	0.7
39	A	11	7	1.	10	0.7
40	A	8	8	1.	10	0.8
41	A	19	10	1.	10	1.
42	A	7	3	1.	10	0.3
43	A	6	3	1.	10	0.3
44	A	6	3	1.	10	0.3
45	A	5	3	1.	10	0.3
46	A	5	3	1.	10	0.3
47	A	4	4	1.	8	0.5

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	2	2	1.	6	0.333
49	A	0	0	0.	0	0.
50	A	0	0	0.	0	0.
51	A	5	2	1.	10	0.2
52	A	4	2	1.	10	0.2
53	A	4	2	1.	10	0.2
54	A	2	2	1.	8	0.25
55	A	3	3	1.	6	0.5
56	A	0	0	0.	0	0.
57	A	0	0	0.	0	0.
58	A	14	5	1.	10	0.5
59	A	12	6	1.	10	0.6
60	A	10	6	1.	10	0.6
61	A	7	7	1.	8	0.875
62	A	4	4	1.	6	0.667
63	A	0	0	0.	0	0.
64	A	0	0	0.	0	0.
65	A	12	4	1.	10	0.4
66	A	9	4	1.	10	0.4
67	A	10	6	1.	10	0.6
68	A	5	5	1.	8	0.625
69	A	5	4	1.	6	0.667
70	A	0	0	0.	0	0.
71	A	0	0	0.	0	0.
72	A	19	7	1.	12	0.583
73	A	14	7	1.	12	0.583
74	A	14	7	1.	12	0.583
75	A	9	7	1.	10	0.7
76	A	7	6	1.	8	0.75
77	A	0	0	0.	0	0.
78	A	41	10	1.	12	0.833
79	A	25	10	1.	12	0.833

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	22	10	1.	12	0.833
81	A	11	10	1.	10	1.
82	A	8	7	1.	8	0.875
83	A	0	0	0.	0	0.
84	A	44	10	1.	12	0.833
85	A	27	9	1.	12	0.75
86	A	24	10	1.	12	0.833
87	A	12	9	1.	10	0.9
88	A	9	7	1.	8	0.875
89	A	0	0	0.	0	0.
90	A	18	6	1.	12	0.5
91	A	13	6	1.	12	0.5
92	A	13	6	1.	12	0.5
93	A	8	7	1.	10	0.7
94	A	6	5	1.	8	0.625
95	A	0	0	0.	0	0.
96	A	0	0	0.	0	0.
97	A	17	5	1.	12	0.417
98	A	12	5	1.	12	0.417
99	A	12	5	1.	12	0.417
100	A	6	5	1.	10	0.5
101	A	7	6	1.	8	0.75
102	A	0	0	0.	0	0.
103	A	34	8	1.	12	0.667
104	A	24	9	1.	12	0.75
105	A	22	9	1.	12	0.75
106	A	11	10	1.	10	1.
107	A	8	7	1.	8	0.875
108	A	0	0	0.	0	0.
109	A	32	7	1.	12	0.583
110	A	21	7	1.	12	0.583
111	A	22	9	1.	12	0.75

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
112	A	9	8	1.	10	0.8
113	A	9	7	1.	8	0.875
114	A	0	0	0.	0	0.
115	A	0	0	0.	0	0.
116	A	0	0	0.	0	0.
117	A	2	2	1.08	10	0.2
118	A	4	4	1.	8	0.5
119	A	0	0	0.	0	0.
120	A	0	0	0.	0	0.
121	A	0	0	0.	0	0.
122	A	0	0	0.	0	0.
123	A	0	0	0.	0	0.
124	A	0	0	0.	0	0.
125	A	0	0	0.	0	0.
126	A	0	0	0.	0	0.
127	A	12	4	1.	10	0.4
128	A	9	4	1.	10	0.4
129	A	9	4	1.	10	0.4
130	A	6	5	1.	8	0.625
131	A	4	3	1.	6	0.5
132	A	0	0	0.	0	0.
133	A	5	5	1.	12	0.417
134	A	4	4	1.	12	0.333
135	A	3	3	1.	10	0.3
136	A	3	2	1.	8	0.25
137	A	5	5	1.	12	0.417
138	A	3	3	1.	12	0.25
139	A	2	2	1.	12	0.167
140	A	5	5	1.	12	0.417
141	A	4	4	1.	12	0.333
142	A	14	7	1.	16	0.438
143	A	9	7	1.	14	0.5

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	7	6	1.	12	0.5
145	A	22	10	1.	16	0.625
146	A	11	10	1.	14	0.714
147	A	8	7	1.	12	0.583
148	A	24	10	1.	16	0.625
149	A	12	9	1.	14	0.643
150	A	9	7	1.	12	0.583
151	A	13	6	1.	16	0.375
152	A	8	7	1.	14	0.5
153	A	6	5	1.	12	0.417
154	A	12	5	1.	16	0.312
155	A	6	5	1.	14	0.357
156	A	7	6	1.	12	0.5
157	A	22	9	1.	16	0.562
158	A	11	10	1.	14	0.714
159	A	8	7	1.	12	0.583
160	A	22	9	1.	16	0.562
161	A	9	8	1.	14	0.571
162	A	9	7	1.	12	0.583
163	A	2	2	1.1	18	0.111
164	A	2	2	1.07	16	0.125
165	A	4	4	1.	14	0.286
166	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int x^4 \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=93

$$-\frac{4x^2\sqrt{ax-1}\sqrt{ax+1}}{75a^3}-\frac{8\sqrt{ax-1}\sqrt{ax+1}}{75a^5}-\frac{x^4\sqrt{ax-1}\sqrt{ax+1}}{25a}+\frac{1}{5}x^5\cosh^{-1}(ax)$$

```
[Out] (-8*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(75*a^5) - (4*x^2*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x])/(75*a^3) - (x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(25*a) + (x^5*ArcCosh[
a*x])/5
```

Rubi [A] time = 0.0374411, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5662, 100, 12, 74}

$$-\frac{4x^2\sqrt{ax-1}\sqrt{ax+1}}{75a^3}-\frac{8\sqrt{ax-1}\sqrt{ax+1}}{75a^5}-\frac{x^4\sqrt{ax-1}\sqrt{ax+1}}{25a}+\frac{1}{5}x^5\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCosh[a*x],x]
```

```
[Out] (-8*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(75*a^5) - (4*x^2*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x])/(75*a^3) - (x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(25*a) + (x^5*ArcCosh[
a*x])/5
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$
\begin{aligned}
\int x^4\cosh^{-1}(ax)\,dx &= \frac{1}{5}x^5\cosh^{-1}(ax)-\frac{1}{5}a\int\frac{x^5}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{25a}+\frac{1}{5}x^5\cosh^{-1}(ax)-\frac{\int\frac{4x^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{25a}\\
&= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{25a}+\frac{1}{5}x^5\cosh^{-1}(ax)-\frac{4\int\frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{25a}\\
&= -\frac{4x^2\sqrt{-1+ax}\sqrt{1+ax}}{75a^3}-\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{25a}+\frac{1}{5}x^5\cosh^{-1}(ax)-\frac{4\int\frac{2x}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{75a^3}\\
&= -\frac{4x^2\sqrt{-1+ax}\sqrt{1+ax}}{75a^3}-\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{25a}+\frac{1}{5}x^5\cosh^{-1}(ax)-\frac{8\int\frac{x}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{75a^3}\\
&= -\frac{8\sqrt{-1+ax}\sqrt{1+ax}}{75a^5}-\frac{4x^2\sqrt{-1+ax}\sqrt{1+ax}}{75a^3}-\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{25a}+\frac{1}{5}x^5\cosh^{-1}(ax)
\end{aligned}
$$

Mathematica [A] time = 0.0328575, size = 55, normalized size = 0.59

$$
\frac{1}{5}x^5\cosh^{-1}(ax)-\frac{\sqrt{ax-1}\sqrt{ax+1}\left(3a^4x^4+4a^2x^2+8\right)}{75a^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCosh[a*x],x]
```

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(8 + 4*a^2*x^2 + 3*a^4*x^4))/(75*a^5) + (x^5
*ArcCosh[a*x])/5
```

Maple [A] time = 0.014, size = 52, normalized size = 0.6

$$
\frac{1}{a^5}\left(\frac{a^5x^5\operatorname{arccosh}(ax)}{5}-\frac{3\,x^4a^4+4\,a^2x^2+8}{75}\sqrt{ax-1}\sqrt{ax+1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x),x)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arccosh(a*x)-1/75*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(3*a^4*x^4
+4*a^2*x^2+8))
```

Maxima [A] time = 1.09191, size = 92, normalized size = 0.99

$$\frac{1}{5}\,x^{5}\operatorname{arcosh}\left(ax\right)-\frac{1}{75}\left(\frac{3\,\sqrt{a^{2}x^{2}-1}x^{4}}{a^{2}}+\frac{4\,\sqrt{a^{2}x^{2}-1}x^{2}}{a^{4}}+\frac{8\,\sqrt{a^{2}x^{2}-1}}{a^{6}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccosh(a*x) - 1/75*(3*sqrt(a^2*x^2 - 1)*x^4/a^2 + 4*sqrt(a^2*x^2 -
 1)*x^2/a^4 + 8*sqrt(a^2*x^2 - 1)/a^6)*a
```

Fricas [A] time = 2.5049, size = 135, normalized size = 1.45

$$\frac{15\,a^{5}x^{5}\log\left(ax+\sqrt{a^{2}x^{2}-1}\right)-\left(3\,a^{4}x^{4}+4\,a^{2}x^{2}+8\right)\sqrt{a^{2}x^{2}-1}}{75\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/75*(15*a^5*x^5*log(a*x + sqrt(a^2*x^2 - 1)) - (3*a^4*x^4 + 4*a^2*x^2 + 8)
*sqrt(a^2*x^2 - 1))/a^5
```

Sympy [A] time = 2.48224, size = 76, normalized size = 0.82

$$\begin{cases}\frac{x^{5}\operatorname{acosh}\left(ax\right)}{5}-\frac{x^{4}\sqrt{a^{2}x^{2}-1}}{25a}-\frac{4x^{2}\sqrt{a^{2}x^{2}-1}}{75a^{3}}-\frac{8\sqrt{a^{2}x^{2}-1}}{75a^{5}} & \text{for}\: a\neq 0\\\frac{i\pi x^{5}}{10} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x),x)
```

```
[Out] Piecewise((x**5*acosh(a*x)/5 - x**4*sqrt(a**2*x**2 - 1)/(25*a) - 4*x**2*sqr
t(a**2*x**2 - 1)/(75*a**3) - 8*sqrt(a**2*x**2 - 1)/(75*a**5), Ne(a, 0)), (I
```

```
*pi*x**5/10, True))
```

Giac [A] time = 1.22981, size = 90, normalized size = 0.97

$$\frac{1}{5}\,x^5\log\left(ax+\sqrt{a^2x^2-1}\right)-\frac{3\left(a^2x^2-1\right)^{\frac{5}{2}}+10\left(a^2x^2-1\right)^{\frac{3}{2}}+15\,\sqrt{a^2x^2-1}}{75\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/5*x^5*log(a*x + sqrt(a^2*x^2 - 1)) - 1/75*(3*(a^2*x^2 - 1)^(5/2) + 10*(a^
2*x^2 - 1)^(3/2) + 15*sqrt(a^2*x^2 - 1))/a^5
```

3.2 $\int x^3 \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=77

$$-\frac{3x\sqrt{ax-1}\sqrt{ax+1}}{32a^3}-\frac{3\cosh^{-1}(ax)}{32a^4}-\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{16a}+\frac{1}{4}x^4\cosh^{-1}(ax)$$

```
[Out] (-3*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(32*a^3) - (x^3*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x])/(16*a) - (3*ArcCosh[a*x])/(32*a^4) + (x^4*ArcCosh[a*x])/4
```

Rubi [A] time = 0.0299139, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5662, 100, 12, 90, 52}

$$-\frac{3x\sqrt{ax-1}\sqrt{ax+1}}{32a^3}-\frac{3\cosh^{-1}(ax)}{32a^4}-\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{16a}+\frac{1}{4}x^4\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCosh[a*x],x]
```

```
[Out] (-3*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(32*a^3) - (x^3*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x])/(16*a) - (3*ArcCosh[a*x])/(32*a^4) + (x^4*ArcCosh[a*x])/4
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3 \cosh^{-1}(ax)\,dx &= \frac{1}{4}x^4\cosh^{-1}(ax) - \frac{1}{4}a\int \frac{x^4}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx \\
&= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{16a} + \frac{1}{4}x^4\cosh^{-1}(ax) - \frac{\int \frac{3x^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{16a} \\
&= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{16a} + \frac{1}{4}x^4\cosh^{-1}(ax) - \frac{3\int \frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{16a} \\
&= -\frac{3x\sqrt{-1+ax}\sqrt{1+ax}}{32a^3} - \frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{16a} + \frac{1}{4}x^4\cosh^{-1}(ax) - \frac{3\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{32a^3} \\
&= -\frac{3x\sqrt{-1+ax}\sqrt{1+ax}}{32a^3} - \frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{16a} - \frac{3\cosh^{-1}(ax)}{32a^4} + \frac{1}{4}x^4\cosh^{-1}(ax)
\end{aligned}$$

Mathematica [A] time = 0.0622652, size = 71, normalized size = 0.92

$$-\frac{ax\sqrt{ax-1}\sqrt{ax+1}\left(2a^2x^2+3\right) - 8a^4x^4\cosh^{-1}(ax) + 6\tanh^{-1}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{32a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*ArcCosh[a*x],x]
```

```
[Out] -(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(3 + 2*a^2*x^2) - 8*a^4*x^4*ArcCosh[a*x]
 + 6*ArcTanh[Sqrt[(-1 + a*x)/(1 + a*x)]])/(32*a^4)
```

Maple [A] time = 0.017, size = 99, normalized size = 1.3

$$\frac{x^4\mathrm{arccosh}\,(ax)}{4}-\frac{x^3}{16\,a}\sqrt{ax-1}\sqrt{ax+1}-\frac{3\,x}{32\,a^3}\sqrt{ax-1}\sqrt{ax+1}-\frac{3}{32\,a^4}\sqrt{ax-1}\sqrt{ax+1}\ln\left(ax+\sqrt{a^2x^2-1}\right)\frac{1}{\sqrt{a^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x),x)
```

```
[Out] 1/4*x^4*arccosh(a*x)-1/16*x^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a-3/32*x*(a*x-1)^
(1/2)*(a*x+1)^(1/2)/a^3-3/32/a^4*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)^(1
/2)*ln(a*x+(a^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.14385, size = 116, normalized size = 1.51

$$\frac{1}{4}\,x^4\,\mathrm{arcosh}\,(ax)-\frac{1}{32}\left(\frac{2\,\sqrt{a^2x^2-1}x^3}{a^2}+\frac{3\,\sqrt{a^2x^2-1}x}{a^4}+\frac{3\,\log\left(2\,a^2x+2\,\sqrt{a^2x^2-1}\sqrt{a^2}\right)}{\sqrt{a^2}a^4}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccosh(a*x) - 1/32*(2*sqrt(a^2*x^2 - 1)*x^3/a^2 + 3*sqrt(a^2*x^2 -
 1)*x/a^4 + 3*log(2*a^2*x + 2*sqrt(a^2*x^2 - 1)*sqrt(a^2))/(sqrt(a^2)*a^4))
*a
```

Fricas [A] time = 2.27403, size = 131, normalized size = 1.7

$$\frac{\left(8\,a^4x^4-3\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\left(2\,a^3x^3+3\,ax\right)\sqrt{a^2x^2-1}}{32\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/32*((8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 - 1)) - (2*a^3*x^3 + 3*a*x)*sq
rt(a^2*x^2 - 1))/a^4
```

Sympy [A] time = 1.34646, size = 68, normalized size = 0.88

$$\begin{cases} \frac{x^4 \operatorname{acosh}(ax)}{4} - \frac{x^3\sqrt{a^2x^2-1}}{16a} - \frac{3x\sqrt{a^2x^2-1}}{32a^3} - \frac{3\operatorname{acosh}(ax)}{32a^4} & \text{for } a \neq 0 \\ \frac{i\pi x^4}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x),x)
```

```
[Out] Piecewise((x**4*acosh(a*x)/4 - x**3*sqrt(a**2*x**2 - 1)/(16*a) - 3*x*sqrt(a
**2*x**2 - 1)/(32*a**3) - 3*acosh(a*x)/(32*a**4), Ne(a, 0)), (I*pi*x**4/8,
True))
```

Giac [A] time = 1.35273, size = 109, normalized size = 1.42

$$\frac{1}{4}x^4 \log\left(ax + \sqrt{a^2x^2-1}\right) - \frac{1}{32}\left(\sqrt{a^2x^2-1}\,x\left(\frac{2x^2}{a^2} + \frac{3}{a^4}\right) - \frac{3\log\left(\left|-x|a| + \sqrt{a^2x^2-1}\right|\right)}{a^4|a|}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(a*x + sqrt(a^2*x^2 - 1)) - 1/32*(sqrt(a^2*x^2 - 1)*x*(2*x^2/a^2
 + 3/a^4) - 3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(a^4*abs(a)))*a
```

3.3 $\int x^2 \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=65

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{9a^3}-\frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{9a}+\frac{1}{3}x^3\cosh^{-1}(ax)$$

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(9*a^3) - (x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*
x])/(9*a) + (x^3*ArcCosh[a*x])/3
```

Rubi [A] time = 0.0225147, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5662, 100, 12, 74}

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{9a^3}-\frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{9a}+\frac{1}{3}x^3\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCosh[a*x],x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(9*a^3) - (x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*
x])/(9*a) + (x^3*ArcCosh[a*x])/3
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}
\int x^2\cosh^{-1}(ax)\,dx &= \frac{1}{3}x^3\cosh^{-1}(ax)-\frac{1}{3}a\int\frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{9a}+\frac{1}{3}x^3\cosh^{-1}(ax)-\frac{\int\frac{2x}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{9a}\\
&= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{9a}+\frac{1}{3}x^3\cosh^{-1}(ax)-\frac{2\int\frac{x}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{9a}\\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{9a^3}-\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{9a}+\frac{1}{3}x^3\cosh^{-1}(ax)
\end{aligned}$$

Mathematica [A] time = 0.0229695, size = 46, normalized size = 0.71

$$\frac{1}{3}x^3\cosh^{-1}(ax)-\frac{\sqrt{ax-1}\sqrt{ax+1}\left(a^2x^2+2\right)}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCosh[a*x],x]
```

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(2 + a^2*x^2))/(9*a^3) + (x^3*ArcCosh[a*x])/
3
```

Maple [A] time = 0.009, size = 43, normalized size = 0.7

$$\frac{1}{a^3}\left(\frac{a^3x^3\operatorname{arccosh}(ax)}{3}-\frac{a^2x^2+2}{9}\sqrt{ax-1}\sqrt{ax+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x),x)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arccosh(a*x)-1/9*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(a^2*x^2+2)
)
```

Maxima [A] time = 1.11093, size = 65, normalized size = 1.

$$\frac{1}{3}x^3\operatorname{arcosh}(ax) - \frac{1}{9}a\left(\frac{\sqrt{a^2x^2-1}x^2}{a^2} + \frac{2\sqrt{a^2x^2-1}}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccosh(a*x) - 1/9*a*(sqrt(a^2*x^2 - 1)*x^2/a^2 + 2*sqrt(a^2*x^2 -
1)/a^4)
```

Fricas [A] time = 2.40158, size = 113, normalized size = 1.74

$$\frac{3a^3x^3\log\left(ax+\sqrt{a^2x^2-1}\right) - \left(a^2x^2+2\right)\sqrt{a^2x^2-1}}{9a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a^3*x^3*log(a*x + sqrt(a^2*x^2 - 1)) - (a^2*x^2 + 2)*sqrt(a^2*x^2 -
1))/a^3
```

Sympy [A] time = 0.619472, size = 54, normalized size = 0.83

$$\begin{cases} \frac{x^3\operatorname{acosh}(ax)}{3} - \frac{x^2\sqrt{a^2x^2-1}}{9a} - \frac{2\sqrt{a^2x^2-1}}{9a^3} & \text{for } a \neq 0 \\ \frac{i\pi x^3}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x),x)
```

```
[Out] Piecewise((x**3*acosh(a*x)/3 - x**2*sqrt(a**2*x**2 - 1)/(9*a) - 2*sqrt(a**2
*x**2 - 1)/(9*a**3), Ne(a, 0)), (I*pi*x**3/6, True))
```

Giac [A] time = 1.24391, size = 70, normalized size = 1.08

$$\frac{1}{3}\,x^3\log\left(ax + \sqrt{a^2x^2 - 1}\right) - \frac{\left(a^2x^2 - 1\right)^{\frac{3}{2}} + 3\,\sqrt{a^2x^2 - 1}}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(a*x + sqrt(a^2*x^2 - 1)) - 1/9*((a^2*x^2 - 1)^(3/2) + 3*sqrt(a^
2*x^2 - 1))/a^3
```

3.4 $\int x \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=49

$$-\frac{\cosh^{-1}(ax)}{4a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)-\frac{x\sqrt{ax-1}\sqrt{ax+1}}{4a}$$

```
[Out] -(x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(4*a) - ArcCosh[a*x]/(4*a^2) + (x^2*ArcCo
sh[a*x])/2
```

Rubi [A] time = 0.0152043, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5662, 90, 52}

$$-\frac{\cosh^{-1}(ax)}{4a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)-\frac{x\sqrt{ax-1}\sqrt{ax+1}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCosh[a*x],x]
```

```
[Out] -(x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(4*a) - ArcCosh[a*x]/(4*a^2) + (x^2*ArcCo
sh[a*x])/2
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int x\cosh^{-1}(ax)\,dx &= \frac{1}{2}x^2\cosh^{-1}(ax)-\frac{1}{2}a\int\frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\ &= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{4a}+\frac{1}{2}x^2\cosh^{-1}(ax)-\frac{\int\frac{1}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{4a}\\ &= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{4a}-\frac{\cosh^{-1}(ax)}{4a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)\end{aligned}$$

Mathematica [A] time = 0.0246326, size = 61, normalized size = 1.24

$$-\frac{-2a^2x^2\cosh^{-1}(ax)+ax\sqrt{ax-1}\sqrt{ax+1}+2\tanh^{-1}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcCosh[a*x],x]
```

```
[Out] -(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x] - 2*a^2*x^2*ArcCosh[a*x] + 2*ArcTanh[Sqr
t[(-1 + a*x)/(1 + a*x)]])/(4*a^2)
```

Maple [A] time = 0.011, size = 77, normalized size = 1.6

$$\frac{x^2\operatorname{arccosh}(ax)}{2}-\frac{x}{4\,a}\sqrt{ax-1}\sqrt{ax+1}-\frac{1}{4\,a^2}\sqrt{ax-1}\sqrt{ax+1}\ln\left(ax+\sqrt{a^2x^2-1}\right)\frac{1}{\sqrt{a^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x),x)
```

```
[Out] 1/2*x^2*arccosh(a*x)-1/4*x*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a-1/4/a^2*(a*x-1)^(1
/2)*(a*x+1)^(1/2)/(a^2*x^2-1)^(1/2)*ln(a*x+(a^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.13153, size = 88, normalized size = 1.8

$$\frac{1}{2}x^2\operatorname{arcosh}(ax) - \frac{1}{4}a\left(\frac{\sqrt{a^2x^2-1}x}{a^2} + \frac{\log\left(2a^2x + 2\sqrt{a^2x^2-1}\sqrt{a^2}\right)}{\sqrt{a^2}a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccosh(a*x) - 1/4*a*(sqrt(a^2*x^2 - 1)*x/a^2 + log(2*a^2*x + 2*sqr
t(a^2*x^2 - 1)*sqrt(a^2))/(sqrt(a^2)*a^2))
```

Fricas [A] time = 2.34593, size = 109, normalized size = 2.22

$$-\frac{\sqrt{a^2x^2-1}ax - \left(2a^2x^2-1\right)\log\left(ax + \sqrt{a^2x^2-1}\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(a^2*x^2 - 1)*a*x - (2*a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1)))
/a^2
```

Sympy [A] time = 0.265342, size = 44, normalized size = 0.9

$$\begin{cases} \frac{x^2\operatorname{acosh}(ax)}{2} - \frac{x\sqrt{a^2x^2-1}}{4a} - \frac{\operatorname{acosh}(ax)}{4a^2} & \text{for } a \neq 0 \\ \frac{i\pi x^2}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x),x)
```

```
[Out] Piecewise((x**2*acosh(a*x)/2 - x*sqrt(a**2*x**2 - 1)/(4*a) - acosh(a*x)/(4*
a**2), Ne(a, 0)), (I*pi*x**2/4, True))
```

Giac [A] time = 1.27737, size = 95, normalized size = 1.94

$$\frac{1}{2}\,x^2\log\left(ax+\sqrt{a^2x^2-1}\right)-\frac{1}{4}\,a\left(\frac{\sqrt{a^2x^2-1}x}{a^2}-\frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{a^2|a|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(a*x + sqrt(a^2*x^2 - 1)) - 1/4*a*(sqrt(a^2*x^2 - 1)*x/a^2 - log
(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(a^2*abs(a)))
```

3.5 $\int \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=30

$$x\cosh^{-1}(ax)-\frac{\sqrt{ax-1}\sqrt{ax+1}}{a}$$

```
[Out] -((Sqrt[-1 + a*x]*Sqrt[1 + a*x])/a) + x*ArcCosh[a*x]
```

Rubi [A] time = 0.006411, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5654, 74}

$$x\cosh^{-1}(ax)-\frac{\sqrt{ax-1}\sqrt{ax+1}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x],x]
```

```
[Out] -((Sqrt[-1 + a*x]*Sqrt[1 + a*x])/a) + x*ArcCosh[a*x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \cosh^{-1}(ax)\,dx&=x\cosh^{-1}(ax)-a\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\&=-\frac{\sqrt{-1+ax}\sqrt{1+ax}}{a}+x\cosh^{-1}(ax)\end{aligned}$$

Mathematica [A] time = 0.016369, size = 30, normalized size = 1.

$$x\cosh^{-1}(ax)-\frac{\sqrt{ax-1}\sqrt{ax+1}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x],x]
```

```
[Out] -((Sqrt[-1 + a*x]*Sqrt[1 + a*x])/a) + x*ArcCosh[a*x]
```

Maple [A] time = 0.001, size = 29, normalized size = 1.

$$\frac{1}{a}\left(ax\mathrm{arccosh}\,(ax)-\sqrt{ax-1}\sqrt{ax+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x),x)
```

```
[Out] 1/a*(a*x*arccosh(a*x)-(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

Maxima [A] time = 1.15911, size = 34, normalized size = 1.13

$$\frac{ax\,\mathrm{arcosh}\,(ax)-\sqrt{a^2x^2-1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x),x, algorithm="maxima")
```

```
[Out] (a*x*arccosh(a*x) - sqrt(a^2*x^2 - 1))/a
```

Fricas [A] time = 2.36269, size = 78, normalized size = 2.6

$$\frac{ax\log\left(ax+\sqrt{a^2x^2-1}\right)-\sqrt{a^2x^2-1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x),x, algorithm="fricas")
```

```
[Out] (a*x*log(a*x + sqrt(a^2*x^2 - 1)) - sqrt(a^2*x^2 - 1))/a
```

Sympy [A] time = 0.208176, size = 26, normalized size = 0.87

$$\begin{cases} x \operatorname{acosh}(ax) - \frac{\sqrt{a^2x^2-1}}{a} & \text{for } a \neq 0 \\ \frac{i\pi x}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x),x)
```

```
[Out] Piecewise((x*acosh(a*x) - sqrt(a**2*x**2 - 1)/a, Ne(a, 0)), (I*pi*x/2, True
))
```

Giac [A] time = 1.37339, size = 47, normalized size = 1.57

$$x \log\left(ax + \sqrt{a^2x^2 - 1}\right) - \frac{\sqrt{a^2x^2 - 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x),x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 - 1)) - sqrt(a^2*x^2 - 1)/a
```

3.6 $\int \frac{\cosh^{-1}(ax)}{x} \, dx$

Optimal. Leaf size=43

$$\frac{1}{2}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)-\frac{1}{2}\cosh^{-1}(ax)^2+\cosh^{-1}(ax)\log\left(e^{2\cosh^{-1}(ax)}+1\right)$$

```
[Out] -ArcCosh[a*x]^2/2 + ArcCosh[a*x]*Log[1 + E^(2*ArcCosh[a*x])] + PolyLog[2, -
E^(2*ArcCosh[a*x])]/2
```

Rubi [A] time = 0.058621, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5660, 3718, 2190, 2279, 2391}

$$\frac{1}{2}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)-\frac{1}{2}\cosh^{-1}(ax)^2+\cosh^{-1}(ax)\log\left(e^{2\cosh^{-1}(ax)}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/x,x]
```

```
[Out] -ArcCosh[a*x]^2/2 + ArcCosh[a*x]*Log[1 + E^(2*ArcCosh[a*x])] + PolyLog[2, -
E^(2*ArcCosh[a*x])]/2
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)}{x}\,dx &= \text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}(ax)\right)\\
&= -\frac{1}{2}\cosh^{-1}(ax)^2 + 2\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}(ax)\right)\\
&= -\frac{1}{2}\cosh^{-1}(ax)^2 + \cosh^{-1}(ax)\log\left(1+e^{2\cosh^{-1}(ax)}\right) - \text{Subst}\left(\int \log\left(1+e^{2x}\right)dx, x, \cosh^{-1}(ax)\right)\\
&= -\frac{1}{2}\cosh^{-1}(ax)^2 + \cosh^{-1}(ax)\log\left(1+e^{2\cosh^{-1}(ax)}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\cosh^{-1}(ax)}\right)\\
&= -\frac{1}{2}\cosh^{-1}(ax)^2 + \cosh^{-1}(ax)\log\left(1+e^{2\cosh^{-1}(ax)}\right) + \frac{1}{2}\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right)
\end{aligned}$$

Mathematica [A] time = 0.0340923, size = 42, normalized size = 0.98

$$\frac{1}{2}\left(\cosh^{-1}(ax)\left(\cosh^{-1}(ax)+2\log\left(e^{-2\cosh^{-1}(ax)}+1\right)\right)-\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(ax)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/x,x]
```

```
[Out] (ArcCosh[a*x]*(ArcCosh[a*x] + 2*Log[1 + E^(-2*ArcCosh[a*x])]) - PolyLog[2,
-E^(-2*ArcCosh[a*x])])/2
```

Maple [A] time = 0.034, size = 66, normalized size = 1.5

$$-\frac{(\operatorname{arccosh}(ax))^2}{2}+\operatorname{arccosh}(ax)\ln\left(1+\left(ax+\sqrt{ax-1}\sqrt{ax+1}\right)^2\right)+\frac{1}{2}\operatorname{polylog}\left(2,-\left(ax+\sqrt{ax-1}\sqrt{ax+1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/x,x)
```

```
[Out] -1/2*arccosh(a*x)^2+arccosh(a*x)*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)+
1/2*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcosh}(ax)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/x,x)
```

```
[Out] Integral(acosh(a*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/x, x)
```

3.7 $\int \frac{\cosh^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=32

$$a\tan^{-1}\left(\sqrt{ax-1}\sqrt{ax+1}\right)-\frac{\cosh^{-1}(ax)}{x}$$

```
[Out] -(ArcCosh[a*x]/x) + a*ArcTan[Sqrt[-1 + a*x]*Sqrt[1 + a*x]]
```

Rubi [A] time = 0.0168724, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5662, 92, 205}

$$a\tan^{-1}\left(\sqrt{ax-1}\sqrt{ax+1}\right)-\frac{\cosh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/x^2,x]
```

```
[Out] -(ArcCosh[a*x]/x) + a*ArcTan[Sqrt[-1 + a*x]*Sqrt[1 + a*x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{x^2} dx = -\frac{\cosh^{-1}(ax)}{x} + a \int \frac{1}{x\sqrt{-1+ax}\sqrt{1+ax}} dx$$
$$= -\frac{\cosh^{-1}(ax)}{x} + a^2 \operatorname{Subst}\left(\int \frac{1}{a+ax^2} dx, x, \sqrt{-1+ax}\sqrt{1+ax}\right)$$
$$= -\frac{\cosh^{-1}(ax)}{x} + a \tan^{-1}\left(\sqrt{-1+ax}\sqrt{1+ax}\right)$$

Mathematica [A] time = 0.0245135, size = 57, normalized size = 1.78

$$\frac{a\sqrt{a^2x^2-1}\tan^{-1}\left(\sqrt{a^2x^2-1}\right)}{\sqrt{ax-1}\sqrt{ax+1}} - \frac{\cosh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/x^2,x]
```

```
[Out] -(ArcCosh[a*x]/x) + (a*Sqrt[-1 + a^2*x^2]*ArcTan[Sqrt[-1 + a^2*x^2]])/(Sqrt
[-1 + a*x]*Sqrt[1 + a*x])
```

Maple [A] time = 0.013, size = 51, normalized size = 1.6

$$-\frac{\operatorname{arccosh}(ax)}{x} - a\sqrt{ax-1}\sqrt{ax+1}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\frac{1}{\sqrt{a^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/x^2,x)
```

```
[Out] -arccosh(a*x)/x-a*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)^(1/2)*arctan(1/(a
^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.78092, size = 32, normalized size = 1.

$$-a \arcsin\left(\frac{1}{\sqrt{a^2}|x|}\right) - \frac{\operatorname{arcosh}(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] -a*arcsin(1/(sqrt(a^2)*abs(x))) - arccosh(a*x)/x
```

Fricas [B] time = 2.40256, size = 158, normalized size = 4.94

$$\frac{2\,ax\arctan\left(-ax+\sqrt{a^2x^2-1}\right)+(x-1)\log\left(ax+\sqrt{a^2x^2-1}\right)+x\log\left(-ax+\sqrt{a^2x^2-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*arctan(-a*x + sqrt(a^2*x^2 - 1)) + (x - 1)*log(a*x + sqrt(a^2*x^2 -
1)) + x*log(-a*x + sqrt(a^2*x^2 - 1)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acosh}(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/x**2,x)
```

```
[Out] Integral(acosh(a*x)/x**2, x)
```

Giac [A] time = 1.39625, size = 49, normalized size = 1.53

$$a\arctan\left(\sqrt{a^2x^2-1}\right)-\frac{\log\left(ax+\sqrt{a^2x^2-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^2,x, algorithm="giac")

[Out] a*arctan(sqrt(a^2*x^2 - 1)) - log(a*x + sqrt(a^2*x^2 - 1))/x
```

3.8 $\int \frac{\cosh^{-1}(ax)}{x^3} \, dx$

Optimal. Leaf size=38

$$\frac{a\sqrt{ax-1}\sqrt{ax+1}}{2x} - \frac{\cosh^{-1}(ax)}{2x^2}$$

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*x) - ArcCosh[a*x]/(2*x^2)
```

Rubi [A] time = 0.0134166, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5662, 95}

$$\frac{a\sqrt{ax-1}\sqrt{ax+1}}{2x} - \frac{\cosh^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/x^3,x]
```

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*x) - ArcCosh[a*x]/(2*x^2)
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{x^3}\,dx = -\frac{\cosh^{-1}(ax)}{2x^2} + \frac{1}{2}a\int \frac{1}{x^2\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$
$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{2x} - \frac{\cosh^{-1}(ax)}{2x^2}$$

Mathematica [A] time = 0.007299, size = 35, normalized size = 0.92

$$\frac{ax\sqrt{ax-1}\sqrt{ax+1} - \cosh^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/x^3,x]
```

```
[Out] (a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x] - ArcCosh[a*x])/(2*x^2)
```

Maple [A] time = 0.012, size = 40, normalized size = 1.1

$$a^2\left(-\frac{\operatorname{arccosh}(ax)}{2\,a^2x^2} + \frac{1}{2\,ax}\sqrt{ax-1}\sqrt{ax+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/x^3,x)
```

```
[Out] a^2*(-1/2*arccosh(a*x)/a^2/x^2+1/2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/x)
```

Maxima [A] time = 1.72844, size = 36, normalized size = 0.95

$$\frac{\sqrt{a^2x^2-1}\,a}{2\,x} - \frac{\operatorname{arcosh}(ax)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a^2*x^2 - 1)*a/x - 1/2*arccosh(a*x)/x^2
```

Fricas [A] time = 2.4195, size = 86, normalized size = 2.26

$$\frac{\sqrt{a^2x^2-1}ax - \log\left(ax + \sqrt{a^2x^2-1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2*x^2 - 1)*a*x - log(a*x + sqrt(a^2*x^2 - 1)))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/x**3,x)
```

```
[Out] Integral(acosh(a*x)/x**3, x)
```

Giac [A] time = 1.20903, size = 68, normalized size = 1.79

$$\frac{a|a|}{\left(x|a| - \sqrt{a^2x^2-1}\right)^2 + 1} - \frac{\log\left(ax + \sqrt{a^2x^2-1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^3,x, algorithm="giac")
```

```
[Out] a*abs(a)/((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1) - 1/2*log(a*x + sqrt(a^2*x^
2 - 1))/x^2
```

3.9 $\int \frac{\cosh^{-1}(ax)}{x^4} \, dx$

Optimal. Leaf size=65

$$\frac{1}{6}a^3\tan^{-1}\left(\sqrt{ax-1}\sqrt{ax+1}\right)+\frac{a\sqrt{ax-1}\sqrt{ax+1}}{6x^2}-\frac{\cosh^{-1}(ax)}{3x^3}$$

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(6*x^2) - ArcCosh[a*x]/(3*x^3) + (a^3*ArcT
an[Sqrt[-1 + a*x]*Sqrt[1 + a*x]])/6
```

Rubi [A] time = 0.027259, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5662, 103, 12, 92, 205}

$$\frac{1}{6}a^3\tan^{-1}\left(\sqrt{ax-1}\sqrt{ax+1}\right)+\frac{a\sqrt{ax-1}\sqrt{ax+1}}{6x^2}-\frac{\cosh^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/x^4,x]
```

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(6*x^2) - ArcCosh[a*x]/(3*x^3) + (a^3*ArcT
an[Sqrt[-1 + a*x]*Sqrt[1 + a*x]])/6
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}(ax)}{x^4}\,dx &= -\frac{\cosh^{-1}(ax)}{3x^3}+\frac{1}{3}a\int \frac{1}{x^3\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{6x^2}-\frac{\cosh^{-1}(ax)}{3x^3}+\frac{1}{6}a\int \frac{a^2}{x\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{6x^2}-\frac{\cosh^{-1}(ax)}{3x^3}+\frac{1}{6}a^3\int \frac{1}{x\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{6x^2}-\frac{\cosh^{-1}(ax)}{3x^3}+\frac{1}{6}a^4\,\mathrm{Subst}\left(\int \frac{1}{a+ax^2}\,dx,x,\sqrt{-1+ax}\sqrt{1+ax}\right)\\
&= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{6x^2}-\frac{\cosh^{-1}(ax)}{3x^3}+\frac{1}{6}a^3\tan^{-1}\left(\sqrt{-1+ax}\sqrt{1+ax}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0752107, size = 78, normalized size = 1.2

$$
\frac{\frac{ax\left(a^2x^2+a^2x^2\sqrt{a^2x^2-1}\tan^{-1}\left(\sqrt{a^2x^2-1}\right)-1\right)}{\sqrt{ax-1}\sqrt{ax+1}}-2\cosh^{-1}(ax)}{6x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/x^4,x]
```

```
[Out] (-2*ArcCosh[a*x] + (a*x*(-1 + a^2*x^2 + a^2*x^2*Sqrt[-1 + a^2*x^2]*ArcTan[S
qrt[-1 + a^2*x^2]]))/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]))/(6*x^3)
```

Maple [A] time = 0.013, size = 73, normalized size = 1.1

$$-\frac{\operatorname{arccosh}(ax)}{3x^3} - \frac{a^3}{6}\sqrt{ax-1}\sqrt{ax+1}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\frac{1}{\sqrt{a^2x^2-1}} + \frac{a}{6x^2}\sqrt{ax-1}\sqrt{ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/x^4,x)
```

```
[Out] -1/3*arccosh(a*x)/x^3-1/6*a^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)^(1/2)
*arctan(1/(a^2*x^2-1)^(1/2))+1/6*a*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x^2
```

Maxima [A] time = 1.82707, size = 61, normalized size = 0.94

$$-\frac{1}{6}\left(a^2\arcsin\left(\frac{1}{\sqrt{a^2}|x|}\right) - \frac{\sqrt{a^2x^2-1}}{x^2}\right)a - \frac{\operatorname{arcosh}(ax)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(a^2*arcsin(1/(sqrt(a^2)*abs(x))) - sqrt(a^2*x^2 - 1)/x^2)*a - 1/3*arc
cosh(a*x)/x^3
```

Fricas [A] time = 2.45052, size = 215, normalized size = 3.31

$$\frac{2a^3x^3\arctan\left(-ax+\sqrt{a^2x^2-1}\right) + 2x^3\log\left(-ax+\sqrt{a^2x^2-1}\right) + \sqrt{a^2x^2-1}ax + 2\left(x^3-1\right)\log\left(ax+\sqrt{a^2x^2-1}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*x^3*arctan(-a*x + sqrt(a^2*x^2 - 1)) + 2*x^3*log(-a*x + sqrt(a^2
*x^2 - 1)) + sqrt(a^2*x^2 - 1)*a*x + 2*(x^3 - 1)*log(a*x + sqrt(a^2*x^2 - 1
)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/x**4,x)
```

```
[Out] Integral(acosh(a*x)/x**4, x)
```

Giac [A] time = 1.33463, size = 78, normalized size = 1.2

$$\frac{1}{6}a^3\left(\frac{\sqrt{a^2x^2-1}}{a^2x^2}+\arctan\left(\sqrt{a^2x^2-1}\right)\right)-\frac{\log\left(ax+\sqrt{a^2x^2-1}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*a^3*(sqrt(a^2*x^2 - 1)/(a^2*x^2) + arctan(sqrt(a^2*x^2 - 1))) - 1/3*log
(a*x + sqrt(a^2*x^2 - 1))/x^3
```

3.10 $\int \frac{\cosh^{-1}(ax)}{x^5}\,dx$

Optimal. Leaf size=66

$$\frac{a^3\sqrt{ax-1}\sqrt{ax+1}}{6x}+\frac{a\sqrt{ax-1}\sqrt{ax+1}}{12x^3}-\frac{\cosh^{-1}(ax)}{4x^4}$$

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(12*x^3) + (a^3*Sqrt[-1 + a*x]*Sqrt[1 + a*
x])/(6*x) - ArcCosh[a*x]/(4*x^4)
```

Rubi [A] time = 0.0244926, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5662, 103, 12, 95}

$$\frac{a^3\sqrt{ax-1}\sqrt{ax+1}}{6x}+\frac{a\sqrt{ax-1}\sqrt{ax+1}}{12x^3}-\frac{\cosh^{-1}(ax)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/x^5,x]
```

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(12*x^3) + (a^3*Sqrt[-1 + a*x]*Sqrt[1 + a*
x])/(6*x) - ArcCosh[a*x]/(4*x^4)
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)}{x^5}\,dx &= -\frac{\cosh^{-1}(ax)}{4x^4}+\frac{1}{4}a\int \frac{1}{x^4\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{12x^3}-\frac{\cosh^{-1}(ax)}{4x^4}+\frac{1}{12}a\int \frac{2a^2}{x^2\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{12x^3}-\frac{\cosh^{-1}(ax)}{4x^4}+\frac{1}{6}a^3\int \frac{1}{x^2\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{12x^3}+\frac{a^3\sqrt{-1+ax}\sqrt{1+ax}}{6x}-\frac{\cosh^{-1}(ax)}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.0229603, size = 45, normalized size = 0.68

$$\frac{ax\sqrt{ax-1}\sqrt{ax+1}\left(2a^2x^2+1\right)-3\cosh^{-1}(ax)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/x^5,x]
```

```
[Out] (a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(1 + 2*a^2*x^2) - 3*ArcCosh[a*x])/(12*x^4
)
```

Maple [A] time = 0.012, size = 50, normalized size = 0.8

$$a^4\left(-\frac{\operatorname{arccosh}(ax)}{4\,x^4a^4}+\frac{2\,a^2x^2+1}{12\,x^3a^3}\sqrt{ax-1}\sqrt{ax+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/x^5,x)
```

```
[Out] a^4*(-1/4*arccosh(a*x)/a^4/x^4+1/12*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(2*a^2*x^2+
1)/a^3/x^3)
```

Maxima [A] time = 1.77233, size = 65, normalized size = 0.98

$$\frac{1}{12}\left(\frac{2\sqrt{a^2x^2-1}a^2}{x}+\frac{\sqrt{a^2x^2-1}}{x^3}\right)a-\frac{\operatorname{arcosh}(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*(2*sqrt(a^2*x^2 - 1)*a^2/x + sqrt(a^2*x^2 - 1)/x^3)*a - 1/4*arccosh(a*
x)/x^4
```

Fricas [A] time = 2.46552, size = 109, normalized size = 1.65

$$\frac{\left(2a^3x^3+ax\right)\sqrt{a^2x^2-1}-3\log\left(ax+\sqrt{a^2x^2-1}\right)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*((2*a^3*x^3 + a*x)*sqrt(a^2*x^2 - 1) - 3*log(a*x + sqrt(a^2*x^2 - 1)))
/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acosh}(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/x**5,x)
```

```
[Out] Integral(acosh(a*x)/x**5, x)
```

Giac [A] time = 1.34505, size = 104, normalized size = 1.58

$$\frac{\left(3\left(x|a| - \sqrt{a^2x^2 - 1}\right)^2 + 1\right)a^3|a|}{3\left(\left(x|a| - \sqrt{a^2x^2 - 1}\right)^2 + 1\right)^3} - \frac{\log\left(ax + \sqrt{a^2x^2 - 1}\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^5,x, algorithm="giac")
```

```
[Out] 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*a^3*abs(a)/((x*abs(a) - sqrt(a
^2*x^2 - 1))^2 + 1)^3 - 1/4*log(a*x + sqrt(a^2*x^2 - 1))/x^4
```

3.11 $\int \frac{\cosh^{-1}(ax)}{x^6} \, dx$

Optimal. Leaf size=93

$$\frac{3a^3\sqrt{ax-1}\sqrt{ax+1}}{40x^2}+\frac{3}{40}a^5\tan^{-1}\left(\sqrt{ax-1}\sqrt{ax+1}\right)+\frac{a\sqrt{ax-1}\sqrt{ax+1}}{20x^4}-\frac{\cosh^{-1}(ax)}{5x^5}$$

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(20*x^4) + (3*a^3*Sqrt[-1 + a*x]*Sqrt[1 +
a*x])/(40*x^2) - ArcCosh[a*x]/(5*x^5) + (3*a^5*ArcTan[Sqrt[-1 + a*x]*Sqrt[1
 + a*x]])/40
```

Rubi [A] time = 0.0407888, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5662, 103, 12, 92, 205}

$$\frac{3a^3\sqrt{ax-1}\sqrt{ax+1}}{40x^2}+\frac{3}{40}a^5\tan^{-1}\left(\sqrt{ax-1}\sqrt{ax+1}\right)+\frac{a\sqrt{ax-1}\sqrt{ax+1}}{20x^4}-\frac{\cosh^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/x^6,x]
```

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(20*x^4) + (3*a^3*Sqrt[-1 + a*x]*Sqrt[1 +
a*x])/(40*x^2) - ArcCosh[a*x]/(5*x^5) + (3*a^5*ArcTan[Sqrt[-1 + a*x]*Sqrt[1
 + a*x]])/40
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}(ax)}{x^6}\,dx &= -\frac{\cosh^{-1}(ax)}{5x^5}+\frac{1}{5}a\int \frac{1}{x^5\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{20x^4}-\frac{\cosh^{-1}(ax)}{5x^5}+\frac{1}{20}a\int \frac{3a^2}{x^3\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{20x^4}-\frac{\cosh^{-1}(ax)}{5x^5}+\frac{1}{20}\left(3a^3\right)\int \frac{1}{x^3\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{20x^4}+\frac{3a^3\sqrt{-1+ax}\sqrt{1+ax}}{40x^2}-\frac{\cosh^{-1}(ax)}{5x^5}+\frac{1}{40}\left(3a^3\right)\int \frac{a^2}{x\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{20x^4}+\frac{3a^3\sqrt{-1+ax}\sqrt{1+ax}}{40x^2}-\frac{\cosh^{-1}(ax)}{5x^5}+\frac{1}{40}\left(3a^5\right)\int \frac{1}{x\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{20x^4}+\frac{3a^3\sqrt{-1+ax}\sqrt{1+ax}}{40x^2}-\frac{\cosh^{-1}(ax)}{5x^5}+\frac{1}{40}\left(3a^6\right)\text{Subst}\left(\int \frac{1}{a+ax^2}\,dx,x,\sqrt{-\right.\\
&= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{20x^4}+\frac{3a^3\sqrt{-1+ax}\sqrt{1+ax}}{40x^2}-\frac{\cosh^{-1}(ax)}{5x^5}+\frac{3}{40}a^5\tan^{-1}\left(\sqrt{-1+ax}\sqrt{1+ax}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0436838, size = 104, normalized size = 1.12

$$-\frac{-3a^5x^5+a^3x^3-3a^5x^5\sqrt{a^2x^2-1}\tan^{-1}\left(\sqrt{a^2x^2-1}\right)+2ax+8\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{40x^5\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/x^6,x]
```

```
[Out] -(2*a*x + a^3*x^3 - 3*a^5*x^5 + 8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]
 - 3*a^5*x^5*Sqrt[-1 + a^2*x^2]*ArcTan[Sqrt[-1 + a^2*x^2]])/(40*x^5*Sqrt[-1
 + a*x]*Sqrt[1 + a*x])
```

Maple [A] time = 0.016, size = 95, normalized size = 1.

$$-\frac{\operatorname{arccosh}(ax)}{5x^5}-\frac{3a^5}{40}\sqrt{ax-1}\sqrt{ax+1}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\frac{1}{\sqrt{a^2x^2-1}}+\frac{3a^3}{40x^2}\sqrt{ax-1}\sqrt{ax+1}+\frac{a}{20x^4}\sqrt{ax-1}\sqrt{ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/x^6,x)
```

```
[Out] -1/5*arccosh(a*x)/x^5-3/40*a^5*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)^(1/2
)*arctan(1/(a^2*x^2-1)^(1/2))+3/40*a^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x^2+1/20
*a*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x^4
```

Maxima [A] time = 1.76618, size = 88, normalized size = 0.95

$$-\frac{1}{40}\left(3a^4\arcsin\left(\frac{1}{\sqrt{a^2}|x|}\right)-\frac{3\sqrt{a^2x^2-1}a^2}{x^2}-\frac{2\sqrt{a^2x^2-1}}{x^4}\right)a-\frac{\operatorname{arcosh}(ax)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/40*(3*a^4*arcsin(1/(sqrt(a^2)*abs(x))) - 3*sqrt(a^2*x^2 - 1)*a^2/x^2 - 2
*sqrt(a^2*x^2 - 1)/x^4)*a - 1/5*arccosh(a*x)/x^5
```

Fricas [A] time = 2.51877, size = 238, normalized size = 2.56

$$\frac{6\,a^5x^5\arctan\left(-ax+\sqrt{a^2x^2-1}\right)+8\,x^5\log\left(-ax+\sqrt{a^2x^2-1}\right)+8\left(x^5-1\right)\log\left(ax+\sqrt{a^2x^2-1}\right)+\left(3\,a^3x^3+2\,ax\right)\sqrt{a^2}}{40\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/40*(6*a^5*x^5*arctan(-a*x + sqrt(a^2*x^2 - 1)) + 8*x^5*log(-a*x + sqrt(a^
2*x^2 - 1)) + 8*(x^5 - 1)*log(a*x + sqrt(a^2*x^2 - 1)) + (3*a^3*x^3 + 2*a*x
)*sqrt(a^2*x^2 - 1))/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acosh}(ax)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/x**6,x)
```

```
[Out] Integral(acosh(a*x)/x**6, x)
```

Giac [A] time = 1.3486, size = 103, normalized size = 1.11

$$\frac{1}{40}\,a^5\left(\frac{3\left(a^2x^2-1\right)^{\frac{3}{2}}+5\,\sqrt{a^2x^2-1}}{a^4x^4}+3\,\arctan\left(\sqrt{a^2x^2-1}\right)\right)-\frac{\log\left(ax+\sqrt{a^2x^2-1}\right)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^6,x, algorithm="giac")
```

```
[Out] 1/40*a^5*((3*(a^2*x^2 - 1)^(3/2) + 5*sqrt(a^2*x^2 - 1))/(a^4*x^4) + 3*arcta
n(sqrt(a^2*x^2 - 1))) - 1/5*log(a*x + sqrt(a^2*x^2 - 1))/x^5
```

3.12 $\int x^4 \cosh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=132

$$\frac{8x^3}{225a^2} - \frac{8x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{75a^3} + \frac{16x}{75a^4} - \frac{16\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{75a^5} + \frac{1}{5}x^5\cosh^{-1}(ax)^2 - \frac{2x^4\sqrt{ax-1}\sqrt{ax}}{25}$$

```
[Out] (16*x)/(75*a^4) + (8*x^3)/(225*a^2) + (2*x^5)/125 - (16*Sqrt[-1 + a*x]*Sqrt
[1 + a*x]*ArcCosh[a*x])/(75*a^5) - (8*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcC
osh[a*x])/(75*a^3) - (2*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(25*
a) + (x^5*ArcCosh[a*x]^2)/5
```

Rubi [A] time = 0.489759, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5662, 5759, 5718, 8, 30}

$$\frac{8x^3}{225a^2} - \frac{8x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{75a^3} + \frac{16x}{75a^4} - \frac{16\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{75a^5} + \frac{1}{5}x^5\cosh^{-1}(ax)^2 - \frac{2x^4\sqrt{ax-1}\sqrt{ax}}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCosh[a*x]^2,x]
```

```
[Out] (16*x)/(75*a^4) + (8*x^3)/(225*a^2) + (2*x^5)/125 - (16*Sqrt[-1 + a*x]*Sqrt
[1 + a*x]*ArcCosh[a*x])/(75*a^5) - (8*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcC
osh[a*x])/(75*a^3) - (2*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(25*
a) + (x^5*ArcCosh[a*x]^2)/5
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
```

```
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^4 \cosh^{-1}(ax)^2\,dx = \frac{1}{5}x^5\cosh^{-1}(ax)^2 - \frac{1}{5}(2a)\int \frac{x^5\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{25a} + \frac{1}{5}x^5\cosh^{-1}(ax)^2 + \frac{2\int x^4\,dx}{25} - \frac{8\int \frac{x^3\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{25a}$$

$$= \frac{2x^5}{125} - \frac{8x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{75a^3} - \frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{25a} + \frac{1}{5}x^5\cosh^{-1}(ax)^2$$

$$= \frac{8x^3}{225a^2} + \frac{2x^5}{125} - \frac{16\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{75a^5} - \frac{8x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{75a^3} - \frac{2x^4\sqrt{-1}}{}$$

$$= \frac{16x}{75a^4} + \frac{8x^3}{225a^2} + \frac{2x^5}{125} - \frac{16\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{75a^5} - \frac{8x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{75a^3} -$$

Mathematica [A] time = 0.106839, size = 80, normalized size = 0.61

$$\frac{\frac{40x^3}{a^2} - \frac{30\sqrt{ax-1}\sqrt{ax+1}\left(3a^4x^4+4a^2x^2+8\right)\cosh^{-1}(ax)}{a^5} + \frac{240x}{a^4} + 225x^5\cosh^{-1}(ax)^2 + 18x^5}{1125}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCosh[a*x]^2,x]
```

```
[Out] ((240*x)/a^4 + (40*x^3)/a^2 + 18*x^5 - (30*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(8
+ 4*a^2*x^2 + 3*a^4*x^4)*ArcCosh[a*x])/a^5 + 225*x^5*ArcCosh[a*x]^2)/1125
```

Maple [A] time = 0.04, size = 168, normalized size = 1.3

$$\frac{1}{a^5}\left(\frac{(\operatorname{arccosh}(ax))^2 a^3x^3 (ax-1)(ax+1)}{5} + \frac{(\operatorname{arccosh}(ax))^2 (ax-1)(ax+1)\,ax}{5} + \frac{(\operatorname{arccosh}(ax))^2 ax}{5} - \frac{2\operatorname{arccosh}(ax)\,a}{25}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)^2,x)
```

```
[Out] 1/a^5*(1/5*arccosh(a*x)^2*a^3*x^3*(a*x-1)*(a*x+1)+1/5*arccosh(a*x)^2*(a*x-1
)*(a*x+1)*a*x+1/5*arccosh(a*x)^2*a*x-2/25*arccosh(a*x)*a^4*x^4*(a*x-1)^(1/2
)*(a*x+1)^(1/2)-16/75*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)-8/75*arccosh
(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^2*x^2+2/125*(a*x-1)*(a*x+1)*a^3*x^3+58/
1125*(a*x-1)*(a*x+1)*a*x+298/1125*a*x)
```

Maxima [A] time = 1.28225, size = 134, normalized size = 1.02

$$\frac{1}{5}x^5\operatorname{arcosh}(ax)^2 - \frac{2}{75}\left(\frac{3\sqrt{a^2x^2-1}x^4}{a^2} + \frac{4\sqrt{a^2x^2-1}x^2}{a^4} + \frac{8\sqrt{a^2x^2-1}}{a^6}\right)a\operatorname{arcosh}(ax) + \frac{2\left(9a^4x^5 + 20a^2x^3 + 120x\right)}{1125a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccosh(a*x)^2 - 2/75*(3*sqrt(a^2*x^2 - 1)*x^4/a^2 + 4*sqrt(a^2*x^2
 - 1)*x^2/a^4 + 8*sqrt(a^2*x^2 - 1)/a^6)*a*arccosh(a*x) + 2/1125*(9*a^4*x^5
```

```
 + 20*a^2*x^3 + 120*x)/a^4
```

Fricas [A] time = 2.53045, size = 234, normalized size = 1.77

$$\frac{225\,a^5x^5\log\left(ax+\sqrt{a^2x^2-1}\right)^2+18\,a^5x^5+40\,a^3x^3-30\left(3\,a^4x^4+4\,a^2x^2+8\right)\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)+240\,ax}{1125\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/1125*(225*a^5*x^5*log(a*x + sqrt(a^2*x^2 - 1))^2 + 18*a^5*x^5 + 40*a^3*x^
3 - 30*(3*a^4*x^4 + 4*a^2*x^2 + 8)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2
 - 1)) + 240*a*x)/a^5
```

Sympy [A] time = 4.91408, size = 122, normalized size = 0.92

$$\begin{cases} \frac{x^5\operatorname{acosh}^2(ax)}{5}+\frac{2x^5}{125}-\frac{2x^4\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{25a}+\frac{8x^3}{225a^2}-\frac{8x^2\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{75a^3}+\frac{16x}{75a^4}-\frac{16\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{75a^5} & \text{for } a\neq 0 \\ -\frac{\pi^2x^5}{20} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)**2,x)
```

```
[Out] Piecewise((x**5*acosh(a*x)**2/5 + 2*x**5/125 - 2*x**4*sqrt(a**2*x**2 - 1)*a
cosh(a*x)/(25*a) + 8*x**3/(225*a**2) - 8*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x
)/(75*a**3) + 16*x/(75*a**4) - 16*sqrt(a**2*x**2 - 1)*acosh(a*x)/(75*a**5),
 Ne(a, 0)), (-pi**2*x**5/20, True))
```

Giac [A] time = 1.33187, size = 153, normalized size = 1.16

$$\frac{1}{5}\,x^5\log\left(ax+\sqrt{a^2x^2-1}\right)^2+\frac{2}{1125}\,a\left(\frac{9\,a^4x^5+20\,a^2x^3+120\,x}{a^5}-\frac{15\left(3\left(a^2x^2-1\right)^{\frac{5}{2}}+10\left(a^2x^2-1\right)^{\frac{3}{2}}+15\sqrt{a^2x^2-1}\right)\log}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/5*x^5*log(a*x + sqrt(a^2*x^2 - 1))^2 + 2/1125*a*((9*a^4*x^5 + 20*a^2*x^3
+ 120*x)/a^5 - 15*(3*(a^2*x^2 - 1)^(5/2) + 10*(a^2*x^2 - 1)^(3/2) + 15*sqrt
(a^2*x^2 - 1))*log(a*x + sqrt(a^2*x^2 - 1))/a^6)
```

3.13 $\int x^3 \cosh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=106

$$\frac{3x^2}{32a^2} - \frac{3x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{16a^3} - \frac{3\cosh^{-1}(ax)^2}{32a^4} + \frac{1}{4}x^4\cosh^{-1}(ax)^2 - \frac{x^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{8a} + \frac{x^4}{32}$$

```
[Out] (3*x^2)/(32*a^2) + x^4/32 - (3*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])
/(16*a^3) - (x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(8*a) - (3*ArcC
osh[a*x]^2)/(32*a^4) + (x^4*ArcCosh[a*x]^2)/4
```

Rubi [A] time = 0.440192, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5662, 5759, 5676, 30}

$$\frac{3x^2}{32a^2} - \frac{3x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{16a^3} - \frac{3\cosh^{-1}(ax)^2}{32a^4} + \frac{1}{4}x^4\cosh^{-1}(ax)^2 - \frac{x^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{8a} + \frac{x^4}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCosh[a*x]^2,x]
```

```
[Out] (3*x^2)/(32*a^2) + x^4/32 - (3*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])
/(16*a^3) - (x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(8*a) - (3*ArcC
osh[a*x]^2)/(32*a^4) + (x^4*ArcCosh[a*x]^2)/4
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
```

```
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3\cosh^{-1}(ax)^2\,dx = \frac{1}{4}x^4\cosh^{-1}(ax)^2 - \frac{1}{2}a\int\frac{x^4\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{8a} + \frac{1}{4}x^4\cosh^{-1}(ax)^2 + \frac{\int x^3\,dx}{8} - \frac{3\int\frac{x^2\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{8a}$$

$$= \frac{x^4}{32} - \frac{3x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{16a^3} - \frac{x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{8a} + \frac{1}{4}x^4\cosh^{-1}(ax)^2 - \frac{3}{\ldots}$$

$$= \frac{3x^2}{32a^2} + \frac{x^4}{32} - \frac{3x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{16a^3} - \frac{x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{8a} - \frac{3\cosh^{-1}(ax)^2}{32a^4}\ldots$$

Mathematica [A] time = 0.0715097, size = 77, normalized size = 0.73

$$\frac{a^2x^2\left(a^2x^2+3\right) - 2ax\sqrt{ax-1}\sqrt{ax+1}\left(2a^2x^2+3\right)\cosh^{-1}(ax) + \left(8a^4x^4-3\right)\cosh^{-1}(ax)^2}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCosh[a*x]^2,x]
```

```
[Out] (a^2*x^2*(3 + a^2*x^2) - 2*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(3 + 2*a^2*x^2)
*ArcCosh[a*x] + (-3 + 8*a^4*x^4)*ArcCosh[a*x]^2)/(32*a^4)
```

Maple [A] time = 0.035, size = 126, normalized size = 1.2

$$\frac{1}{a^4}\left(\frac{(\operatorname{arccosh}(ax))^2(ax-1)(ax+1)a^2x^2}{4}+\frac{(\operatorname{arccosh}(ax))^2a^2x^2}{4}-\frac{a^3x^3\operatorname{arccosh}(ax)}{8}\sqrt{ax-1}\sqrt{ax+1}-\frac{3\,ax\operatorname{arccosh}(}{16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)^2,x)
```

```
[Out] 1/a^4*(1/4*arccosh(a*x)^2*(a*x-1)*(a*x+1)*a^2*x^2+1/4*arccosh(a*x)^2*a^2*x^
2-1/8*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^3*x^3-3/16*arccosh(a*x)*a*
x*(a*x-1)^(1/2)*(a*x+1)^(1/2)-3/32*arccosh(a*x)^2+1/32*(a*x-1)*(a*x+1)*a^2*
x^2+1/8*a^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}x^4\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^2-\int\frac{\left(a^3x^6+\sqrt{ax+1}\sqrt{ax-1}a^2x^5-ax^4\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)}{2\left(a^3x^3+\left(a^2x^2-1\right)\sqrt{ax+1}\sqrt{ax-1}-ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2 - integrate(1/2*(a^3*x^6 +
 sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x^5 - a*x^4)*log(a*x + sqrt(a*x + 1)*sqrt(
a*x - 1))/(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x), x)
```

Fricas [A] time = 2.36711, size = 205, normalized size = 1.93

$$\frac{a^4x^4+3\,a^2x^2+\left(8\,a^4x^4-3\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^2-2\left(2\,a^3x^3+3\,ax\right)\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)}{32\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(a^4*x^4 + 3*a^2*x^2 + (8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 - 1))^2
- 2*(2*a^3*x^3 + 3*a*x)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1)))/a^4
```

Sympy [A] time = 2.87612, size = 99, normalized size = 0.93

$$\begin{cases} \frac{x^4 \operatorname{acosh}^2(ax)}{4} + \frac{x^4}{32} - \frac{x^3\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{8a} + \frac{3x^2}{32a^2} - \frac{3x\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{16a^3} - \frac{3\operatorname{acosh}^2(ax)}{32a^4} & \text{for } a \neq 0 \\ -\frac{\pi^2 x^4}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**2,x)
```

```
[Out] Piecewise((x**4*acosh(a*x)**2/4 + x**4/32 - x**3*sqrt(a**2*x**2 - 1)*acosh(
a*x)/(8*a) + 3*x**2/(32*a**2) - 3*x*sqrt(a**2*x**2 - 1)*acosh(a*x)/(16*a**3
) - 3*acosh(a*x)**2/(32*a**4), Ne(a, 0)), (-pi**2*x**4/16, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arcosh}(ax)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccosh(a*x)^2, x)
```

3.14 $\int x^2 \cosh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=90

$$\frac{4x}{9a^2} - \frac{4\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{9a^3} + \frac{1}{3}x^3\cosh^{-1}(ax)^2 - \frac{2x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{9a} + \frac{2x^3}{27}$$

```
[Out] (4*x)/(9*a^2) + (2*x^3)/27 - (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/
(9*a^3) - (2*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(9*a) + (x^3*Ar
cCosh[a*x]^2)/3
```

Rubi [A] time = 0.311197, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5662, 5759, 5718, 8, 30}

$$\frac{4x}{9a^2} - \frac{4\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{9a^3} + \frac{1}{3}x^3\cosh^{-1}(ax)^2 - \frac{2x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{9a} + \frac{2x^3}{27}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCosh[a*x]^2,x]
```

```
[Out] (4*x)/(9*a^2) + (2*x^3)/27 - (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/
(9*a^3) - (2*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(9*a) + (x^3*Ar
cCosh[a*x]^2)/3
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
```

```
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\cosh^{-1}(ax)^2\,dx = \frac{1}{3}x^3\cosh^{-1}(ax)^2 - \frac{1}{3}(2a)\int \frac{x^3\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{9a} + \frac{1}{3}x^3\cosh^{-1}(ax)^2 + \frac{2\int x^2\,dx}{9} - \frac{4\int \frac{x\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{9a}$$

$$= \frac{2x^3}{27} - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{9a^3} - \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{9a} + \frac{1}{3}x^3\cosh^{-1}(ax)^2 + \frac{4}{\cdots}$$

$$= \frac{4x}{9a^2} + \frac{2x^3}{27} - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{9a^3} - \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{9a} + \frac{1}{3}x^3\cosh^{-1}(a\cdots$$

Mathematica [A] time = 0.0950779, size = 64, normalized size = 0.71

$$\frac{1}{27}\left(2x\left(\frac{6}{a^2}+x^2\right) - \frac{6\sqrt{ax-1}\sqrt{ax+1}\left(a^2x^2+2\right)\cosh^{-1}(ax)}{a^3} + 9x^3\cosh^{-1}(ax)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCosh[a*x]^2,x]
```

```
[Out] (2*x*(6/a^2 + x^2) - (6*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(2 + a^2*x^2)*ArcCosh[
a*x])/a^3 + 9*x^3*ArcCosh[a*x]^2)/27
```

Maple [A] time = 0.031, size = 100, normalized size = 1.1

$$\frac{1}{a^3}\left(\frac{(\mathrm{arccosh}\,(ax))^2\,(ax-1)\,(ax+1)\,ax}{3}+\frac{(\mathrm{arccosh}\,(ax))^2\,ax}{3}-\frac{2\,\mathrm{arccosh}\,(ax)\,a^2x^2}{9}\sqrt{ax-1}\sqrt{ax+1}-\frac{4\,\mathrm{arccosh}\,(ax)}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)^2,x)
```

```
[Out] 1/a^3*(1/3*arccosh(a*x)^2*(a*x-1)*(a*x+1)*a*x+1/3*arccosh(a*x)^2*a*x-2/9*ar
ccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^2*x^2-4/9*arccosh(a*x)*(a*x-1)^(1/
2)*(a*x+1)^(1/2)+2/27*(a*x-1)*(a*x+1)*a*x+14/27*a*x)
```

Maxima [A] time = 1.1807, size = 95, normalized size = 1.06

$$\frac{1}{3}\,x^3\,\mathrm{arcosh}\,(ax)^2-\frac{2}{9}\,a\left(\frac{\sqrt{a^2x^2-1}x^2}{a^2}+\frac{2\,\sqrt{a^2x^2-1}}{a^4}\right)\mathrm{arcosh}\,(ax)+\frac{2\left(a^2x^3+6\,x\right)}{27\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccosh(a*x)^2 - 2/9*a*(sqrt(a^2*x^2 - 1)*x^2/a^2 + 2*sqrt(a^2*x^2
- 1)/a^4)*arccosh(a*x) + 2/27*(a^2*x^3 + 6*x)/a^2
```

Fricas [A] time = 2.25661, size = 188, normalized size = 2.09

$$\frac{9\,a^3x^3\log\left(ax+\sqrt{a^2x^2-1}\right)^2+2\,a^3x^3-6\left(a^2x^2+2\right)\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)+12\,ax}{27\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*a^3*x^3*log(a*x + sqrt(a^2*x^2 - 1))^2 + 2*a^3*x^3 - 6*(a^2*x^2 + 2
)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1)) + 12*a*x)/a^3
```

Sympy [A] time = 1.2952, size = 85, normalized size = 0.94

$$\begin{cases} \frac{x^3 \operatorname{acosh}^2(ax)}{3} + \frac{2x^3}{27} - \frac{2x^2\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{9a} + \frac{4x}{9a^2} - \frac{4\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{9a^3} & \text{for } a \neq 0 \\ -\frac{\pi^2 x^3}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)**2,x)
```

```
[Out] Piecewise((x**3*acosh(a*x)**2/3 + 2*x**3/27 - 2*x**2*sqrt(a**2*x**2 - 1)*ac
osh(a*x)/(9*a) + 4*x/(9*a**2) - 4*sqrt(a**2*x**2 - 1)*acosh(a*x)/(9*a**3),
Ne(a, 0)), (-pi**2*x**3/12, True))
```

Giac [A] time = 1.32305, size = 120, normalized size = 1.33

$$\frac{1}{3}x^3\log\left(ax + \sqrt{a^2x^2-1}\right)^2 + \frac{2}{27}a\left(\frac{a^2x^3+6x}{a^3} - \frac{3\left(\left(a^2x^2-1\right)^{\frac{3}{2}} + 3\sqrt{a^2x^2-1}\right)\log\left(ax+\sqrt{a^2x^2-1}\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(a*x + sqrt(a^2*x^2 - 1))^2 + 2/27*a*((a^2*x^3 + 6*x)/a^3 - 3*((
a^2*x^2 - 1)^(3/2) + 3*sqrt(a^2*x^2 - 1))*log(a*x + sqrt(a^2*x^2 - 1))/a^4)
```

3.15 $\int x \cosh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=64

$$-\frac{\cosh^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^2-\frac{x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{2a}+\frac{x^2}{4}$$

```
[Out] x^2/4 - (x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(2*a) - ArcCosh[a*x]^
2/(4*a^2) + (x^2*ArcCosh[a*x]^2)/2
```

Rubi [A] time = 0.250665, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5662, 5759, 5676, 30}

$$-\frac{\cosh^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^2-\frac{x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{2a}+\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCosh[a*x]^2,x]
```

```
[Out] x^2/4 - (x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(2*a) - ArcCosh[a*x]^
2/(4*a^2) + (x^2*ArcCosh[a*x]^2)/2
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
```

```
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\cosh^{-1}(ax)^2\,dx &= \frac{1}{2}x^2\cosh^{-1}(ax)^2 - a\int\frac{x^2\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\ &= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{2a} + \frac{1}{2}x^2\cosh^{-1}(ax)^2 + \frac{\int x\,dx}{2} - \frac{\int\frac{\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{2a}\\ &= \frac{x^2}{4} - \frac{x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{2a} - \frac{\cosh^{-1}(ax)^2}{4a^2} + \frac{1}{2}x^2\cosh^{-1}(ax)^2\end{aligned}$$

Mathematica [A] time = 0.0498287, size = 58, normalized size = 0.91

$$\frac{a^2x^2 + \left(2a^2x^2-1\right)\cosh^{-1}(ax)^2 - 2ax\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCosh[a*x]^2,x]
```

```
[Out] (a^2*x^2 - 2*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x] + (-1 + 2*a^2*x^
2)*ArcCosh[a*x]^2)/(4*a^2)
```

Maple [A] time = 0.029, size = 58, normalized size = 0.9

$$\frac{1}{a^2}\left(\frac{(\operatorname{arccosh}(ax))^2 a^2x^2}{2}-\frac{ax\operatorname{arccosh}(ax)}{2}\sqrt{ax-1}\sqrt{ax+1}-\frac{(\operatorname{arccosh}(ax))^2}{4}+\frac{a^2x^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^2,x)
```

```
[Out] 1/a^2*(1/2*arccosh(a*x)^2*a^2*x^2-1/2*arccosh(a*x)*a*x*(a*x-1)^(1/2)*(a*x+1
)^(1/2)-1/4*arccosh(a*x)^2+1/4*a^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^2-\int\frac{\left(a^3x^4+\sqrt{ax+1}\sqrt{ax-1}a^2x^3-ax^2\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)}{a^3x^3+\left(a^2x^2-1\right)\sqrt{ax+1}\sqrt{ax-1}-ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2 - integrate((a^3*x^4 + sqr
t(a*x + 1)*sqrt(a*x - 1)*a^2*x^3 - a*x^2)*log(a*x + sqrt(a*x + 1)*sqrt(a*x
- 1))/(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x), x)
```

Fricas [A] time = 2.19339, size = 166, normalized size = 2.59

$$\frac{a^2x^2-2\sqrt{a^2x^2-1}ax\log\left(ax+\sqrt{a^2x^2-1}\right)+\left(2a^2x^2-1\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^2}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 - 1)*a*x*log(a*x + sqrt(a^2*x^2 - 1)) + (2*a^
2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^2)/a^2
```

Sympy [A] time = 0.639111, size = 60, normalized size = 0.94

$$\begin{cases} \frac{x^2 \operatorname{acosh}^2(ax)}{2} + \frac{x^2}{4} - \frac{x\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{2a} - \frac{\operatorname{acosh}^2(ax)}{4a^2} & \text{for } a \neq 0 \\ -\frac{\pi^2x^2}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**2,x)
```

```
[Out] Piecewise((x**2*acosh(a*x)**2/2 + x**2/4 - x*sqrt(a**2*x**2 - 1)*acosh(a*x)
/(2*a) - acosh(a*x)**2/(4*a**2), Ne(a, 0)), (-pi**2*x**2/8, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcosh}(ax)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arccosh(a*x)^2, x)
```

3.16 $\int \cosh^{-1}(ax)^2\,dx$

Optimal. Leaf size=39

$$x\cosh^{-1}(ax)^2-\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{a}+2x$$

```
[Out] 2*x - (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/a + x*ArcCosh[a*x]^2
```

Rubi [A] time = 0.127432, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5654, 5718, 8}

$$x\cosh^{-1}(ax)^2-\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{a}+2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2,x]
```

```
[Out] 2*x - (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/a + x*ArcCosh[a*x]^2
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \cosh^{-1}(ax)^2\,dx &= x\cosh^{-1}(ax)^2-(2a)\int \frac{x\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{a}+x\cosh^{-1}(ax)^2+2\int 1\,dx\\
&= 2x-\frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{a}+x\cosh^{-1}(ax)^2
\end{aligned}
$$

Mathematica [A] time = 0.0198588, size = 39, normalized size = 1.

$$x\cosh^{-1}(ax)^2-\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{a}+2x$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^2,x]
```

```
[Out] 2*x - (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/a + x*ArcCosh[a*x]^2
```

Maple [A] time = 0.026, size = 39, normalized size = 1.

$$\frac{1}{a}\left((\operatorname{arccosh}(ax))^2ax-2\operatorname{arccosh}(ax)\sqrt{ax-1}\sqrt{ax+1}+2ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2,x)
```

```
[Out] 1/a*(arccosh(a*x)^2*a*x-2*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+2*a*x)
```

Maxima [A] time = 1.13679, size = 43, normalized size = 1.1

$$x\operatorname{arcosh}(ax)^2+2x-\frac{2\sqrt{a^2x^2-1}\operatorname{arcosh}(ax)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] x*arccosh(a*x)^2 + 2*x - 2*sqrt(a^2*x^2 - 1)*arccosh(a*x)/a
```

Fricas [A] time = 2.32478, size = 134, normalized size = 3.44

$$\frac{ax\log\left(ax+\sqrt{a^2x^2-1}\right)^2+2\,ax-2\,\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] (a*x*log(a*x + sqrt(a^2*x^2 - 1))^2 + 2*a*x - 2*sqrt(a^2*x^2 - 1)*log(a*x +
 sqrt(a^2*x^2 - 1)))/a
```

Sympy [A] time = 0.24437, size = 39, normalized size = 1.

$$\begin{cases} x\,\mathrm{acosh}^2\,(ax)+2x-\frac{2\sqrt{a^2x^2-1}\,\mathrm{acosh}\,(ax)}{a} & \text{for } a\neq 0\\ -\frac{\pi^2x}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2,x)
```

```
[Out] Piecewise((x*acosh(a*x)**2 + 2*x - 2*sqrt(a**2*x**2 - 1)*acosh(a*x)/a, Ne(a
, 0)), (-pi**2*x/4, True))
```

Giac [A] time = 1.35347, size = 84, normalized size = 2.15

$$x\log\left(ax+\sqrt{a^2x^2-1}\right)^2+2\,a\left(\frac{x}{a}-\frac{\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 - 1))^2 + 2*a*(x/a - sqrt(a^2*x^2 - 1)*log(a*x + s
qrt(a^2*x^2 - 1))/a^2)
```

3.17 $\int \frac{\cosh^{-1}(ax)^2}{x}\,dx$

Optimal. Leaf size=62

$$\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)-\frac{1}{2}\text{PolyLog}\left(3,-e^{2\cosh^{-1}(ax)}\right)-\frac{1}{3}\cosh^{-1}(ax)^3+\cosh^{-1}(ax)^2\log\left(e^{2\cosh^{-1}(ax)}+\right.$$

```
[Out] -ArcCosh[a*x]^3/3 + ArcCosh[a*x]^2*Log[1 + E^(2*ArcCosh[a*x])] + ArcCosh[a*
x]*PolyLog[2, -E^(2*ArcCosh[a*x])] - PolyLog[3, -E^(2*ArcCosh[a*x])]/2
```

Rubi [A] time = 0.092509, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5660, 3718, 2190, 2531, 2282, 6589}

$$\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)-\frac{1}{2}\text{PolyLog}\left(3,-e^{2\cosh^{-1}(ax)}\right)-\frac{1}{3}\cosh^{-1}(ax)^3+\cosh^{-1}(ax)^2\log\left(e^{2\cosh^{-1}(ax)}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/x,x]
```


```
[Out] -ArcCosh[a*x]^3/3 + ArcCosh[a*x]^2*Log[1 + E^(2*ArcCosh[a*x])] + ArcCosh[a*
x]*PolyLog[2, -E^(2*ArcCosh[a*x])] - PolyLog[3, -E^(2*ArcCosh[a*x])]/2
```


Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{x}\,dx = \text{Subst}\left(\int x^2\tanh(x)\,dx, x, \cosh^{-1}(ax)\right)$$

$$= -\frac{1}{3}\cosh^{-1}(ax)^3 + 2\,\text{Subst}\left(\int \frac{e^{2x}x^2}{1+e^{2x}}\,dx, x, \cosh^{-1}(ax)\right)$$

$$= -\frac{1}{3}\cosh^{-1}(ax)^3 + \cosh^{-1}(ax)^2\log\left(1+e^{2\cosh^{-1}(ax)}\right) - 2\,\text{Subst}\left(\int x\log\left(1+e^{2x}\right)dx, x, \cosh^{-1}(ax)\right.$$

$$= -\frac{1}{3}\cosh^{-1}(ax)^3 + \cosh^{-1}(ax)^2\log\left(1+e^{2\cosh^{-1}(ax)}\right) + \cosh^{-1}(ax)\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right) - \text{Subst}\left(\int \text{L}\right.$$

$$= -\frac{1}{3}\cosh^{-1}(ax)^3 + \cosh^{-1}(ax)^2\log\left(1+e^{2\cosh^{-1}(ax)}\right) + \cosh^{-1}(ax)\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right) - \frac{1}{2}\,\text{Subst}\left(\int\right.$$

$$= -\frac{1}{3}\cosh^{-1}(ax)^3 + \cosh^{-1}(ax)^2\log\left(1+e^{2\cosh^{-1}(ax)}\right) + \cosh^{-1}(ax)\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right) - \frac{1}{2}\text{Li}_3\left(-e^{2\cos}\right.$$

Mathematica [A] time = 0.0280018, size = 63, normalized size = 1.02

$$-\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(ax)}\right)-\frac{1}{2}\text{PolyLog}\left(3,-e^{-2\cosh^{-1}(ax)}\right)+\frac{1}{3}\cosh^{-1}(ax)^3+\cosh^{-1}(ax)^2\log\left(e^{-2\cosh^{-1}(ax)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/x,x]
```

```
[Out] ArcCosh[a*x]^3/3 + ArcCosh[a*x]^2*Log[1 + E^(-2*ArcCosh[a*x])] - ArcCosh[a*
x]*PolyLog[2, -E^(-2*ArcCosh[a*x])] - PolyLog[3, -E^(-2*ArcCosh[a*x])]/2
```

Maple [A] time = 0.029, size = 98, normalized size = 1.6

$$-\frac{(\text{arccosh}\,(ax))^3}{3}+(\text{arccosh}\,(ax))^2\ln\left(1+\left(ax+\sqrt{ax-1}\sqrt{ax+1}\right)^2\right)+\text{arccosh}\,(ax)\,\text{polylog}\left(2,-\left(ax+\sqrt{ax-1}\sqrt{ax\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/x,x)
```

```
[Out] -1/3*arccosh(a*x)^3+arccosh(a*x)^2*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2
)+arccosh(a*x)*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)-1/2*polylog(
3,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{arcosh}\,(ax)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^2/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arcosh}\,(ax)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^2/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{acosh}^2\,(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/x,x)
```

```
[Out] Integral(acosh(a*x)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arcosh}\,(ax)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^2/x, x)
```

3.18 $\int \frac{\cosh^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=60

$$-2iaPolyLog\left(2,-ie^{\cosh^{-1}(ax)}\right)+2iaPolyLog\left(2,ie^{\cosh^{-1}(ax)}\right)-\frac{\cosh^{-1}(ax)^2}{x}+4a\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)$$

```
[Out] -(ArcCosh[a*x]^2/x) + 4*a*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]] - (2*I)*a*Pol
yLog[2, (-I)*E^ArcCosh[a*x]] + (2*I)*a*PolyLog[2, I*E^ArcCosh[a*x]]
```

Rubi [A] time = 0.208523, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5662, 5761, 4180, 2279, 2391}

$$-2iaPolyLog\left(2,-ie^{\cosh^{-1}(ax)}\right)+2iaPolyLog\left(2,ie^{\cosh^{-1}(ax)}\right)-\frac{\cosh^{-1}(ax)^2}{x}+4a\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/x^2,x]
```

```
[Out] -(ArcCosh[a*x]^2/x) + 4*a*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]] - (2*I)*a*Pol
yLog[2, (-I)*E^ArcCosh[a*x]] + (2*I)*a*PolyLog[2, I*E^ArcCosh[a*x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^2}{x^2}\,dx &= -\frac{\cosh^{-1}(ax)^2}{x} + (2a)\int \frac{\cosh^{-1}(ax)}{x\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\ &= -\frac{\cosh^{-1}(ax)^2}{x} + (2a)\,\text{Subst}\left(\int x\,\mathbf{sech}(x)\,dx, x, \cosh^{-1}(ax)\right)\\ &= -\frac{\cosh^{-1}(ax)^2}{x} + 4a\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - (2ia)\,\text{Subst}\left(\int \log\left(1-ie^{x}\right)\,dx, x, \cosh^{-1}(ax)\right)\\ &= -\frac{\cosh^{-1}(ax)^2}{x} + 4a\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - (2ia)\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\cosh^{-1}(ax)}\right) + \\ &= -\frac{\cosh^{-1}(ax)^2}{x} + 4a\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 2ia\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) + 2ia\text{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)\end{aligned}$$

Mathematica [A] time = 0.250518, size = 92, normalized size = 1.53

$$-ia\left(2\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(ax)}\right) - 2\text{PolyLog}\left(2,ie^{-\cosh^{-1}(ax)}\right) + \cosh^{-1}(ax)\left(-\frac{i\cosh^{-1}(ax)}{ax} + 2\log\left(1-ie^{-\cosh^{-1}(ax)}\right) - \right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/x^2,x]
```

```
[Out] (-I)*a*(ArcCosh[a*x]*(((-I)*ArcCosh[a*x])/(a*x) + 2*Log[1 - I/E^ArcCosh[a*x
]] - 2*Log[1 + I/E^ArcCosh[a*x]]) + 2*PolyLog[2, (-I)/E^ArcCosh[a*x]] - 2*P
olyLog[2, I/E^ArcCosh[a*x]])
```

Maple [A] time = 0.061, size = 137, normalized size = 2.3

$$-\frac{(\operatorname{arccosh}(ax))^2}{x} - 2\,ia\operatorname{arccosh}(ax)\ln\left(1+i\left(ax+\sqrt{ax-1}\sqrt{ax+1}\right)\right) + 2\,ia\operatorname{arccosh}(ax)\ln\left(1-i\left(ax+\sqrt{ax-1}\sqrt{ax+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/x^2,x)
```

```
[Out] -arccosh(a*x)^2/x-2*I*a*arccosh(a*x)*ln(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2
)))+2*I*a*arccosh(a*x)*ln(1-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-2*I*a*dilo
g(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))+2*I*a*dilog(1-I*(a*x+(a*x-1)^(1/2)
*(a*x+1)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^2}{x} + \int \frac{2\left(a^3x^2+\sqrt{ax+1}\sqrt{ax-1}a^2x-a\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)}{a^3x^4-ax^2+\left(a^2x^3-x\right)\sqrt{ax+1}\sqrt{ax-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] -log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/x + integrate(2*(a^3*x^2 + sqrt(a
*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))/(a^
3*x^4 - a*x^2 + (a^2*x^3 - x)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcosh}(ax)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^2(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/x**2,x)
```

```
[Out] Integral(acosh(a*x)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^2/x^2, x)
```

3.19 $\int \frac{\cosh^{-1}(ax)^2}{x^3} dx$

Optimal. Leaf size=48

$$a^2(-\log(x)) - \frac{\cosh^{-1}(ax)^2}{2x^2} + \frac{a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{x}$$

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/x - ArcCosh[a*x]^2/(2*x^2) -
a^2*Log[x]
```

Rubi [A] time = 0.192071, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5662, 5724, 29}

$$a^2(-\log(x)) - \frac{\cosh^{-1}(ax)^2}{2x^2} + \frac{a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/x^3,x]
```

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/x - ArcCosh[a*x]^2/(2*x^2) -
a^2*Log[x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
```

```
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^2}{x^3}\,dx &= -\frac{\cosh^{-1}(ax)^2}{2x^2}+a\int \frac{\cosh^{-1}(ax)}{x^2\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\ &= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{x}-\frac{\cosh^{-1}(ax)^2}{2x^2}-a^2\int \frac{1}{x}\,dx\\ &= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{x}-\frac{\cosh^{-1}(ax)^2}{2x^2}-a^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.0191179, size = 48, normalized size = 1.

$$a^2(-\log(x))-\frac{\cosh^{-1}(ax)^2}{2x^2}+\frac{a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^2/x^3,x]
```

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/x - ArcCosh[a*x]^2/(2*x^2) -
a^2*Log[x]
```

Maple [A] time = 0.063, size = 73, normalized size = 1.5

$$a^2\operatorname{arccosh}(ax)+\frac{a\operatorname{arccosh}(ax)}{x}\sqrt{ax-1}\sqrt{ax+1}-\frac{(\operatorname{arccosh}(ax))^2}{2x^2}-a^2\ln\left(1+\left(ax+\sqrt{ax-1}\sqrt{ax+1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/x^3,x)
```

```
[Out] a^2*arccosh(a*x)+a*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x-1/2*arccosh(a
*x)^2/x^2-a^2*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)
```

Maxima [A] time = 1.73286, size = 53, normalized size = 1.1

$$-a^2\log(x)+\frac{\sqrt{a^2x^2-1}a\operatorname{arcosh}(ax)}{x}-\frac{\operatorname{arcosh}(ax)^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] -a^2*log(x) + sqrt(a^2*x^2 - 1)*a*arccosh(a*x)/x - 1/2*arccosh(a*x)^2/x^2
```

Fricas [A] time = 2.51547, size = 158, normalized size = 3.29

$$-\frac{2a^2x^2\log(x)-2\sqrt{a^2x^2-1}ax\log\left(ax+\sqrt{a^2x^2-1}\right)+\log\left(ax+\sqrt{a^2x^2-1}\right)^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*x^2*log(x) - 2*sqrt(a^2*x^2 - 1)*a*x*log(a*x + sqrt(a^2*x^2 - 1
)) + log(a*x + sqrt(a^2*x^2 - 1))^2)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acosh}^2(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/x**3,x)
```

```
[Out] Integral(acosh(a*x)**2/x**3, x)
```

Giac [B] time = 1.42507, size = 149, normalized size = 3.1

$$\left(a\left(\frac{\log\left(\left|-x|a| + \sqrt{a^2x^2-1}\right|\right)}{|a|} - \frac{|a|\log\left(|x|\right)}{a^2}\right)|a| + \frac{2\,|a|\log\left(ax + \sqrt{a^2x^2-1}\right)}{\left(x|a| - \sqrt{a^2x^2-1}\right)^2 + 1}\right)a - \frac{\log\left(ax + \sqrt{a^2x^2-1}\right)^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] (a*(log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/abs(a) - abs(a)*log(abs(x))/a^2
)*abs(a) + 2*abs(a)*log(a*x + sqrt(a^2*x^2 - 1))/((x*abs(a) - sqrt(a^2*x^2
- 1))^2 + 1))*a - 1/2*log(a*x + sqrt(a^2*x^2 - 1))^2/x^2
```

3.20 $\int \frac{\cosh^{-1}(ax)^2}{x^4} \, dx$

Optimal. Leaf size=114

$$-\frac{1}{3} i a^3 \text{PolyLog}\left(2,-i e^{\cosh^{-1}(ax)}\right)+\frac{1}{3} i a^3 \text{PolyLog}\left(2,i e^{\cosh^{-1}(ax)}\right)+\frac{a^2}{3x}+\frac{2}{3} a^3 \cosh^{-1}(ax) \tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)+\frac{a\sqrt{ax-1}}{}$$

```
[Out] a^2/(3*x) + (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(3*x^2) - ArcCosh
[a*x]^2/(3*x^3) + (2*a^3*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]])/3 - (I/3)*a^3
*PolyLog[2, (-I)*E^ArcCosh[a*x]] + (I/3)*a^3*PolyLog[2, I*E^ArcCosh[a*x]]
```

Rubi [A] time = 0.392414, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5662, 5748, 5761, 4180, 2279, 2391, 30}

$$-\frac{1}{3} i a^3 \text{PolyLog}\left(2,-i e^{\cosh^{-1}(ax)}\right)+\frac{1}{3} i a^3 \text{PolyLog}\left(2,i e^{\cosh^{-1}(ax)}\right)+\frac{a^2}{3x}+\frac{2}{3} a^3 \cosh^{-1}(ax) \tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)+\frac{a\sqrt{ax-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/x^4,x]
```

```
[Out] a^2/(3*x) + (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(3*x^2) - ArcCosh
[a*x]^2/(3*x^3) + (2*a^3*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]])/3 - (I/3)*a^3
*PolyLog[2, (-I)*E^ArcCosh[a*x]] + (I/3)*a^3*PolyLog[2, I*E^ArcCosh[a*x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
```

```
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{x^4}\,dx = -\frac{\cosh^{-1}(ax)^2}{3x^3} + \frac{1}{3}(2a)\int \frac{\cosh^{-1}(ax)}{x^3\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$
$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3x^2} - \frac{\cosh^{-1}(ax)^2}{3x^3} - \frac{1}{3}a^2\int \frac{1}{x^2}\,dx + \frac{1}{3}a^3\int \frac{\cosh^{-1}(ax)}{x\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$
$$= \frac{a^2}{3x} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3x^2} - \frac{\cosh^{-1}(ax)^2}{3x^3} + \frac{1}{3}a^3\,\mathrm{Subst}\left(\int x\mathrm{sech}(x)\,dx, x, \cosh^{-1}(ax)\right)$$
$$= \frac{a^2}{3x} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3x^2} - \frac{\cosh^{-1}(ax)^2}{3x^3} + \frac{2}{3}a^3\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - \frac{1}{3}\left(ia^3\right.$$
$$= \frac{a^2}{3x} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3x^2} - \frac{\cosh^{-1}(ax)^2}{3x^3} + \frac{2}{3}a^3\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - \frac{1}{3}\left(ia^3\right.$$
$$= \frac{a^2}{3x} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3x^2} - \frac{\cosh^{-1}(ax)^2}{3x^3} + \frac{2}{3}a^3\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - \frac{1}{3}ia^3\mathrm{L}$$

Mathematica [A] time = 0.242953, size = 144, normalized size = 1.26

$$\frac{1}{3}a^3\left(-i\mathrm{PolyLog}\left(2, -ie^{-\cosh^{-1}(ax)}\right) + i\mathrm{PolyLog}\left(2, ie^{-\cosh^{-1}(ax)}\right) - \frac{\cosh^{-1}(ax)^2}{a^3x^3} + \frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\cosh^{-1}(ax)}{a^2x^2} + \frac{1}{ax} - i\,c\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/x^4,x]
```

```
[Out] (a^3*(1/(a*x) + (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x])/(a^2*x^
2) - ArcCosh[a*x]^2/(a^3*x^3) - I*ArcCosh[a*x]*Log[1 - I/E^ArcCosh[a*x]] +
I*ArcCosh[a*x]*Log[1 + I/E^ArcCosh[a*x]] - I*PolyLog[2, (-I)/E^ArcCosh[a*x]
] + I*PolyLog[2, I/E^ArcCosh[a*x]]))/3
```

Maple [A] time = 0.106, size = 177, normalized size = 1.6

$$\frac{a\mathrm{arccosh}\,(ax)}{3\,x^2}\sqrt{ax-1}\sqrt{ax+1} + \frac{a^2}{3\,x} - \frac{(\mathrm{arccosh}\,(ax))^2}{3\,x^3} - \frac{i}{3}a^3\mathrm{arccosh}\,(ax)\ln\left(1 + i\left(ax + \sqrt{ax-1}\sqrt{ax+1}\right)\right) + \frac{i}{3}a^3\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/x^4,x)
```

```
[Out] 1/3*a*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x^2+1/3*a^2/x-1/3*arccosh(a*
x)^2/x^3-1/3*I*a^3*arccosh(a*x)*ln(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))+1
/3*I*a^3*arccosh(a*x)*ln(1-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-1/3*I*a^3*d
ilog(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))+1/3*I*a^3*dilog(1-I*(a*x+(a*x-1
)^(1/2)*(a*x+1)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^2}{3\,x^3}+\int\frac{2\left(a^3x^2+\sqrt{ax+1}\sqrt{ax-1}a^2x-a\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)}{3\left(a^3x^6-ax^4+\left(a^2x^5-x^3\right)\sqrt{ax+1}\sqrt{ax-1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/x^3 + integrate(2/3*(a^3*x^2
+ sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x -
 1))/(a^3*x^6 - a*x^4 + (a^2*x^5 - x^3)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arcosh}\left(ax\right)^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^2/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{acosh}^2\left(ax\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/x**4,x)
```

```
[Out] Integral(acosh(a*x)**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^2/x^4, x)
```

3.21 $\int \frac{\cosh^{-1}(ax)^2}{x^5}\,dx$

Optimal. Leaf size=95

$$\frac{a^2}{12x^2}-\frac{1}{3}a^4\log(x)+\frac{a^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{3x}+\frac{a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{6x^3}-\frac{\cosh^{-1}(ax)^2}{4x^4}$$

```
[Out] a^2/(12*x^2) + (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(6*x^3) + (a^3
*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(3*x) - ArcCosh[a*x]^2/(4*x^4)
- (a^4*Log[x])/3
```

Rubi [A] time = 0.364883, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5662, 5748, 5724, 29, 30}

$$\frac{a^2}{12x^2}-\frac{1}{3}a^4\log(x)+\frac{a^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{3x}+\frac{a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{6x^3}-\frac{\cosh^{-1}(ax)^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/x^5,x]
```

```
[Out] a^2/(12*x^2) + (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(6*x^3) + (a^3
*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(3*x) - ArcCosh[a*x]^2/(4*x^4)
- (a^4*Log[x])/3
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
```

```
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{x^5}\,dx = -\frac{\cosh^{-1}(ax)^2}{4x^4} + \frac{1}{2}a\int \frac{\cosh^{-1}(ax)}{x^4\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{6x^3} - \frac{\cosh^{-1}(ax)^2}{4x^4} - \frac{1}{6}a^2\int \frac{1}{x^3}\,dx + \frac{1}{3}a^3\int \frac{\cosh^{-1}(ax)}{x^2\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= \frac{a^2}{12x^2} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{6x^3} + \frac{a^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3x} - \frac{\cosh^{-1}(ax)^2}{4x^4} - \frac{1}{3}a^4\int$$

$$= \frac{a^2}{12x^2} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{6x^3} + \frac{a^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3x} - \frac{\cosh^{-1}(ax)^2}{4x^4} - \frac{1}{3}a^4\,\text{lo}$$

Mathematica [A] time = 0.0776289, size = 69, normalized size = 0.73

$$\frac{a^2x^2 - 4a^4x^4\log(x) + 2ax\sqrt{ax-1}\sqrt{ax+1}\left(2a^2x^2+1\right)\cosh^{-1}(ax) - 3\cosh^{-1}(ax)^2}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^2/x^5,x]
```

```
[Out] (a^2*x^2 + 2*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(1 + 2*a^2*x^2)*ArcCosh[a*x]
- 3*ArcCosh[a*x]^2 - 4*a^4*x^4*Log[x])/(12*x^4)
```

Maple [A] time = 0.107, size = 109, normalized size = 1.2

$$\frac{a^4\mathrm{arccosh}\,(ax)}{3}+\frac{a^3\mathrm{arccosh}\,(ax)}{3\,x}\sqrt{ax-1}\sqrt{ax+1}+\frac{a\mathrm{arccosh}\,(ax)}{6\,x^3}\sqrt{ax-1}\sqrt{ax+1}+\frac{a^2}{12\,x^2}-\frac{(\mathrm{arccosh}\,(ax))^2}{4\,x^4}-\frac{a^4}{3}\ln\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/x^5,x)
```

```
[Out] 1/3*a^4*arccosh(a*x)+1/3*a^3*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x+1/6
*a*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x^3+1/12*a^2/x^2-1/4*arccosh(a*
x)^2/x^4-1/3*a^4*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)
```

Maxima [A] time = 1.55523, size = 97, normalized size = 1.02

$$-\frac{1}{12}\left(4\,a^2\log\left(x\right)-\frac{1}{x^2}\right)a^2+\frac{1}{6}\left(\frac{2\,\sqrt{a^2x^2-1}a^2}{x}+\frac{\sqrt{a^2x^2-1}}{x^3}\right)a\,\mathrm{arcosh}\left(ax\right)-\frac{\mathrm{arcosh}\left(ax\right)^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(4*a^2*log(x) - 1/x^2)*a^2 + 1/6*(2*sqrt(a^2*x^2 - 1)*a^2/x + sqrt(a^
2*x^2 - 1)/x^3)*a*arccosh(a*x) - 1/4*arccosh(a*x)^2/x^4
```

Fricas [A] time = 2.54507, size = 194, normalized size = 2.04

$$-\frac{4\,a^4x^4\log\left(x\right)-a^2x^2-2\left(2\,a^3x^3+ax\right)\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)+3\,\log\left(ax+\sqrt{a^2x^2-1}\right)^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*a^4*x^4*log(x) - a^2*x^2 - 2*(2*a^3*x^3 + a*x)*sqrt(a^2*x^2 - 1)*l
og(a*x + sqrt(a^2*x^2 - 1)) + 3*log(a*x + sqrt(a^2*x^2 - 1))^2)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^2(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/x**5,x)
```

```
[Out] Integral(acosh(a*x)**2/x**5, x)
```

Giac [A] time = 1.62617, size = 198, normalized size = 2.08

$$-\frac{1}{12}\left(2\,a^3\log\left(x^2\right)-4\,a^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)-\frac{8\left(3\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)a^2|a|\log\left(ax+\sqrt{a^2x^2-1}\right)}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3}-\frac{2\,a^3x^2+}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(2*a^3*log(x^2) - 4*a^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1))) - 8*(
3*(x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*a^2*abs(a)*log(a*x + sqrt(a^2*x^2 -
 1))/((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3 - (2*a^3*x^2 + a)/x^2)*a - 1/
4*log(a*x + sqrt(a^2*x^2 - 1))^2/x^4
```

3.22 $\int x^4 \cosh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=231

$$-\frac{272x^2\sqrt{ax-1}\sqrt{ax+1}}{5625a^3}+\frac{8x^3\cosh^{-1}(ax)}{75a^2}-\frac{4x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{25a^3}-\frac{4144\sqrt{ax-1}\sqrt{ax+1}}{5625a^5}+\frac{16x\cosh^{-1}(ax)}{25a^4}$$

```
[Out] (-4144*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(5625*a^5) - (272*x^2*Sqrt[-1 + a*x]*S
qrt[1 + a*x])/(5625*a^3) - (6*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(625*a) + (
16*x*ArcCosh[a*x])/(25*a^4) + (8*x^3*ArcCosh[a*x])/(75*a^2) + (6*x^5*ArcCos
h[a*x])/125 - (8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(25*a^5) - (4
*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(25*a^3) - (3*x^4*Sqrt[-1
 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(25*a) + (x^5*ArcCosh[a*x]^3)/5
```

Rubi [A] time = 0.771434, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5662, 5759, 5718, 5654, 74, 100, 12}

$$-\frac{272x^2\sqrt{ax-1}\sqrt{ax+1}}{5625a^3}+\frac{8x^3\cosh^{-1}(ax)}{75a^2}-\frac{4x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{25a^3}-\frac{4144\sqrt{ax-1}\sqrt{ax+1}}{5625a^5}+\frac{16x\cosh^{-1}(ax)}{25a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCosh[a*x]^3,x]
```


```
[Out] (-4144*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(5625*a^5) - (272*x^2*Sqrt[-1 + a*x]*S
qrt[1 + a*x])/(5625*a^3) - (6*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(625*a) + (
16*x*ArcCosh[a*x])/(25*a^4) + (8*x^3*ArcCosh[a*x])/(75*a^2) + (6*x^5*ArcCos
h[a*x])/125 - (8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(25*a^5) - (4
*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(25*a^3) - (3*x^4*Sqrt[-1
 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(25*a) + (x^5*ArcCosh[a*x]^3)/5
```


Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int x^4\cosh^{-1}(ax)^3\,dx = \frac{1}{5}x^5\cosh^{-1}(ax)^3 - \frac{1}{5}(3a)\int \frac{x^5\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{3x^4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{25a} + \frac{1}{5}x^5\cosh^{-1}(ax)^3 + \frac{6}{25}\int x^4\cosh^{-1}(ax)\,dx - \frac{12\int \frac{x^3\cosh^{-1}}{\sqrt{-1+ax}\sqrt{}}}{25a}$$

$$= \frac{6}{125}x^5\cosh^{-1}(ax) - \frac{4x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{25a^3} - \frac{3x^4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{25a} + \frac{1}{5}$$

$$= -\frac{6x^4\sqrt{-1+ax}\sqrt{1+ax}}{625a} + \frac{8x^3\cosh^{-1}(ax)}{75a^2} + \frac{6}{125}x^5\cosh^{-1}(ax) - \frac{8\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{25a^5}$$

$$= -\frac{8x^2\sqrt{-1+ax}\sqrt{1+ax}}{225a^3} - \frac{6x^4\sqrt{-1+ax}\sqrt{1+ax}}{625a} + \frac{16x\cosh^{-1}(ax)}{25a^4} + \frac{8x^3\cosh^{-1}(ax)}{75a^2} + \frac{6}{125}x^5\,\mathrm{co}$$

$$= -\frac{16\sqrt{-1+ax}\sqrt{1+ax}}{25a^5} - \frac{272x^2\sqrt{-1+ax}\sqrt{1+ax}}{5625a^3} - \frac{6x^4\sqrt{-1+ax}\sqrt{1+ax}}{625a} + \frac{16x\cosh^{-1}(ax)}{25a^4} + \frac{8}{}$$

$$= -\frac{32\sqrt{-1+ax}\sqrt{1+ax}}{45a^5} - \frac{272x^2\sqrt{-1+ax}\sqrt{1+ax}}{5625a^3} - \frac{6x^4\sqrt{-1+ax}\sqrt{1+ax}}{625a} + \frac{16x\cosh^{-1}(ax)}{25a^4} + \frac{8}{}$$

$$= -\frac{4144\sqrt{-1+ax}\sqrt{1+ax}}{5625a^5} - \frac{272x^2\sqrt{-1+ax}\sqrt{1+ax}}{5625a^3} - \frac{6x^4\sqrt{-1+ax}\sqrt{1+ax}}{625a} + \frac{16x\cosh^{-1}(ax)}{25a^4} +$$

Mathematica [A] time = 0.116565, size = 130, normalized size = 0.56

$$\frac{-2\sqrt{ax-1}\sqrt{ax+1}\left(27a^4x^4+136a^2x^2+2072\right)+1125a^5x^5\cosh^{-1}(ax)^3+30ax\left(9a^4x^4+20a^2x^2+120\right)\cosh^{-1}(ax)-225}{5625a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCosh[a*x]^3,x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(2072 + 136*a^2*x^2 + 27*a^4*x^4) + 30*a*x
*(120 + 20*a^2*x^2 + 9*a^4*x^4)*ArcCosh[a*x] - 225*Sqrt[-1 + a*x]*Sqrt[1 +
a*x]*(8 + 4*a^2*x^2 + 3*a^4*x^4)*ArcCosh[a*x]^2 + 1125*a^5*x^5*ArcCosh[a*x]
```

```
^3)/(5625*a^5)
```

Maple [A] time = 0.048, size = 246, normalized size = 1.1

$$\frac{1}{a^5}\left(\frac{(\operatorname{arccosh}(ax))^3 a^3x^3 (ax-1)(ax+1)}{5}+\frac{(\operatorname{arccosh}(ax))^3 (ax-1)(ax+1)\,ax}{5}+\frac{(\operatorname{arccosh}(ax))^3\,ax}{5}-\frac{3\,a^4\,(\operatorname{arccosh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)^3,x)
```

```
[Out] 1/a^5*(1/5*arccosh(a*x)^3*a^3*x^3*(a*x-1)*(a*x+1)+1/5*arccosh(a*x)^3*(a*x-1
)*(a*x+1)*a*x+1/5*arccosh(a*x)^3*a*x-3/25*arccosh(a*x)^2*a^4*x^4*(a*x-1)^(1
/2)*(a*x+1)^(1/2)-8/25*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-4/25*arcc
osh(a*x)^2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+6/125*arccosh(a*x)*(a*x-1)*(
a*x+1)*a^3*x^3+58/375*arccosh(a*x)*(a*x-1)*(a*x+1)*a*x+298/375*a*x*arccosh(
a*x)-6/625*a^4*x^4*(a*x-1)^(1/2)*(a*x+1)^(1/2)-4144/5625*(a*x-1)^(1/2)*(a*x
+1)^(1/2)-272/5625*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

Maxima [A] time = 1.19804, size = 223, normalized size = 0.97

$$\frac{1}{5}\,x^5\operatorname{arcosh}(ax)^3-\frac{1}{25}\left(\frac{3\,\sqrt{a^2x^2-1}x^4}{a^2}+\frac{4\,\sqrt{a^2x^2-1}x^2}{a^4}+\frac{8\,\sqrt{a^2x^2-1}}{a^6}\right)a\operatorname{arcosh}(ax)^2-\frac{2}{5625}\,a\left(\frac{27\,\sqrt{a^2x^2-1}a^2x^4+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccosh(a*x)^3 - 1/25*(3*sqrt(a^2*x^2 - 1)*x^4/a^2 + 4*sqrt(a^2*x^2
 - 1)*x^2/a^4 + 8*sqrt(a^2*x^2 - 1)/a^6)*a*arccosh(a*x)^2 - 2/5625*a*((27*s
qrt(a^2*x^2 - 1)*a^2*x^4 + 136*sqrt(a^2*x^2 - 1)*x^2 + 2072*sqrt(a^2*x^2 -
1)/a^2)/a^4 - 15*(9*a^4*x^5 + 20*a^2*x^3 + 120*x)*arccosh(a*x)/a^5)
```

Fricas [A] time = 2.41865, size = 359, normalized size = 1.55

$$\frac{1125\,a^5x^5\log\left(ax+\sqrt{a^2x^2-1}\right)^3-225\left(3\,a^4x^4+4\,a^2x^2+8\right)\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)^2+30\left(9\,a^5x^5+20\,a^3x^3+1\right.}{5625\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/5625*(1125*a^5*x^5*log(a*x + sqrt(a^2*x^2 - 1))^3 - 225*(3*a^4*x^4 + 4*a^
2*x^2 + 8)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^2 + 30*(9*a^5*x^5
 + 20*a^3*x^3 + 120*a*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 2*(27*a^4*x^4 + 136
*a^2*x^2 + 2072)*sqrt(a^2*x^2 - 1))/a^5
```

Sympy [A] time = 8.78706, size = 206, normalized size = 0.89

$$\begin{cases} \frac{x^5\operatorname{acosh}^3(ax)}{5} + \frac{6x^5\operatorname{acosh}(ax)}{125} - \frac{3x^4\sqrt{a^2x^2-1}\operatorname{acosh}^2(ax)}{25a} - \frac{6x^4\sqrt{a^2x^2-1}}{625a} + \frac{8x^3\operatorname{acosh}(ax)}{75a^2} - \frac{4x^2\sqrt{a^2x^2-1}\operatorname{acosh}^2(ax)}{25a^3} - \frac{272x^2\sqrt{a^2x^2-1}}{5625a^3} + \frac{16x\operatorname{acc}}{25} \\ -\frac{i\pi^3x^5}{40} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)**3,x)
```

```
[Out] Piecewise((x**5*acosh(a*x)**3/5 + 6*x**5*acosh(a*x)/125 - 3*x**4*sqrt(a**2*
x**2 - 1)*acosh(a*x)**2/(25*a) - 6*x**4*sqrt(a**2*x**2 - 1)/(625*a) + 8*x**
3*acosh(a*x)/(75*a**2) - 4*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(25*a**3)
 - 272*x**2*sqrt(a**2*x**2 - 1)/(5625*a**3) + 16*x*acosh(a*x)/(25*a**4) - 8
*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(25*a**5) - 4144*sqrt(a**2*x**2 - 1)/(56
25*a**5), Ne(a, 0)), (-I*pi**3*x**5/40, True))
```

Giac [A] time = 1.60727, size = 243, normalized size = 1.05

$$\frac{1}{5}x^5\log\left(ax+\sqrt{a^2x^2-1}\right)^3 - \frac{1}{5625}a\left(\frac{225\left(3\left(a^2x^2-1\right)^{\frac{5}{2}}+10\left(a^2x^2-1\right)^{\frac{3}{2}}+15\sqrt{a^2x^2-1}\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^2}{a^6} - \frac{2\left(15\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/5*x^5*log(a*x + sqrt(a^2*x^2 - 1))^3 - 1/5625*a*(225*(3*(a^2*x^2 - 1)^(5/
2) + 10*(a^2*x^2 - 1)^(3/2) + 15*sqrt(a^2*x^2 - 1))*log(a*x + sqrt(a^2*x^2
- 1))^2/a^6 - 2*(15*(9*a^4*x^5 + 20*a^2*x^3 + 120*x)*log(a*x + sqrt(a^2*x^2
 - 1)) - (27*(a^2*x^2 - 1)^(5/2) + 190*(a^2*x^2 - 1)^(3/2) + 2235*sqrt(a^2*
x^2 - 1))/a)/a^5)
```

3.23 $\int x^3 \cosh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=183

$$\frac{9x^2\cosh^{-1}(ax)}{32a^2} - \frac{45x\sqrt{ax-1}\sqrt{ax+1}}{256a^3} - \frac{9x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{32a^3} - \frac{3\cosh^{-1}(ax)^3}{32a^4} - \frac{45\cosh^{-1}(ax)}{256a^4} - \frac{3x^3\sqrt{ax-}}{128}$$

```
[Out] (-45*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(256*a^3) - (3*x^3*Sqrt[-1 + a*x]*Sqrt
[1 + a*x])/(128*a) - (45*ArcCosh[a*x])/(256*a^4) + (9*x^2*ArcCosh[a*x])/(32
*a^2) + (3*x^4*ArcCosh[a*x])/32 - (9*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh
[a*x]^2)/(32*a^3) - (3*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(16
*a) - (3*ArcCosh[a*x]^3)/(32*a^4) + (x^4*ArcCosh[a*x]^3)/4
```

Rubi [A] time = 0.666215, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5662, 5759, 5676, 90, 52, 100, 12}

$$\frac{9x^2\cosh^{-1}(ax)}{32a^2} - \frac{45x\sqrt{ax-1}\sqrt{ax+1}}{256a^3} - \frac{9x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{32a^3} - \frac{3\cosh^{-1}(ax)^3}{32a^4} - \frac{45\cosh^{-1}(ax)}{256a^4} - \frac{3x^3\sqrt{ax-}}{128}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCosh[a*x]^3,x]
```


```
[Out] (-45*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(256*a^3) - (3*x^3*Sqrt[-1 + a*x]*Sqrt
[1 + a*x])/(128*a) - (45*ArcCosh[a*x])/(256*a^4) + (9*x^2*ArcCosh[a*x])/(32
*a^2) + (3*x^4*ArcCosh[a*x])/32 - (9*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh
[a*x]^2)/(32*a^3) - (3*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(16
*a) - (3*ArcCosh[a*x]^3)/(32*a^4) + (x^4*ArcCosh[a*x]^3)/4
```


Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
```

```
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int x^3\cosh^{-1}(ax)^3\,dx = \frac{1}{4}x^4\cosh^{-1}(ax)^3 - \frac{1}{4}(3a)\int \frac{x^4\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{16a} + \frac{1}{4}x^4\cosh^{-1}(ax)^3 + \frac{3}{8}\int x^3\cosh^{-1}(ax)\,dx - \frac{9\int \frac{x^2\cosh^{-1}(ax}{\sqrt{-1+ax}\sqrt{1+}}}{16a}$$

$$= \frac{3}{32}x^4\cosh^{-1}(ax) - \frac{9x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{32a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{16a} + \frac{1}{4}x^4$$

$$= -\frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}}{128a} + \frac{9x^2\cosh^{-1}(ax)}{32a^2} + \frac{3}{32}x^4\cosh^{-1}(ax) - \frac{9x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{32a^3}$$

$$= -\frac{9x\sqrt{-1+ax}\sqrt{1+ax}}{64a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}}{128a} + \frac{9x^2\cosh^{-1}(ax)}{32a^2} + \frac{3}{32}x^4\cosh^{-1}(ax) - \frac{9x\sqrt{-1+}}{}$$

$$= -\frac{45x\sqrt{-1+ax}\sqrt{1+ax}}{256a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}}{128a} - \frac{9\cosh^{-1}(ax)}{64a^4} + \frac{9x^2\cosh^{-1}(ax)}{32a^2} + \frac{3}{32}x^4\cosh^{-}$$

$$= -\frac{45x\sqrt{-1+ax}\sqrt{1+ax}}{256a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}}{128a} - \frac{45\cosh^{-1}(ax)}{256a^4} + \frac{9x^2\cosh^{-1}(ax)}{32a^2} + \frac{3}{32}x^4\cosh$$

Mathematica [A] time = 0.131483, size = 143, normalized size = 0.78

$$\frac{-3ax\sqrt{ax-1}\sqrt{ax+1}\left(2a^2x^2+15\right)+8\left(8a^4x^4-3\right)\cosh^{-1}(ax)^3-24ax\sqrt{ax-1}\sqrt{ax+1}\left(2a^2x^2+3\right)\cosh^{-1}(ax)^2+24a^2x}{256a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCosh[a*x]^3,x]
```

```
[Out] (-3*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(15 + 2*a^2*x^2) + 24*a^2*x^2*(3 + a^2
*x^2)*ArcCosh[a*x] - 24*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(3 + 2*a^2*x^2)*Ar
cCosh[a*x]^2 + 8*(-3 + 8*a^4*x^4)*ArcCosh[a*x]^3 - 45*Log[a*x + Sqrt[-1 + a
*x]*Sqrt[1 + a*x]])/(256*a^4)
```

Maple [A] time = 0.042, size = 184, normalized size = 1.

$$\frac{1}{a^4}\left(\frac{(\mathrm{arccosh}\,(ax))^3(ax-1)(ax+1)a^2x^2}{4} + \frac{(\mathrm{arccosh}\,(ax))^3a^2x^2}{4} - \frac{3\,(\mathrm{arccosh}\,(ax))^2a^3x^3}{16}\sqrt{ax-1}\sqrt{ax+1} - \frac{9\,(\mathrm{arccos}}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)^3,x)
```

```
[Out] 1/a^4*(1/4*arccosh(a*x)^3*(a*x-1)*(a*x+1)*a^2*x^2+1/4*arccosh(a*x)^3*a^2*x^
2-3/16*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^3*x^3-9/32*arccosh(a*x)
^2*a*x*(a*x-1)^(1/2)*(a*x+1)^(1/2)-3/32*arccosh(a*x)^3+3/32*arccosh(a*x)*(a
*x-1)*(a*x+1)*a^2*x^2-3/128*a^3*x^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)-45/256*(a*x
+1)^(1/2)*(a*x-1)^(1/2)*a*x-45/256*arccosh(a*x)+3/8*a^2*x^2*arccosh(a*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}x^4\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^3-\int\frac{3\left(a^3x^6+\sqrt{ax+1}\sqrt{ax-1}a^2x^5-ax^4\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^2}{4\left(a^3x^3+\left(a^2x^2-1\right)\sqrt{ax+1}\sqrt{ax-1}-ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3 - integrate(3/4*(a^3*x^6 +
 sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x^5 - a*x^4)*log(a*x + sqrt(a*x + 1)*sqrt(
a*x - 1))^2/(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x), x)
```

Fricas [A] time = 2.52813, size = 327, normalized size = 1.79

$$\frac{8\left(8a^4x^4-3\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^3-24\left(2a^3x^3+3ax\right)\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)^2+3\left(8a^4x^4+24a^2x^2-15\right)\text{lo}}{256a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(8*(8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 - 1))^3 - 24*(2*a^3*x^3 + 3
*a*x)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^2 + 3*(8*a^4*x^4 + 24*
a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 - 1)) - 3*(2*a^3*x^3 + 15*a*x)*sqrt(a^
2*x^2 - 1))/a^4
```

Sympy [A] time = 5.03371, size = 170, normalized size = 0.93

$$\begin{cases} \frac{x^4 \operatorname{acosh}^3(ax)}{4} + \frac{3x^4 \operatorname{acosh}(ax)}{32} - \frac{3x^3\sqrt{a^2x^2-1}\operatorname{acosh}^2(ax)}{16a} - \frac{3x^3\sqrt{a^2x^2-1}}{128a} + \frac{9x^2 \operatorname{acosh}(ax)}{32a^2} - \frac{9x\sqrt{a^2x^2-1}\operatorname{acosh}^2(ax)}{32a^3} - \frac{45x\sqrt{a^2x^2-1}}{256a^3} - \frac{3\operatorname{acosh}^3(a \\ -\frac{i\pi^3x^4}{32} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**3,x)
```

```
[Out] Piecewise((x**4*acosh(a*x)**3/4 + 3*x**4*acosh(a*x)/32 - 3*x**3*sqrt(a**2*x
**2 - 1)*acosh(a*x)**2/(16*a) - 3*x**3*sqrt(a**2*x**2 - 1)/(128*a) + 9*x**2
*acosh(a*x)/(32*a**2) - 9*x*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(32*a**3) - 4
5*x*sqrt(a**2*x**2 - 1)/(256*a**3) - 3*acosh(a*x)**3/(32*a**4) - 45*acosh(a
*x)/(256*a**4), Ne(a, 0)), (-I*pi**3*x**4/32, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arcosh}(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccosh(a*x)^3, x)
```

3.24 $\int x^2 \cosh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=155

$$-\frac{40\sqrt{ax-1}\sqrt{ax+1}}{27a^3}+\frac{4x\cosh^{-1}(ax)}{3a^2}-\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{3a^3}-\frac{2x^2\sqrt{ax-1}\sqrt{ax+1}}{27a}+\frac{1}{3}x^3\cosh^{-1}(ax)^3+\frac{2}{9}x$$

```
[Out] (-40*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(27*a^3) - (2*x^2*Sqrt[-1 + a*x]*Sqrt[1
+ a*x])/(27*a) + (4*x*ArcCosh[a*x])/(3*a^2) + (2*x^3*ArcCosh[a*x])/9 - (2*S
qrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(3*a^3) - (x^2*Sqrt[-1 + a*x]*S
qrt[1 + a*x]*ArcCosh[a*x]^2)/(3*a) + (x^3*ArcCosh[a*x]^3)/3
```

Rubi [A] time = 0.4742, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5662, 5759, 5718, 5654, 74, 100, 12}

$$-\frac{40\sqrt{ax-1}\sqrt{ax+1}}{27a^3}+\frac{4x\cosh^{-1}(ax)}{3a^2}-\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{3a^3}-\frac{2x^2\sqrt{ax-1}\sqrt{ax+1}}{27a}+\frac{1}{3}x^3\cosh^{-1}(ax)^3+\frac{2}{9}x$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCosh[a*x]^3,x]
```

```
[Out] (-40*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(27*a^3) - (2*x^2*Sqrt[-1 + a*x]*Sqrt[1
+ a*x])/(27*a) + (4*x*ArcCosh[a*x])/(3*a^2) + (2*x^3*ArcCosh[a*x])/9 - (2*S
qrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(3*a^3) - (x^2*Sqrt[-1 + a*x]*S
qrt[1 + a*x]*ArcCosh[a*x]^2)/(3*a) + (x^3*ArcCosh[a*x]^3)/3
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
```

```
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int x^2\cosh^{-1}(ax)^3\,dx=\frac{1}{3}x^3\cosh^{-1}(ax)^3-a\int\frac{x^3\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$=-\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{3a}+\frac{1}{3}x^3\cosh^{-1}(ax)^3+\frac{2}{3}\int x^2\cosh^{-1}(ax)\,dx-\frac{2\int\frac{x\cosh^{-1}(ax}{\sqrt{-1+ax}\sqrt{1+}}}{3a}$$

$$=\frac{2}{9}x^3\cosh^{-1}(ax)-\frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{3a^3}-\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{3a}+\frac{1}{3}x^3\text{co}$$

$$=-\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{27a}+\frac{4x\cosh^{-1}(ax)}{3a^2}+\frac{2}{9}x^3\cosh^{-1}(ax)-\frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{3a^3}-$$

$$=-\frac{4\sqrt{-1+ax}\sqrt{1+ax}}{3a^3}-\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{27a}+\frac{4x\cosh^{-1}(ax)}{3a^2}+\frac{2}{9}x^3\cosh^{-1}(ax)-\frac{2\sqrt{-1+ax}}{}$$

$$=-\frac{40\sqrt{-1+ax}\sqrt{1+ax}}{27a^3}-\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{27a}+\frac{4x\cosh^{-1}(ax)}{3a^2}+\frac{2}{9}x^3\cosh^{-1}(ax)-\frac{2\sqrt{-1+a}}{}$$

Mathematica [A] time = 0.0925802, size = 103, normalized size = 0.66

$$\frac{-2\sqrt{ax-1}\sqrt{ax+1}\left(a^2x^2+20\right)+9a^3x^3\cosh^{-1}(ax)^3-9\sqrt{ax-1}\sqrt{ax+1}\left(a^2x^2+2\right)\cosh^{-1}(ax)^2+6ax\left(a^2x^2+6\right)\cosh}{27a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCosh[a*x]^3,x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(20 + a^2*x^2) + 6*a*x*(6 + a^2*x^2)*ArcCo
sh[a*x] - 9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(2 + a^2*x^2)*ArcCosh[a*x]^2 + 9*a
^3*x^3*ArcCosh[a*x]^3)/(27*a^3)
```

Maple [A] time = 0.037, size = 150, normalized size = 1.

$$\frac{1}{a^3}\left(\frac{(\mathrm{arccosh}\,(ax))^3(ax-1)(ax+1)ax}{3}+\frac{(\mathrm{arccosh}\,(ax))^3ax}{3}-\frac{(\mathrm{arccosh}\,(ax))^2a^2x^2}{3}\sqrt{ax-1}\sqrt{ax+1}-\frac{2\,(\mathrm{arccosh}\,(ax}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)^3,x)
```

```
[Out] 1/a^3*(1/3*arccosh(a*x)^3*(a*x-1)*(a*x+1)*a*x+1/3*arccosh(a*x)^3*a*x-1/3*ar
ccosh(a*x)^2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-2/3*arccosh(a*x)^2*(a*x-1)
^(1/2)*(a*x+1)^(1/2)+2/9*arccosh(a*x)*(a*x-1)*(a*x+1)*a*x+14/9*a*x*arccosh(
a*x)-2/27*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-40/27*(a*x-1)^(1/2)*(a*x+1)^(
1/2))
```

Maxima [A] time = 1.252, size = 157, normalized size = 1.01

$$\frac{1}{3}x^3\operatorname{arcosh}(ax)^3 - \frac{1}{3}a\left(\frac{\sqrt{a^2x^2-1}x^2}{a^2} + \frac{2\sqrt{a^2x^2-1}}{a^4}\right)\operatorname{arcosh}(ax)^2 - \frac{2}{27}a\left(\frac{\sqrt{a^2x^2-1}x^2 + \frac{20\sqrt{a^2x^2-1}}{a^2}}{a^2} - \frac{3\left(a^2x^3+6x\right)\operatorname{arc}}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccosh(a*x)^3 - 1/3*a*(sqrt(a^2*x^2 - 1)*x^2/a^2 + 2*sqrt(a^2*x^2
- 1)/a^4)*arccosh(a*x)^2 - 2/27*a*((sqrt(a^2*x^2 - 1)*x^2 + 20*sqrt(a^2*x^2
 - 1)/a^2)/a^2 - 3*(a^2*x^3 + 6*x)*arccosh(a*x)/a^3)
```

Fricas [A] time = 2.50902, size = 281, normalized size = 1.81

$$\frac{9a^3x^3\log\left(ax+\sqrt{a^2x^2-1}\right)^3 - 9\left(a^2x^2+2\right)\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)^2 + 6\left(a^3x^3+6ax\right)\log\left(ax+\sqrt{a^2x^2-1}\right) - 2\left(\right.}{27a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/27*(9*a^3*x^3*log(a*x + sqrt(a^2*x^2 - 1))^3 - 9*(a^2*x^2 + 2)*sqrt(a^2*x
^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^2 + 6*(a^3*x^3 + 6*a*x)*log(a*x + sqrt
(a^2*x^2 - 1)) - 2*(a^2*x^2 + 20)*sqrt(a^2*x^2 - 1))/a^3
```

Sympy [A] time = 2.59883, size = 138, normalized size = 0.89

$$\begin{cases} \frac{x^3\operatorname{acosh}^3(ax)}{3} + \frac{2x^3\operatorname{acosh}(ax)}{9} - \frac{x^2\sqrt{a^2x^2-1}\operatorname{acosh}^2(ax)}{3a} - \frac{2x^2\sqrt{a^2x^2-1}}{27a} + \frac{4x\operatorname{acosh}(ax)}{3a^2} - \frac{2\sqrt{a^2x^2-1}\operatorname{acosh}^2(ax)}{3a^3} - \frac{40\sqrt{a^2x^2-1}}{27a^3} & \text{for } a \neq 0 \\ -\frac{i\pi^3x^3}{24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)**3,x)
```

```
[Out] Piecewise((x**3*acosh(a*x)**3/3 + 2*x**3*acosh(a*x)/9 - x**2*sqrt(a**2*x**2
 - 1)*acosh(a*x)**2/(3*a) - 2*x**2*sqrt(a**2*x**2 - 1)/(27*a) + 4*x*acosh(a
*x)/(3*a**2) - 2*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(3*a**3) - 40*sqrt(a**2*
x**2 - 1)/(27*a**3), Ne(a, 0)), (-I*pi**3*x**3/24, True))
```

Giac [A] time = 1.58303, size = 190, normalized size = 1.23

$$\frac{1}{3}x^3\log\left(ax+\sqrt{a^2x^2-1}\right)^3-\frac{1}{27}a\left(\frac{9\left(\left(a^2x^2-1\right)^{\frac{3}{2}}+3\sqrt{a^2x^2-1}\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^2}{a^4}-\frac{2\left(3\left(a^2x^3+6x\right)\log\left(ax+\sqrt{\right.\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(a*x + sqrt(a^2*x^2 - 1))^3 - 1/27*a*(9*((a^2*x^2 - 1)^(3/2) + 3
*sqrt(a^2*x^2 - 1))*log(a*x + sqrt(a^2*x^2 - 1))^2/a^4 - 2*(3*(a^2*x^3 + 6*
x)*log(a*x + sqrt(a^2*x^2 - 1)) - ((a^2*x^2 - 1)^(3/2) + 21*sqrt(a^2*x^2 -
1))/a)/a^3)
```

3.25 $\int x \cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=107

$$-\frac{\cosh^{-1}(ax)^3}{4a^2}-\frac{3\cosh^{-1}(ax)}{8a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^3+\frac{3}{4}x^2\cosh^{-1}(ax)-\frac{3x\sqrt{ax-1}\sqrt{ax+1}}{8a}-\frac{3x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(a}{4a}$$

```
[Out] (-3*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(8*a) - (3*ArcCosh[a*x])/(8*a^2) + (3*x
^2*ArcCosh[a*x])/4 - (3*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(4*a
) - ArcCosh[a*x]^3/(4*a^2) + (x^2*ArcCosh[a*x]^3)/2
```

Rubi [A] time = 0.380996, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5662, 5759, 5676, 90, 52}

$$-\frac{\cosh^{-1}(ax)^3}{4a^2}-\frac{3\cosh^{-1}(ax)}{8a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^3+\frac{3}{4}x^2\cosh^{-1}(ax)-\frac{3x\sqrt{ax-1}\sqrt{ax+1}}{8a}-\frac{3x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(a}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCosh[a*x]^3,x]
```

```
[Out] (-3*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(8*a) - (3*ArcCosh[a*x])/(8*a^2) + (3*x
^2*ArcCosh[a*x])/4 - (3*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(4*a
) - ArcCosh[a*x]^3/(4*a^2) + (x^2*ArcCosh[a*x]^3)/2
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
```

```
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int x\cosh^{-1}(ax)^3\,dx = \frac{1}{2}x^2\cosh^{-1}(ax)^3 - \frac{1}{2}(3a)\int \frac{x^2\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{3x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4a} + \frac{1}{2}x^2\cosh^{-1}(ax)^3 + \frac{3}{2}\int x\cosh^{-1}(ax)\,dx - \frac{3\int \frac{\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}}{4a}$$

$$= \frac{3}{4}x^2\cosh^{-1}(ax) - \frac{3x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4a} - \frac{\cosh^{-1}(ax)^3}{4a^2} + \frac{1}{2}x^2\cosh^{-1}(ax)^3 - \frac{1}{4}(3a)$$

$$= -\frac{3x\sqrt{-1+ax}\sqrt{1+ax}}{8a} + \frac{3}{4}x^2\cosh^{-1}(ax) - \frac{3x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4a} - \frac{\cosh^{-1}(ax)^3}{4a^2} + \frac{1}{2}$$

$$= -\frac{3x\sqrt{-1+ax}\sqrt{1+ax}}{8a} - \frac{3\cosh^{-1}(ax)}{8a^2} + \frac{3}{4}x^2\cosh^{-1}(ax) - \frac{3x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4a} -$$

Mathematica [A] time = 0.0816682, size = 113, normalized size = 1.06

$$\frac{\left(4a^2x^2-2\right)\cosh^{-1}(ax)^3+6a^2x^2\cosh^{-1}(ax)-3\left(ax\sqrt{ax-1}\sqrt{ax+1}+\log\left(ax+\sqrt{ax-1}\sqrt{ax+1}\right)\right)-6ax\sqrt{ax-1}\sqrt{ax+}}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCosh[a*x]^3,x]
```

```
[Out] (6*a^2*x^2*ArcCosh[a*x] - 6*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2
 + (-2 + 4*a^2*x^2)*ArcCosh[a*x]^3 - 3*(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x] +
Log[a*x + Sqrt[-1 + a*x]*Sqrt[1 + a*x]]))/(8*a^2)
```

Maple [A] time = 0.03, size = 88, normalized size = 0.8

$$\frac{1}{a^2}\left(\frac{(\operatorname{arccosh}(ax))^3a^2x^2}{2}-\frac{3(\operatorname{arccosh}(ax))^2ax}{4}\sqrt{ax-1}\sqrt{ax+1}-\frac{(\operatorname{arccosh}(ax))^3}{4}+\frac{3a^2x^2\operatorname{arccosh}(ax)}{4}-\frac{3ax}{8}\sqrt{ax-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^3,x)
```

```
[Out] 1/a^2*(1/2*arccosh(a*x)^3*a^2*x^2-3/4*arccosh(a*x)^2*a*x*(a*x-1)^(1/2)*(a*x
+1)^(1/2)-1/4*arccosh(a*x)^3+3/4*a^2*x^2*arccosh(a*x)-3/8*(a*x+1)^(1/2)*(a*
x-1)^(1/2)*a*x-3/8*arccosh(a*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^3-\int\frac{3\left(a^3x^4+\sqrt{ax+1}\sqrt{ax-1}a^2x^3-ax^2\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^2}{2\left(a^3x^3+\left(a^2x^2-1\right)\sqrt{ax+1}\sqrt{ax-1}-ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3 - integrate(3/2*(a^3*x^4 +
 sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x^3 - a*x^2)*log(a*x + sqrt(a*x + 1)*sqrt(
```

```
a*x - 1))^2/(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x), x)
```

Fricas [A] time = 2.45023, size = 261, normalized size = 2.44

$$-\frac{6\sqrt{a^2x^2-1}ax\log\left(ax+\sqrt{a^2x^2-1}\right)^2-2\left(2a^2x^2-1\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^3+3\sqrt{a^2x^2-1}ax-3\left(2a^2x^2-1\right)\log\left(ax\right.}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*sqrt(a^2*x^2 - 1)*a*x*log(a*x + sqrt(a^2*x^2 - 1))^2 - 2*(2*a^2*x^2
 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^3 + 3*sqrt(a^2*x^2 - 1)*a*x - 3*(2*a^2*x
^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1)))/a^2
```

Sympy [A] time = 1.25823, size = 102, normalized size = 0.95

$$\begin{cases} \frac{x^2\operatorname{acosh}^3(ax)}{2}+\frac{3x^2\operatorname{acosh}(ax)}{4}-\frac{3x\sqrt{a^2x^2-1}\operatorname{acosh}^2(ax)}{4a}-\frac{3x\sqrt{a^2x^2-1}}{8a}-\frac{\operatorname{acosh}^3(ax)}{4a^2}-\frac{3\operatorname{acosh}(ax)}{8a^2} & \text{for } a\neq 0 \\ -\frac{i\pi^3x^2}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**3,x)
```

```
[Out] Piecewise((x**2*acosh(a*x)**3/2 + 3*x**2*acosh(a*x)/4 - 3*x*sqrt(a**2*x**2
- 1)*acosh(a*x)**2/(4*a) - 3*x*sqrt(a**2*x**2 - 1)/(8*a) - acosh(a*x)**3/(4
*a**2) - 3*acosh(a*x)/(8*a**2), Ne(a, 0)), (-I*pi**3*x**2/16, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{arcosh}(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arccosh(a*x)^3, x)
```

3.26 $\int \cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=68

$$-\frac{6\sqrt{ax-1}\sqrt{ax+1}}{a}+x\cosh^{-1}(ax)^3-\frac{3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{a}+6x\cosh^{-1}(ax)$$

```
[Out] (-6*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/a + 6*x*ArcCosh[a*x] - (3*Sqrt[-1 + a*x]*
Sqrt[1 + a*x]*ArcCosh[a*x]^2)/a + x*ArcCosh[a*x]^3
```

Rubi [A] time = 0.183306, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5654, 5718, 74}

$$-\frac{6\sqrt{ax-1}\sqrt{ax+1}}{a}+x\cosh^{-1}(ax)^3-\frac{3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{a}+6x\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3,x]
```

```
[Out] (-6*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/a + 6*x*ArcCosh[a*x] - (3*Sqrt[-1 + a*x]*
Sqrt[1 + a*x]*ArcCosh[a*x]^2)/a + x*ArcCosh[a*x]^3
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$
\begin{aligned}
\int \cosh^{-1}(ax)^3\,dx &= x\cosh^{-1}(ax)^3-(3a)\int \frac{x\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= -\frac{3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{a}+x\cosh^{-1}(ax)^3+6\int \cosh^{-1}(ax)\,dx\\
&= 6x\cosh^{-1}(ax)-\frac{3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{a}+x\cosh^{-1}(ax)^3-(6a)\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= -\frac{6\sqrt{-1+ax}\sqrt{1+ax}}{a}+6x\cosh^{-1}(ax)-\frac{3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{a}+x\cosh^{-1}(ax)^3
\end{aligned}
$$

Mathematica [A] time = 0.0254505, size = 68, normalized size = 1.

$$-\frac{6\sqrt{ax-1}\sqrt{ax+1}}{a}+x\cosh^{-1}(ax)^3-\frac{3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{a}+6x\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^3,x]
```

```
[Out] (-6*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/a + 6*x*ArcCosh[a*x] - (3*Sqrt[-1 + a*x]*
Sqrt[1 + a*x]*ArcCosh[a*x]^2)/a + x*ArcCosh[a*x]^3
```

Maple [A] time = 0.031, size = 61, normalized size = 0.9

$$\frac{1}{a}\left((\mathrm{arccosh}\,(ax))^3\,ax-3\,(\mathrm{arccosh}\,(ax))^2\,\sqrt{ax-1}\sqrt{ax+1}+6\,ax\mathrm{arccosh}\,(ax)-6\,\sqrt{ax-1}\sqrt{ax+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3,x)
```

```
[Out] 1/a*(arccosh(a*x)^3*a*x-3*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+6*a*x*
arccosh(a*x)-6*(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

Maxima [A] time = 1.20984, size = 77, normalized size = 1.13

$$x \operatorname{arcosh}(ax)^3 - \frac{3\sqrt{a^2x^2-1}\operatorname{arcosh}(ax)^2}{a} + \frac{6\left(ax\operatorname{arcosh}(ax) - \sqrt{a^2x^2-1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] x*arccosh(a*x)^3 - 3*sqrt(a^2*x^2 - 1)*arccosh(a*x)^2/a + 6*(a*x*arccosh(a*
x) - sqrt(a^2*x^2 - 1))/a
```

Fricas [A] time = 2.47585, size = 205, normalized size = 3.01

$$\frac{ax\log\left(ax+\sqrt{a^2x^2-1}\right)^3 + 6\,ax\log\left(ax+\sqrt{a^2x^2-1}\right) - 3\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)^2 - 6\sqrt{a^2x^2-1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] (a*x*log(a*x + sqrt(a^2*x^2 - 1))^3 + 6*a*x*log(a*x + sqrt(a^2*x^2 - 1)) -
3*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^2 - 6*sqrt(a^2*x^2 - 1))/a
```

Sympy [A] time = 0.570065, size = 63, normalized size = 0.93

$$\begin{cases} x\operatorname{acosh}^3(ax) + 6x\operatorname{acosh}(ax) - \frac{3\sqrt{a^2x^2-1}\operatorname{acosh}^2(ax)}{a} - \frac{6\sqrt{a^2x^2-1}}{a} & \text{for } a \neq 0 \\ -\frac{i\pi^3 x}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3,x)
```

```
[Out] Piecewise((x*acosh(a*x)**3 + 6*x*acosh(a*x) - 3*sqrt(a**2*x**2 - 1)*acosh(a
*x)**2/a - 6*sqrt(a**2*x**2 - 1)/a, Ne(a, 0)), (-I*pi**3*x/8, True))
```

Giac [A] time = 1.36205, size = 132, normalized size = 1.94

$$x\log\left(ax+\sqrt{a^2x^2-1}\right)^3-3\,a\left(\frac{\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)^2}{a^2}-\frac{2\left(x\log\left(ax+\sqrt{a^2x^2-1}\right)-\frac{\sqrt{a^2x^2-1}}{a}\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 - 1))^3 - 3*a*(sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^
2*x^2 - 1))^2/a^2 - 2*(x*log(a*x + sqrt(a^2*x^2 - 1)) - sqrt(a^2*x^2 - 1)/a
)/a)
```

3.27 $\int \frac{\cosh^{-1}(ax)^3}{x}\,dx$

Optimal. Leaf size=87

$$\frac{3}{2}\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)-\frac{3}{2}\cosh^{-1}(ax)\text{PolyLog}\left(3,-e^{2\cosh^{-1}(ax)}\right)+\frac{3}{4}\text{PolyLog}\left(4,-e^{2\cosh^{-1}(ax)}\right)-\frac{1}{4}$$

```
[Out] -ArcCosh[a*x]^4/4 + ArcCosh[a*x]^3*Log[1 + E^(2*ArcCosh[a*x])] + (3*ArcCosh
[a*x]^2*PolyLog[2, -E^(2*ArcCosh[a*x])])/2 - (3*ArcCosh[a*x]*PolyLog[3, -E^
(2*ArcCosh[a*x])])/2 + (3*PolyLog[4, -E^(2*ArcCosh[a*x])])/4
```

Rubi [A] time = 0.105491, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5660, 3718, 2190, 2531, 6609, 2282, 6589}

$$\frac{3}{2}\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)-\frac{3}{2}\cosh^{-1}(ax)\text{PolyLog}\left(3,-e^{2\cosh^{-1}(ax)}\right)+\frac{3}{4}\text{PolyLog}\left(4,-e^{2\cosh^{-1}(ax)}\right)-\frac{1}{4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/x,x]
```

```
[Out] -ArcCosh[a*x]^4/4 + ArcCosh[a*x]^3*Log[1 + E^(2*ArcCosh[a*x])] + (3*ArcCosh
[a*x]^2*PolyLog[2, -E^(2*ArcCosh[a*x])])/2 - (3*ArcCosh[a*x]*PolyLog[3, -E^
(2*ArcCosh[a*x])])/2 + (3*PolyLog[4, -E^(2*ArcCosh[a*x])])/4
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{x}\,dx = \text{Subst}\left(\int x^3\tanh(x)\,dx, x, \cosh^{-1}(ax)\right)$$
$$= -\frac{1}{4}\cosh^{-1}(ax)^4 + 2\,\text{Subst}\left(\int \frac{e^{2x}x^3}{1+e^{2x}}\,dx, x, \cosh^{-1}(ax)\right)$$
$$= -\frac{1}{4}\cosh^{-1}(ax)^4 + \cosh^{-1}(ax)^3\log\left(1+e^{2\cosh^{-1}(ax)}\right) - 3\,\text{Subst}\left(\int x^2\log\left(1+e^{2x}\right)\,dx, x, \cosh^{-1}(a$$
$$= -\frac{1}{4}\cosh^{-1}(ax)^4 + \cosh^{-1}(ax)^3\log\left(1+e^{2\cosh^{-1}(ax)}\right) + \frac{3}{2}\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right) - 3\,\text{Subst}$$
$$= -\frac{1}{4}\cosh^{-1}(ax)^4 + \cosh^{-1}(ax)^3\log\left(1+e^{2\cosh^{-1}(ax)}\right) + \frac{3}{2}\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right) - \frac{3}{2}\cosh$$
$$= -\frac{1}{4}\cosh^{-1}(ax)^4 + \cosh^{-1}(ax)^3\log\left(1+e^{2\cosh^{-1}(ax)}\right) + \frac{3}{2}\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right) - \frac{3}{2}\cosh$$
$$= -\frac{1}{4}\cosh^{-1}(ax)^4 + \cosh^{-1}(ax)^3\log\left(1+e^{2\cosh^{-1}(ax)}\right) + \frac{3}{2}\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right) - \frac{3}{2}\cosh$$

Mathematica [A] time = 0.0496696, size = 82, normalized size = 0.94

$$\frac{1}{4}\left(-6\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(ax)}\right) - 6\cosh^{-1}(ax)\text{PolyLog}\left(3,-e^{-2\cosh^{-1}(ax)}\right) - 3\text{PolyLog}\left(4,-e^{-2\cosh^{-1}(ax}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/x,x]
```

```
[Out] (ArcCosh[a*x]^4 + 4*ArcCosh[a*x]^3*Log[1 + E^(-2*ArcCosh[a*x])] - 6*ArcCosh
[a*x]^2*PolyLog[2, -E^(-2*ArcCosh[a*x])] - 6*ArcCosh[a*x]*PolyLog[3, -E^(-2
*ArcCosh[a*x])] - 3*PolyLog[4, -E^(-2*ArcCosh[a*x])])/4
```

Maple [A] time = 0.037, size = 132, normalized size = 1.5

$$-\frac{(\text{arccosh}\,(ax))^4}{4} + (\text{arccosh}\,(ax))^3\ln\left(1+\left(ax+\sqrt{ax-1}\sqrt{ax+1}\right)^2\right) + \frac{3\,(\text{arccosh}\,(ax))^2}{2}\text{polylog}\left(2,-\left(ax+\sqrt{ax-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/x,x)
```

```
[Out] -1/4*arccosh(a*x)^4+arccosh(a*x)^3*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2
)+3/2*arccosh(a*x)^2*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)-3/2*ar
```

```
ccosh(a*x)*polylog(3,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)+3/4*polylog(4,-(
a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^3/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcosh}(ax)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^3/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^3(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/x,x)
```

```
[Out] Integral(acosh(a*x)**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/x, x)
```

3.28 $\int \frac{\cosh^{-1}(ax)^3}{x^2}\,dx$

Optimal. Leaf size=104

$$-6ia\cosh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)+6ia\cosh^{-1}(ax)\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)+6ia\text{PolyLog}\left(3,-ie^{\cosh^{-1}(ax)}\right)-6i$$

```
[Out] -(ArcCosh[a*x]^3/x) + 6*a*ArcCosh[a*x]^2*ArcTan[E^ArcCosh[a*x]] - (6*I)*a*A
rcCosh[a*x]*PolyLog[2, (-I)*E^ArcCosh[a*x]] + (6*I)*a*ArcCosh[a*x]*PolyLog[
2, I*E^ArcCosh[a*x]] + (6*I)*a*PolyLog[3, (-I)*E^ArcCosh[a*x]] - (6*I)*a*Po
lyLog[3, I*E^ArcCosh[a*x]]
```

Rubi [A] time = 0.310209, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5662, 5761, 4180, 2531, 2282, 6589}

$$-6ia\cosh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)+6ia\cosh^{-1}(ax)\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)+6ia\text{PolyLog}\left(3,-ie^{\cosh^{-1}(ax)}\right)-6i$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/x^2,x]
```

```
[Out] -(ArcCosh[a*x]^3/x) + 6*a*ArcCosh[a*x]^2*ArcTan[E^ArcCosh[a*x]] - (6*I)*a*A
rcCosh[a*x]*PolyLog[2, (-I)*E^ArcCosh[a*x]] + (6*I)*a*ArcCosh[a*x]*PolyLog[
2, I*E^ArcCosh[a*x]] + (6*I)*a*PolyLog[3, (-I)*E^ArcCosh[a*x]] - (6*I)*a*Po
lyLog[3, I*E^ArcCosh[a*x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
```

```
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{x^2}\,dx = -\frac{\cosh^{-1}(ax)^3}{x} + (3a)\int \frac{\cosh^{-1}(ax)^2}{x\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$
$$= -\frac{\cosh^{-1}(ax)^3}{x} + (3a)\,\text{Subst}\left(\int x^2\mathbf{sech}(x)\,dx, x, \cosh^{-1}(ax)\right)$$
$$= -\frac{\cosh^{-1}(ax)^3}{x} + 6a\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - (6ia)\,\text{Subst}\left(\int x\log\left(1-ie^x\right)\,dx, x, \cosh^{-1}(ax\right.$$
$$= -\frac{\cosh^{-1}(ax)^3}{x} + 6a\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 6ia\cosh^{-1}(ax)\mathbf{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) + 6ia\cosh^{-1}($$
$$= -\frac{\cosh^{-1}(ax)^3}{x} + 6a\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 6ia\cosh^{-1}(ax)\mathbf{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) + 6ia\cosh^{-1}($$
$$= -\frac{\cosh^{-1}(ax)^3}{x} + 6a\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 6ia\cosh^{-1}(ax)\mathbf{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) + 6ia\cosh^{-1}($$

Mathematica [A] time = 0.167938, size = 128, normalized size = 1.23

$$-\frac{\cosh^{-1}(ax)^3}{x} + 3ia\left(-2\cosh^{-1}(ax)\left(\text{PolyLog}\left(2, -ie^{-\cosh^{-1}(ax)}\right) - \text{PolyLog}\left(2, ie^{-\cosh^{-1}(ax)}\right)\right) - 2\text{PolyLog}\left(3, -ie^{-\cosh^{-1}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/x^2,x]
```

```
[Out] -(ArcCosh[a*x]^3/x) + (3*I)*a*(-(ArcCosh[a*x]^2*(Log[1 - I/E^ArcCosh[a*x]]
- Log[1 + I/E^ArcCosh[a*x]])) - 2*ArcCosh[a*x]*(PolyLog[2, (-I)/E^ArcCosh[a
*x]] - PolyLog[2, I/E^ArcCosh[a*x]]) - 2*PolyLog[3, (-I)/E^ArcCosh[a*x]] +
2*PolyLog[3, I/E^ArcCosh[a*x]])
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int \frac{\left(\text{arccosh}\,(ax)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/x^2,x)
```

```
[Out] int(arccosh(a*x)^3/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^3}{x}+\int\frac{3\left(a^3x^2+\sqrt{ax+1}\sqrt{ax-1}a^2x-a\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^2}{a^3x^4-ax^2+\left(a^2x^3-x\right)\sqrt{ax+1}\sqrt{ax-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] -log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/x + integrate(3*(a^3*x^2 + sqrt(a
*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/(
a^3*x^4 - a*x^2 + (a^2*x^3 - x)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcosh}(ax)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^3/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acosh}^3(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/x**2,x)
```

```
[Out] Integral(acosh(a*x)**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/x^2, x)
```

3.29 $\int \frac{\cosh^{-1}(ax)^3}{x^3}\,dx$

Optimal. Leaf size=98

$$-\frac{3}{2}a^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)+\frac{3}{2}a^2\cosh^{-1}(ax)^2-3a^2\cosh^{-1}(ax)\log\left(e^{2\cosh^{-1}(ax)}+1\right)-\frac{\cosh^{-1}(ax)^3}{2x^2}+\frac{3a\sqrt{ax-1}}{}$$

```
[Out] (3*a^2*ArcCosh[a*x]^2)/2 + (3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2
)/(2*x) - ArcCosh[a*x]^3/(2*x^2) - 3*a^2*ArcCosh[a*x]*Log[1 + E^(2*ArcCosh[
a*x])] - (3*a^2*PolyLog[2, -E^(2*ArcCosh[a*x])])/2
```

Rubi [A] time = 0.31681, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5662, 5724, 5660, 3718, 2190, 2279, 2391}

$$-\frac{3}{2}a^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)+\frac{3}{2}a^2\cosh^{-1}(ax)^2-3a^2\cosh^{-1}(ax)\log\left(e^{2\cosh^{-1}(ax)}+1\right)-\frac{\cosh^{-1}(ax)^3}{2x^2}+\frac{3a\sqrt{ax-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/x^3,x]
```

```
[Out] (3*a^2*ArcCosh[a*x]^2)/2 + (3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2
)/(2*x) - ArcCosh[a*x]^3/(2*x^2) - 3*a^2*ArcCosh[a*x]*Log[1 + E^(2*ArcCosh[
a*x])] - (3*a^2*PolyLog[2, -E^(2*ArcCosh[a*x])])/2
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
```

```
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}(ax)^3}{x^3}\,dx &= -\frac{\cosh^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3a)\int \frac{\cosh^{-1}(ax)^2}{x^2\sqrt{-1+ax}\sqrt{1+ax}}\,dx \\
&= \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x} - \frac{\cosh^{-1}(ax)^3}{2x^2} - \left(3a^2\right)\int \frac{\cosh^{-1}(ax)}{x}\,dx \\
&= \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x} - \frac{\cosh^{-1}(ax)^3}{2x^2} - \left(3a^2\right)\mathrm{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}(ax)\right) \\
&= \frac{3}{2}a^2\cosh^{-1}(ax)^2 + \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x} - \frac{\cosh^{-1}(ax)^3}{2x^2} - \left(6a^2\right)\mathrm{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx\right. \\
&= \frac{3}{2}a^2\cosh^{-1}(ax)^2 + \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x} - \frac{\cosh^{-1}(ax)^3}{2x^2} - 3a^2\cosh^{-1}(ax)\log\left(1+e^{2\cdots}\right. \\
&= \frac{3}{2}a^2\cosh^{-1}(ax)^2 + \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x} - \frac{\cosh^{-1}(ax)^3}{2x^2} - 3a^2\cosh^{-1}(ax)\log\left(1+e^{2\cdots}\right. \\
&= \frac{3}{2}a^2\cosh^{-1}(ax)^2 + \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x} - \frac{\cosh^{-1}(ax)^3}{2x^2} - 3a^2\cosh^{-1}(ax)\log\left(1+e^{2\cdots}\right.
\end{aligned}
$$

Mathematica [A] time = 0.793931, size = 92, normalized size = 0.94

$$
\frac{3}{2}a^2\left(\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(ax)}\right)+\cosh^{-1}(ax)\left(\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\cosh^{-1}(ax)}{ax}-\cosh^{-1}(ax)-2\log\left(e^{-2\cosh^{-1}(ax)}+1\right)\right)\right)-
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/x^3,x]
```

```
[Out] -ArcCosh[a*x]^3/(2*x^2) + (3*a^2*(ArcCosh[a*x]*(-ArcCosh[a*x] + (Sqrt[(-1 +
 a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x])/(a*x) - 2*Log[1 + E^(-2*ArcCosh[a*
x])]) + PolyLog[2, -E^(-2*ArcCosh[a*x])]))/2
```

Maple [A] time = 0.082, size = 113, normalized size = 1.2

$$
\frac{3a^2(\mathrm{arccosh}(ax))^2}{2} - \frac{(\mathrm{arccosh}(ax))^3}{2x^2} - 3a^2\mathrm{arccosh}(ax)\ln\left(1+\left(ax+\sqrt{ax-1}\sqrt{ax+1}\right)^2\right) - \frac{3a^2}{2}\mathrm{polylog}\left(2,-\left(ax+\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/x^3,x)
```

```
[Out] 3/2*a^2*arccosh(a*x)^2-1/2*arccosh(a*x)^3/x^2-3*a^2*arccosh(a*x)*ln(1+(a*x+
(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)-3/2*a^2*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+
1)^(1/2))^2)+3/2*a*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^3}{2\,x^2}+\int\frac{3\left(a^3x^2+\sqrt{ax+1}\sqrt{ax-1}a^2x-a\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^2}{2\left(a^3x^5-ax^3+\left(a^2x^4-x^2\right)\sqrt{ax+1}\sqrt{ax-1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/x^2 + integrate(3/2*(a^3*x^2
+ sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x -
 1))^2/(a^3*x^5 - a*x^3 + (a^2*x^4 - x^2)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{arcosh}\left(ax\right)^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^3/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acosh}^3\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/x**3,x)
```

```
[Out] Integral(acosh(a*x)**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/x^3, x)
```

3.30 $\int \frac{\cosh^{-1}(ax)^3}{x^4} dx$

Optimal. Leaf size=183

$$-ia^3\cosh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)+ia^3\cosh^{-1}(ax)\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)+ia^3\text{PolyLog}\left(3,-ie^{\cosh^{-1}(ax)}\right)-ia^3$$

```
[Out] (a^2*ArcCosh[a*x])/x + (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(2*x
^2) - ArcCosh[a*x]^3/(3*x^3) + a^3*ArcCosh[a*x]^2*ArcTan[E^ArcCosh[a*x]] -
a^3*ArcTan[Sqrt[-1 + a*x]*Sqrt[1 + a*x]] - I*a^3*ArcCosh[a*x]*PolyLog[2, (-
I)*E^ArcCosh[a*x]] + I*a^3*ArcCosh[a*x]*PolyLog[2, I*E^ArcCosh[a*x]] + I*a^
3*PolyLog[3, (-I)*E^ArcCosh[a*x]] - I*a^3*PolyLog[3, I*E^ArcCosh[a*x]]
```

Rubi [A] time = 0.5773, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.9$, Rules used = {5662, 5748, 5761, 4180, 2531, 2282, 6589, 92, 205}

$$-ia^3\cosh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)+ia^3\cosh^{-1}(ax)\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)+ia^3\text{PolyLog}\left(3,-ie^{\cosh^{-1}(ax)}\right)-ia^3$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/x^4,x]
```


```
[Out] (a^2*ArcCosh[a*x])/x + (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(2*x
^2) - ArcCosh[a*x]^3/(3*x^3) + a^3*ArcCosh[a*x]^2*ArcTan[E^ArcCosh[a*x]] -
a^3*ArcTan[Sqrt[-1 + a*x]*Sqrt[1 + a*x]] - I*a^3*ArcCosh[a*x]*PolyLog[2, (-
I)*E^ArcCosh[a*x]] + I*a^3*ArcCosh[a*x]*PolyLog[2, I*E^ArcCosh[a*x]] + I*a^
3*PolyLog[3, (-I)*E^ArcCosh[a*x]] - I*a^3*PolyLog[3, I*E^ArcCosh[a*x]]
```


Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
```

```
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```

```
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)^3}{x^4}\,dx &= -\frac{\cosh^{-1}(ax)^3}{3x^3} + a\int \frac{\cosh^{-1}(ax)^2}{x^3\sqrt{-1+ax}\sqrt{1+ax}}\,dx \\
&= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x^2} - \frac{\cosh^{-1}(ax)^3}{3x^3} - a^2\int \frac{\cosh^{-1}(ax)}{x^2}\,dx + \frac{1}{2}a^3\int \frac{\cosh^{-1}(ax)^2}{x\sqrt{-1+ax}\sqrt{1+ax}} \\
&= \frac{a^2\cosh^{-1}(ax)}{x} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x^2} - \frac{\cosh^{-1}(ax)^3}{3x^3} + \frac{1}{2}a^3\,\mathrm{Subst}\left(\int x^2\mathbf{sech}(x)\,dx, x,\right. \\
&= \frac{a^2\cosh^{-1}(ax)}{x} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x^2} - \frac{\cosh^{-1}(ax)^3}{3x^3} + a^3\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right. \\
&= \frac{a^2\cosh^{-1}(ax)}{x} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x^2} - \frac{\cosh^{-1}(ax)^3}{3x^3} + a^3\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right. \\
&= \frac{a^2\cosh^{-1}(ax)}{x} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x^2} - \frac{\cosh^{-1}(ax)^3}{3x^3} + a^3\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right. \\
&= \frac{a^2\cosh^{-1}(ax)}{x} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x^2} - \frac{\cosh^{-1}(ax)^3}{3x^3} + a^3\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right.
\end{aligned}$$

Mathematica [A] time = 0.545972, size = 201, normalized size = 1.1

$$\frac{1}{6}\left(-3ia^3\left(2\cosh^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{-\cosh^{-1}(ax)}\right)-2\cosh^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{-\cosh^{-1}(ax)}\right)+2\mathrm{PolyLog}\left(3,-ie^{-\cosh^{-1}(a}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/x^4,x]
```

```
[Out] ((6*a^2*ArcCosh[a*x])/x + (3*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh
[a*x]^2)/x^2 - (2*ArcCosh[a*x]^3)/x^3 - (3*I)*a^3*((-4*I)*ArcTan[Tanh[ArcCo
sh[a*x]/2]] + ArcCosh[a*x]^2*Log[1 - I/E^ArcCosh[a*x]] - ArcCosh[a*x]^2*Log
[1 + I/E^ArcCosh[a*x]] + 2*ArcCosh[a*x]*PolyLog[2, (-I)/E^ArcCosh[a*x]] - 2
*ArcCosh[a*x]*PolyLog[2, I/E^ArcCosh[a*x]] + 2*PolyLog[3, (-I)/E^ArcCosh[a*
x]] - 2*PolyLog[3, I/E^ArcCosh[a*x]]))/6
```

Maple [F] time = 0.169, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{arccosh}(ax))^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/x^4,x)
```

```
[Out] int(arccosh(a*x)^3/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^3}{3\,x^3}+\int \frac{\left(a^3x^2+\sqrt{ax+1}\sqrt{ax-1}a^2x-a\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^2}{a^3x^6-ax^4+\left(a^2x^5-x^3\right)\sqrt{ax+1}\sqrt{ax-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/x^3 + integrate((a^3*x^2 + sq
rt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))
^2/(a^3*x^6 - a*x^4 + (a^2*x^5 - x^3)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcosh}(ax)^3}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^3/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^3(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/x**4,x)
```

```
[Out] Integral(acosh(a*x)**3/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/x^4, x)
```

3.31 $\int \frac{\cosh^{-1}(ax)^3}{x^5}\,dx$

Optimal. Leaf size=174

$$-\frac{1}{2}a^4\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)+\frac{a^2\cosh^{-1}(ax)}{4x^2}-\frac{a^3\sqrt{ax-1}\sqrt{ax+1}}{4x}+\frac{1}{2}a^4\cosh^{-1}(ax)^2+\frac{a^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}}{2x}$$

```
[Out] -(a^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(4*x) + (a^2*ArcCosh[a*x])/(4*x^2) + (a
^4*ArcCosh[a*x]^2)/2 + (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(4*x
^3) + (a^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(2*x) - ArcCosh[a*x
]^3/(4*x^4) - a^4*ArcCosh[a*x]*Log[1 + E^(2*ArcCosh[a*x])] - (a^4*PolyLog[2
, -E^(2*ArcCosh[a*x])])/2
```

Rubi [A] time = 0.577703, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {5662, 5748, 5724, 5660, 3718, 2190, 2279, 2391, 95}

$$-\frac{1}{2}a^4\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)+\frac{a^2\cosh^{-1}(ax)}{4x^2}-\frac{a^3\sqrt{ax-1}\sqrt{ax+1}}{4x}+\frac{1}{2}a^4\cosh^{-1}(ax)^2+\frac{a^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/x^5,x]
```

```
[Out] -(a^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(4*x) + (a^2*ArcCosh[a*x])/(4*x^2) + (a
^4*ArcCosh[a*x]^2)/2 + (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(4*x
^3) + (a^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(2*x) - ArcCosh[a*x
]^3/(4*x^4) - a^4*ArcCosh[a*x]*Log[1 + E^(2*ArcCosh[a*x])] - (a^4*PolyLog[2
, -E^(2*ArcCosh[a*x])])/2
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
```

```
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
```

```
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{x^5}\,dx = -\frac{\cosh^{-1}(ax)^3}{4x^4} + \frac{1}{4}(3a)\int \frac{\cosh^{-1}(ax)^2}{x^4\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4x^3} - \frac{\cosh^{-1}(ax)^3}{4x^4} - \frac{1}{2}a^2\int \frac{\cosh^{-1}(ax)}{x^3}\,dx + \frac{1}{2}a^3\int \frac{\cosh^{-1}(ax)}{x^2\sqrt{-1+ax}\sqrt{1}}$$

$$= \frac{a^2\cosh^{-1}(ax)}{4x^2} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x} - \frac{\cosh^{-1}(a}{4x^4}$$

$$= -\frac{a^3\sqrt{-1+ax}\sqrt{1+ax}}{4x} + \frac{a^2\cosh^{-1}(ax)}{4x^2} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{-1+ax}\sqrt{1+ax}}{2x}$$

$$= -\frac{a^3\sqrt{-1+ax}\sqrt{1+ax}}{4x} + \frac{a^2\cosh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\cosh^{-1}(ax)^2 + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4x^3} + \frac{a}{}$$

$$= -\frac{a^3\sqrt{-1+ax}\sqrt{1+ax}}{4x} + \frac{a^2\cosh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\cosh^{-1}(ax)^2 + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4x^3} + \frac{a}{}$$

$$= -\frac{a^3\sqrt{-1+ax}\sqrt{1+ax}}{4x} + \frac{a^2\cosh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\cosh^{-1}(ax)^2 + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4x^3} + \frac{a}{}$$

$$= -\frac{a^3\sqrt{-1+ax}\sqrt{1+ax}}{4x} + \frac{a^2\cosh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\cosh^{-1}(ax)^2 + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4x^3} + \frac{a}{}$$

Mathematica [A] time = 0.598095, size = 220, normalized size = 1.26

$$\frac{2a^4x^4\sqrt{\frac{ax-1}{ax+1}}(ax+1)\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(ax)}\right) - a^5x^5 + a^3x^3 - ax(ax+1)\left(2a^3x^3\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+2a^2x^2-ax+1\right)\cosh}{4x^4\sqrt{ax-1}\sqrt{ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/x^5,x]
```

```
[Out] (a^3*x^3 - a^5*x^5 - a*x*(1 + a*x)*(1 - a*x + 2*a^2*x^2 + 2*a^3*x^3*(-1 + S
qrt[(-1 + a*x)/(1 + a*x)]))*ArcCosh[a*x]^2 - Sqrt[-1 + a*x]*Sqrt[1 + a*x]*A
rcCosh[a*x]^3 - a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x]*(
-1 + 4*a^2*x^2*Log[1 + E^(-2*ArcCosh[a*x])]) + 2*a^4*x^4*Sqrt[(-1 + a*x)/(1
 + a*x)]*(1 + a*x)*PolyLog[2, -E^(-2*ArcCosh[a*x])])/(4*x^4*Sqrt[-1 + a*x]*
Sqrt[1 + a*x])
```

Maple [A] time = 0.122, size = 180, normalized size = 1.

$$\frac{a^3\left(\operatorname{arccosh}(ax)\right)^2}{2x}\sqrt{ax-1}\sqrt{ax+1}+\frac{a^4\left(\operatorname{arccosh}(ax)\right)^2}{2}-\frac{a^3}{4x}\sqrt{ax-1}\sqrt{ax+1}+\frac{a^4}{4}+\frac{a\left(\operatorname{arccosh}(ax)\right)^2}{4x^3}\sqrt{ax-1}\sqrt{ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/x^5,x)
```

```
[Out] 1/2*a^3*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x+1/2*a^4*arccosh(a*x)^2
-1/4*a^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x+1/4*a^4+1/4*a*arccosh(a*x)^2*(a*x-1)
^(1/2)*(a*x+1)^(1/2)/x^3+1/4*a^2*arccosh(a*x)/x^2-1/4*arccosh(a*x)^3/x^4-a^
4*arccosh(a*x)*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)-1/2*a^4*polylog(2,
-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^3}{4x^4}+\int\frac{3\left(a^3x^2+\sqrt{ax+1}\sqrt{ax-1}a^2x-a\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^2}{4\left(a^3x^7-ax^5+\left(a^2x^6-x^4\right)\sqrt{ax+1}\sqrt{ax-1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/x^4 + integrate(3/4*(a^3*x^2
+ sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x -
 1))^2/(a^3*x^7 - a*x^5 + (a^2*x^6 - x^4)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcosh}(ax)^3}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^5,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^3/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^3(ax)}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/x**5,x)
```

```
[Out] Integral(acosh(a*x)**3/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^5,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/x^5, x)
```

3.32 $\int x^5 \cosh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=306

$$\frac{65x^4}{3456a^2}+\frac{245x^2}{1152a^4}+\frac{5x^4\cosh^{-1}(ax)^2}{48a^2}-\frac{5x^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{36a^3}-\frac{65x^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{864a^3}+\frac{5x^2\cosh}{16a}$$

```
[Out] (245*x^2)/(1152*a^4) + (65*x^4)/(3456*a^2) + x^6/324 - (245*x*Sqrt[-1 + a*x
]*Sqrt[1 + a*x]*ArcCosh[a*x])/(576*a^5) - (65*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a
*x]*ArcCosh[a*x])/(864*a^3) - (x^5*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x
])/(54*a) - (245*ArcCosh[a*x]^2)/(1152*a^6) + (5*x^2*ArcCosh[a*x]^2)/(16*a^
4) + (5*x^4*ArcCosh[a*x]^2)/(48*a^2) + (x^6*ArcCosh[a*x]^2)/18 - (5*x*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(24*a^5) - (5*x^3*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]*ArcCosh[a*x]^3)/(36*a^3) - (x^5*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Ar
cCosh[a*x]^3)/(9*a) - (5*ArcCosh[a*x]^4)/(96*a^6) + (x^6*ArcCosh[a*x]^4)/6
```

Rubi [A] time = 2.19252, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5662, 5759, 5676, 30}

$$\frac{65x^4}{3456a^2}+\frac{245x^2}{1152a^4}+\frac{5x^4\cosh^{-1}(ax)^2}{48a^2}-\frac{5x^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{36a^3}-\frac{65x^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{864a^3}+\frac{5x^2\cosh}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcCosh[a*x]^4,x]
```


```
[Out] (245*x^2)/(1152*a^4) + (65*x^4)/(3456*a^2) + x^6/324 - (245*x*Sqrt[-1 + a*x
]*Sqrt[1 + a*x]*ArcCosh[a*x])/(576*a^5) - (65*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a
*x]*ArcCosh[a*x])/(864*a^3) - (x^5*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x
])/(54*a) - (245*ArcCosh[a*x]^2)/(1152*a^6) + (5*x^2*ArcCosh[a*x]^2)/(16*a^
4) + (5*x^4*ArcCosh[a*x]^2)/(48*a^2) + (x^6*ArcCosh[a*x]^2)/18 - (5*x*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(24*a^5) - (5*x^3*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]*ArcCosh[a*x]^3)/(36*a^3) - (x^5*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Ar
cCosh[a*x]^3)/(9*a) - (5*ArcCosh[a*x]^4)/(96*a^6) + (x^6*ArcCosh[a*x]^4)/6
```


Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
```

```
NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^5 \cosh^{-1}(ax)^4 \, dx = \frac{1}{6}x^6 \cosh^{-1}(ax)^4 - \frac{1}{3}(2a) \int \frac{x^6 \cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}} \, dx$$

$$= -\frac{x^5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{9a} + \frac{1}{6}x^6 \cosh^{-1}(ax)^4 + \frac{1}{3}\int x^5 \cosh^{-1}(ax)^2 \, dx - \frac{5\int \frac{x^4 \cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+a}}}{9a}$$

$$= \frac{1}{18}x^6 \cosh^{-1}(ax)^2 - \frac{5x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{36a^3} - \frac{x^5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{9a} + \frac{1}{6}x^6$$

$$= -\frac{x^5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{54a} + \frac{5x^4\cosh^{-1}(ax)^2}{48a^2} + \frac{1}{18}x^6\cosh^{-1}(ax)^2 - \frac{5x\sqrt{-1+ax}\sqrt{1+ax}}{24a^5}$$

$$= \frac{x^6}{324} - \frac{65x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{864a^3} - \frac{x^5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{54a} + \frac{5x^2\cosh^{-1}(ax)^2}{16a^4}$$

$$= \frac{65x^4}{3456a^2} + \frac{x^6}{324} - \frac{245x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{576a^5} - \frac{65x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{864a^3} - x^5\sqrt{}$$

$$= \frac{245x^2}{1152a^4} + \frac{65x^4}{3456a^2} + \frac{x^6}{324} - \frac{245x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{576a^5} - \frac{65x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(}{864a^3}$$

Mathematica [A] time = 0.152674, size = 175, normalized size = 0.57

$$\frac{a^2x^2\left(32a^4x^4+195a^2x^2+2205\right)+108\left(16a^6x^6-5\right)\cosh^{-1}(ax)^4-144ax\sqrt{ax-1}\sqrt{ax+1}\left(8a^4x^4+10a^2x^2+15\right)\cosh^{-1}(}{1036}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcCosh[a*x]^4,x]
```

```
[Out] (a^2*x^2*(2205 + 195*a^2*x^2 + 32*a^4*x^4) - 6*a*x*Sqrt[-1 + a*x]*Sqrt[1 +
a*x]*(735 + 130*a^2*x^2 + 32*a^4*x^4)*ArcCosh[a*x] + 9*(-245 + 360*a^2*x^2
+ 120*a^4*x^4 + 64*a^6*x^6)*ArcCosh[a*x]^2 - 144*a*x*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]*(15 + 10*a^2*x^2 + 8*a^4*x^4)*ArcCosh[a*x]^3 + 108*(-5 + 16*a^6*x^6)
*ArcCosh[a*x]^4)/(10368*a^6)
```

Maple [A] time = 0.052, size = 344, normalized size = 1.1

$$\frac{1}{a^6}\left(\frac{a^4x^4(\operatorname{arccosh}(ax))^4(ax-1)(ax+1)}{6} + \frac{(\operatorname{arccosh}(ax))^4(ax-1)(ax+1)a^2x^2}{6} + \frac{(\operatorname{arccosh}(ax))^4a^2x^2}{6} - \frac{x^5a^5(\operatorname{arcco}}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arccosh(a*x)^4,x)
```

```
[Out] 1/a^6*(1/6*a^4*x^4*arccosh(a*x)^4*(a*x-1)*(a*x+1)+1/6*arccosh(a*x)^4*(a*x-1
)*(a*x+1)*a^2*x^2+1/6*arccosh(a*x)^4*a^2*x^2-1/9*a^5*x^5*arccosh(a*x)^3*(a*
x-1)^(1/2)*(a*x+1)^(1/2)-5/36*arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^
3*x^3-5/24*arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a*x-5/96*arccosh(a*x)
^4+1/18*arccosh(a*x)^2*(a*x-1)*(a*x+1)*a^4*x^4+23/144*arccosh(a*x)^2*(a*x-1
)*(a*x+1)*a^2*x^2+17/36*arccosh(a*x)^2*a^2*x^2-1/54*arccosh(a*x)*(a*x-1)^(1
/2)*(a*x+1)^(1/2)*a^5*x^5-65/864*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a
^3*x^3-245/576*arccosh(a*x)*a*x*(a*x-1)^(1/2)*(a*x+1)^(1/2)-245/1152*arccos
h(a*x)^2+1/324*(a*x-1)*(a*x+1)*a^4*x^4+227/10368*(a*x-1)*(a*x+1)*a^2*x^2+19
/81*a^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}x^6\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^4-\int\frac{2\left(a^3x^8+\sqrt{ax+1}\sqrt{ax-1}a^2x^7-ax^6\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^3}{3\left(a^3x^3+\left(a^2x^2-1\right)\sqrt{ax+1}\sqrt{ax-1}-ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccosh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^4 - integrate(2/3*(a^3*x^8 +
 sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x^7 - a*x^6)*log(a*x + sqrt(a*x + 1)*sqrt(
a*x - 1))^3/(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x), x)
```

Fricas [A] time = 2.39829, size = 497, normalized size = 1.62

$$\frac{32\,a^6x^6+195\,a^4x^4+108\left(16\,a^6x^6-5\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^4-144\left(8\,a^5x^5+10\,a^3x^3+15\,ax\right)\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccosh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/10368*(32*a^6*x^6 + 195*a^4*x^4 + 108*(16*a^6*x^6 - 5)*log(a*x + sqrt(a^2
*x^2 - 1))^4 - 144*(8*a^5*x^5 + 10*a^3*x^3 + 15*a*x)*sqrt(a^2*x^2 - 1)*log(
a*x + sqrt(a^2*x^2 - 1))^3 + 2205*a^2*x^2 + 9*(64*a^6*x^6 + 120*a^4*x^4 + 3
60*a^2*x^2 - 245)*log(a*x + sqrt(a^2*x^2 - 1))^2 - 6*(32*a^5*x^5 + 130*a^3*
x^3 + 735*a*x)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1)))/a^6
```

Sympy [A] time = 25.0856, size = 275, normalized size = 0.9

$$\begin{cases} \frac{x^6 \operatorname{acosh}^4(ax)}{6} + \frac{x^6 \operatorname{acosh}^2(ax)}{18} + \frac{x^6}{324} - \frac{x^5\sqrt{a^2x^2-1}\operatorname{acosh}^3(ax)}{9a} - \frac{x^5\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{54a} + \frac{5x^4\operatorname{acosh}^2(ax)}{48a^2} + \frac{65x^4}{3456a^2} - \frac{5x^3\sqrt{a^2x^2-1}\operatorname{acosh}^3(ax)}{36a^3} \cdot \\ \frac{\pi^4 x^6}{96} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*acosh(a*x)**4,x)
```

```
[Out] Piecewise((x**6*acosh(a*x)**4/6 + x**6*acosh(a*x)**2/18 + x**6/324 - x**5*s
qrt(a**2*x**2 - 1)*acosh(a*x)**3/(9*a) - x**5*sqrt(a**2*x**2 - 1)*acosh(a*x
)/(54*a) + 5*x**4*acosh(a*x)**2/(48*a**2) + 65*x**4/(3456*a**2) - 5*x**3*sq
rt(a**2*x**2 - 1)*acosh(a*x)**3/(36*a**3) - 65*x**3*sqrt(a**2*x**2 - 1)*aco
sh(a*x)/(864*a**3) + 5*x**2*acosh(a*x)**2/(16*a**4) + 245*x**2/(1152*a**4)
- 5*x*sqrt(a**2*x**2 - 1)*acosh(a*x)**3/(24*a**5) - 245*x*sqrt(a**2*x**2 -
1)*acosh(a*x)/(576*a**5) - 5*acosh(a*x)**4/(96*a**6) - 245*acosh(a*x)**2/(1
152*a**6), Ne(a, 0)), (pi**4*x**6/96, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{arcosh}(ax)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^5*arccosh(a*x)^4, x)
```

3.33 $\int x^4 \cosh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=274

$$\frac{1088x^3}{16875a^2}+\frac{16x^3\cosh^{-1}(ax)^2}{75a^2}-\frac{16x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{75a^3}-\frac{1088x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{5625a^3}+\frac{16576x}{5625a^4}+\frac{3}{}$$

```
[Out] (16576*x)/(5625*a^4) + (1088*x^3)/(16875*a^2) + (24*x^5)/3125 - (16576*Sqrt
[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(5625*a^5) - (1088*x^2*Sqrt[-1 + a*x
]*Sqrt[1 + a*x]*ArcCosh[a*x])/(5625*a^3) - (24*x^4*Sqrt[-1 + a*x]*Sqrt[1 +
a*x]*ArcCosh[a*x])/(625*a) + (32*x*ArcCosh[a*x]^2)/(25*a^4) + (16*x^3*ArcCo
sh[a*x]^2)/(75*a^2) + (12*x^5*ArcCosh[a*x]^2)/125 - (32*Sqrt[-1 + a*x]*Sqrt
[1 + a*x]*ArcCosh[a*x]^3)/(75*a^5) - (16*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*A
rcCosh[a*x]^3)/(75*a^3) - (4*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^
3)/(25*a) + (x^5*ArcCosh[a*x]^4)/5
```

Rubi [A] time = 1.62758, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5662, 5759, 5718, 5654, 8, 30}

$$\frac{1088x^3}{16875a^2}+\frac{16x^3\cosh^{-1}(ax)^2}{75a^2}-\frac{16x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{75a^3}-\frac{1088x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{5625a^3}+\frac{16576x}{5625a^4}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCosh[a*x]^4,x]
```


```
[Out] (16576*x)/(5625*a^4) + (1088*x^3)/(16875*a^2) + (24*x^5)/3125 - (16576*Sqrt
[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(5625*a^5) - (1088*x^2*Sqrt[-1 + a*x
]*Sqrt[1 + a*x]*ArcCosh[a*x])/(5625*a^3) - (24*x^4*Sqrt[-1 + a*x]*Sqrt[1 +
a*x]*ArcCosh[a*x])/(625*a) + (32*x*ArcCosh[a*x]^2)/(25*a^4) + (16*x^3*ArcCo
sh[a*x]^2)/(75*a^2) + (12*x^5*ArcCosh[a*x]^2)/125 - (32*Sqrt[-1 + a*x]*Sqrt
[1 + a*x]*ArcCosh[a*x]^3)/(75*a^5) - (16*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*A
rcCosh[a*x]^3)/(75*a^3) - (4*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^
3)/(25*a) + (x^5*ArcCosh[a*x]^4)/5
```


Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
```

```
NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^4 \cosh^{-1}(ax)^4\,dx = \frac{1}{5}x^5\cosh^{-1}(ax)^4 - \frac{1}{5}(4a)\int \frac{x^5\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{4x^4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{25a} + \frac{1}{5}x^5\cosh^{-1}(ax)^4 + \frac{12}{25}\int x^4\cosh^{-1}(ax)^2\,dx - \frac{16\int \frac{x^3\cos}{\sqrt{-1+a}}}{25}$$

$$= \frac{12}{125}x^5\cosh^{-1}(ax)^2 - \frac{16x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{75a^3} - \frac{4x^4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{25a}$$

$$= -\frac{24x^4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{625a} + \frac{16x^3\cosh^{-1}(ax)^2}{75a^2} + \frac{12}{125}x^5\cosh^{-1}(ax)^2 - \frac{32\sqrt{-1+ax}\sqrt{}}{}$$

$$= \frac{24x^5}{3125} - \frac{1088x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{5625a^3} - \frac{24x^4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{625a} + \frac{32x\cosh^{-}}{25a^4}$$

$$= \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} - \frac{16576\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{5625a^5} - \frac{1088x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{5625a^3}$$

$$= \frac{16576x}{5625a^4} + \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} - \frac{16576\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{5625a^5} - \frac{1088x^2\sqrt{-1+ax}\sqrt{1+ax}\,\mathrm{c}}{5625a^3}$$

Mathematica [A] time = 0.127004, size = 158, normalized size = 0.58

$$\frac{8ax\left(81a^4x^4+680a^2x^2+31080\right)+16875a^5x^5\cosh^{-1}(ax)^4+900ax\left(9a^4x^4+20a^2x^2+120\right)\cosh^{-1}(ax)^2-4500\sqrt{ax-}}{84375a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCosh[a*x]^4,x]
```

```
[Out] (8*a*x*(31080 + 680*a^2*x^2 + 81*a^4*x^4) - 120*Sqrt[-1 + a*x]*Sqrt[1 + a*x
]*(2072 + 136*a^2*x^2 + 27*a^4*x^4)*ArcCosh[a*x] + 900*a*x*(120 + 20*a^2*x^
2 + 9*a^4*x^4)*ArcCosh[a*x]^2 - 4500*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(8 + 4*a^
2*x^2 + 3*a^4*x^4)*ArcCosh[a*x]^3 + 16875*a^5*x^5*ArcCosh[a*x]^4)/(84375*a^
5)
```

Maple [A] time = 0.05, size = 300, normalized size = 1.1

$$\frac{1}{a^5}\left(\frac{a^3x^3\left(\operatorname{arccosh}(ax)\right)^4(ax-1)(ax+1)}{5} + \frac{(ax-1)(ax+1)\left(\operatorname{arccosh}(ax)\right)^4 ax}{5} + \frac{\left(\operatorname{arccosh}(ax)\right)^4 ax}{5} - \frac{4\left(\operatorname{arccosh}(a\right.}{25}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)^4,x)
```

```
[Out] 1/a^5*(1/5*a^3*x^3*arccosh(a*x)^4*(a*x-1)*(a*x+1)+1/5*(a*x-1)*(a*x+1)*arcco
sh(a*x)^4*a*x+1/5*arccosh(a*x)^4*a*x-4/25*arccosh(a*x)^3*a^4*x^4*(a*x-1)^(1
/2)*(a*x+1)^(1/2)-32/75*arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)-16/75*ar
ccosh(a*x)^3*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+12/125*arccosh(a*x)^2*a^3*
x^3*(a*x-1)*(a*x+1)+116/375*arccosh(a*x)^2*(a*x-1)*(a*x+1)*a*x+596/375*arcc
osh(a*x)^2*a*x-24/625*arccosh(a*x)*a^4*x^4*(a*x-1)^(1/2)*(a*x+1)^(1/2)-1657
6/5625*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)-1088/5625*arccosh(a*x)*(a*x
-1)^(1/2)*(a*x+1)^(1/2)*a^2*x^2+24/3125*(a*x-1)*(a*x+1)*a^3*x^3+6088/84375*
(a*x-1)*(a*x+1)*a*x+254728/84375*a*x)
```

Maxima [A] time = 1.23144, size = 271, normalized size = 0.99

$$\frac{1}{5}x^5 \operatorname{arcosh}(ax)^4 - \frac{4}{75}\left(\frac{3\sqrt{a^2x^2-1}x^4}{a^2} + \frac{4\sqrt{a^2x^2-1}x^2}{a^4} + \frac{8\sqrt{a^2x^2-1}}{a^6}\right)a \operatorname{arcosh}(ax)^3 - \frac{4}{84375}\left(2a\left(\frac{15\left(27\sqrt{a^2x^2-1}a^2\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccosh(a*x)^4 - 4/75*(3*sqrt(a^2*x^2 - 1)*x^4/a^2 + 4*sqrt(a^2*x^2
 - 1)*x^2/a^4 + 8*sqrt(a^2*x^2 - 1)/a^6)*a*arccosh(a*x)^3 - 4/84375*(2*a*(1
5*(27*sqrt(a^2*x^2 - 1)*a^2*x^4 + 136*sqrt(a^2*x^2 - 1)*x^2 + 2072*sqrt(a^2
*x^2 - 1)/a^2)*arccosh(a*x)/a^5 - (81*a^4*x^5 + 680*a^2*x^3 + 31080*x)/a^6)
 - 225*(9*a^4*x^5 + 20*a^2*x^3 + 120*x)*arccosh(a*x)^2/a^5)*a
```

Fricas [A] time = 2.61499, size = 466, normalized size = 1.7

$$\frac{16875a^5x^5\log\left(ax+\sqrt{a^2x^2-1}\right)^4 + 648a^5x^5 + 5440a^3x^3 - 4500\left(3a^4x^4+4a^2x^2+8\right)\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)^3 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/84375*(16875*a^5*x^5*log(a*x + sqrt(a^2*x^2 - 1))^4 + 648*a^5*x^5 + 5440*
a^3*x^3 - 4500*(3*a^4*x^4 + 4*a^2*x^2 + 8)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt
(a^2*x^2 - 1))^3 + 900*(9*a^5*x^5 + 20*a^3*x^3 + 120*a*x)*log(a*x + sqrt(a^
2*x^2 - 1))^2 - 120*(27*a^4*x^4 + 136*a^2*x^2 + 2072)*sqrt(a^2*x^2 - 1)*log
(a*x + sqrt(a^2*x^2 - 1)) + 248640*a*x)/a^5
```

Sympy [A] time = 14.6522, size = 248, normalized size = 0.91

$$\begin{cases} \frac{x^5 \operatorname{acosh}^4(ax)}{5} + \frac{12x^5 \operatorname{acosh}^2(ax)}{125} + \frac{24x^5}{3125} - \frac{4x^4\sqrt{a^2x^2-1}\operatorname{acosh}^3(ax)}{25a} - \frac{24x^4\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{625a} + \frac{16x^3\operatorname{acosh}^2(ax)}{75a^2} + \frac{1088x^3}{16875a^2} - \frac{16x^2\sqrt{a^2x^2-1}\,\ldots}{75a^{\ldots}} \\ \frac{\pi^4 x^5}{80} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)**4,x)
```

```
[Out] Piecewise((x**5*acosh(a*x)**4/5 + 12*x**5*acosh(a*x)**2/125 + 24*x**5/3125
- 4*x**4*sqrt(a**2*x**2 - 1)*acosh(a*x)**3/(25*a) - 24*x**4*sqrt(a**2*x**2
- 1)*acosh(a*x)/(625*a) + 16*x**3*acosh(a*x)**2/(75*a**2) + 1088*x**3/(1687
5*a**2) - 16*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)**3/(75*a**3) - 1088*x**2*s
qrt(a**2*x**2 - 1)*acosh(a*x)/(5625*a**3) + 32*x*acosh(a*x)**2/(25*a**4) +
16576*x/(5625*a**4) - 32*sqrt(a**2*x**2 - 1)*acosh(a*x)**3/(75*a**5) - 1657
6*sqrt(a**2*x**2 - 1)*acosh(a*x)/(5625*a**5), Ne(a, 0)), (pi**4*x**5/80, Tr
ue))
```

Giac [A] time = 1.84988, size = 293, normalized size = 1.07

$$\frac{1}{5}x^5\log\left(ax+\sqrt{a^2x^2-1}\right)^4 - \frac{4}{84375}a\left(\frac{1125\left(3\left(a^2x^2-1\right)^{\frac{5}{2}}+10\left(a^2x^2-1\right)^{\frac{3}{2}}+15\sqrt{a^2x^2-1}\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^3}{a^6} - \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/5*x^5*log(a*x + sqrt(a^2*x^2 - 1))^4 - 4/84375*a*(1125*(3*(a^2*x^2 - 1)^(
5/2) + 10*(a^2*x^2 - 1)^(3/2) + 15*sqrt(a^2*x^2 - 1))*log(a*x + sqrt(a^2*x^
```

```
2 - 1))^3/a^6 - (162*a^4*x^5 + 1360*a^2*x^3 + 225*(9*a^4*x^5 + 20*a^2*x^3 +
 120*x)*log(a*x + sqrt(a^2*x^2 - 1))^2 + 62160*x - 30*(27*(a^2*x^2 - 1)^(5/
2) + 190*(a^2*x^2 - 1)^(3/2) + 2235*sqrt(a^2*x^2 - 1))*log(a*x + sqrt(a^2*x
^2 - 1))/a)/a^5)
```

3.34 $\int x^3 \cosh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=214

$$\frac{45x^2}{128a^2}+\frac{9x^2\cosh^{-1}(ax)^2}{16a^2}-\frac{3x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{8a^3}-\frac{45x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{64a^3}-\frac{3\cosh^{-1}(ax)^4}{32a^4}-\frac{45\,\mathrm{c}}{}$$

```
[Out] (45*x^2)/(128*a^2) + (3*x^4)/128 - (45*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCo
sh[a*x])/(64*a^3) - (3*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(32*a
) - (45*ArcCosh[a*x]^2)/(128*a^4) + (9*x^2*ArcCosh[a*x]^2)/(16*a^2) + (3*x^
4*ArcCosh[a*x]^2)/16 - (3*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(8
*a^3) - (x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(4*a) - (3*ArcCos
h[a*x]^4)/(32*a^4) + (x^4*ArcCosh[a*x]^4)/4
```

Rubi [A] time = 1.31439, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5662, 5759, 5676, 30}

$$\frac{45x^2}{128a^2}+\frac{9x^2\cosh^{-1}(ax)^2}{16a^2}-\frac{3x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{8a^3}-\frac{45x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{64a^3}-\frac{3\cosh^{-1}(ax)^4}{32a^4}-\frac{45\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCosh[a*x]^4,x]
```


```
[Out] (45*x^2)/(128*a^2) + (3*x^4)/128 - (45*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCo
sh[a*x])/(64*a^3) - (3*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(32*a
) - (45*ArcCosh[a*x]^2)/(128*a^4) + (9*x^2*ArcCosh[a*x]^2)/(16*a^2) + (3*x^
4*ArcCosh[a*x]^2)/16 - (3*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(8
*a^3) - (x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(4*a) - (3*ArcCos
h[a*x]^4)/(32*a^4) + (x^4*ArcCosh[a*x]^4)/4
```


Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3 \cosh^{-1}(ax)^4 \, dx = \frac{1}{4}x^4 \cosh^{-1}(ax)^4 - a\int \frac{x^4 \cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}} \, dx$$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{4a} + \frac{1}{4}x^4\cosh^{-1}(ax)^4 + \frac{3}{4}\int x^3\cosh^{-1}(ax)^2\,dx - \frac{3\int \frac{x^2\cosh^{-1}(ax}{\sqrt{-1+ax}\sqrt{1+a}}}{4a}$$

$$= \frac{3}{16}x^4\cosh^{-1}(ax)^2 - \frac{3x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{8a^3} - \frac{x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{4a} + \frac{1}{4}x^4$$

$$= -\frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{32a} + \frac{9x^2\cosh^{-1}(ax)^2}{16a^2} + \frac{3}{16}x^4\cosh^{-1}(ax)^2 - \frac{3x\sqrt{-1+ax}\sqrt{1+a}}{8a^3}$$

$$= \frac{3x^4}{128} - \frac{45x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{64a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{32a} + \frac{9x^2\cosh^{-1}(ax)^2}{16a^2}$$

$$= \frac{45x^2}{128a^2} + \frac{3x^4}{128} - \frac{45x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{64a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{32a} - \frac{45\cosh}{128}$$

Mathematica [A] time = 0.109415, size = 143, normalized size = 0.67

$$\frac{3a^2x^2\left(a^2x^2+15\right)+4\left(8a^4x^4-3\right)\cosh^{-1}(ax)^4-16ax\sqrt{ax-1}\sqrt{ax+1}\left(2a^2x^2+3\right)\cosh^{-1}(ax)^3+3\left(8a^4x^4+24a^2x^2-1\right.}{128a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCosh[a*x]^4,x]
```

```
[Out] (3*a^2*x^2*(15 + a^2*x^2) - 6*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(15 + 2*a^2*
x^2)*ArcCosh[a*x] + 3*(-15 + 24*a^2*x^2 + 8*a^4*x^4)*ArcCosh[a*x]^2 - 16*a*
x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(3 + 2*a^2*x^2)*ArcCosh[a*x]^3 + 4*(-3 + 8*a
^4*x^4)*ArcCosh[a*x]^4)/(128*a^4)
```

Maple [A] time = 0.045, size = 224, normalized size = 1.1

$$\frac{1}{a^4}\left(\frac{(\operatorname{arccosh}(ax))^4(ax-1)(ax+1)a^2x^2}{4}+\frac{(\operatorname{arccosh}(ax))^4a^2x^2}{4}-\frac{(\operatorname{arccosh}(ax))^3a^3x^3}{4}\sqrt{ax-1}\sqrt{ax+1}-\frac{3\,(\operatorname{arccos}}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)^4,x)
```

```
[Out] 1/a^4*(1/4*arccosh(a*x)^4*(a*x-1)*(a*x+1)*a^2*x^2+1/4*arccosh(a*x)^4*a^2*x^
2-1/4*arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^3*x^3-3/8*arccosh(a*x)^3
*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a*x-3/32*arccosh(a*x)^4+3/16*arccosh(a*x)^2*(a
*x-1)*(a*x+1)*a^2*x^2+3/4*arccosh(a*x)^2*a^2*x^2-3/32*arccosh(a*x)*(a*x-1)^
(1/2)*(a*x+1)^(1/2)*a^3*x^3-45/64*arccosh(a*x)*a*x*(a*x-1)^(1/2)*(a*x+1)^(1
/2)-45/128*arccosh(a*x)^2+3/128*(a*x-1)*(a*x+1)*a^2*x^2+3/8*a^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}x^4\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^4-\int\frac{\left(a^3x^6+\sqrt{ax+1}\sqrt{ax-1}a^2x^5-ax^4\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^3}{a^3x^3+\left(a^2x^2-1\right)\sqrt{ax+1}\sqrt{ax-1}-ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^4 - integrate((a^3*x^6 + sqr
t(a*x + 1)*sqrt(a*x - 1)*a^2*x^5 - a*x^4)*log(a*x + sqrt(a*x + 1)*sqrt(a*x
- 1))^3/(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x), x)
```

Fricas [A] time = 2.4774, size = 402, normalized size = 1.88

$$\frac{3\,a^4x^4 + 4\left(8\,a^4x^4 - 3\right)\log\left(ax + \sqrt{a^2x^2 - 1}\right)^4 - 16\left(2\,a^3x^3 + 3\,ax\right)\sqrt{a^2x^2 - 1}\log\left(ax + \sqrt{a^2x^2 - 1}\right)^3 + 45\,a^2x^2 + 3\left(8\,a^4x^4\right.}{128\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(3*a^4*x^4 + 4*(8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 - 1))^4 - 16*(2
*a^3*x^3 + 3*a*x)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^3 + 45*a^2
*x^2 + 3*(8*a^4*x^4 + 24*a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 - 1))^2 - 6*(
2*a^3*x^3 + 15*a*x)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1)))/a^4
```

Sympy [A] time = 8.66786, size = 197, normalized size = 0.92

$$\begin{cases} \frac{x^4 \operatorname{acosh}^4(ax)}{4} + \frac{3x^4 \operatorname{acosh}^2(ax)}{16} + \frac{3x^4}{128} - \frac{x^3\sqrt{a^2x^2-1}\operatorname{acosh}^3(ax)}{4a} - \frac{3x^3\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{32a} + \frac{9x^2\operatorname{acosh}^2(ax)}{16a^2} + \frac{45x^2}{128a^2} - \frac{3x\sqrt{a^2x^2-1}\operatorname{acosh}^3(ax)}{8a^3} \\ \frac{\pi^4x^4}{64} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**4,x)
```

```
[Out] Piecewise((x**4*acosh(a*x)**4/4 + 3*x**4*acosh(a*x)**2/16 + 3*x**4/128 - x*
*3*sqrt(a**2*x**2 - 1)*acosh(a*x)**3/(4*a) - 3*x**3*sqrt(a**2*x**2 - 1)*aco
sh(a*x)/(32*a) + 9*x**2*acosh(a*x)**2/(16*a**2) + 45*x**2/(128*a**2) - 3*x*
sqrt(a**2*x**2 - 1)*acosh(a*x)**3/(8*a**3) - 45*x*sqrt(a**2*x**2 - 1)*acosh
(a*x)/(64*a**3) - 3*acosh(a*x)**4/(32*a**4) - 45*acosh(a*x)**2/(128*a**4),
Ne(a, 0)), (pi**4*x**4/64, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arcosh}(ax)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccosh(a*x)^4, x)
```

3.35 $\int x^2 \cosh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=182

$$\frac{160x}{27a^2} - \frac{8\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{9a^3} + \frac{8x\cosh^{-1}(ax)^2}{3a^2} - \frac{160\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{27a^3} + \frac{1}{3}x^3\cosh^{-1}(ax)^4 - \frac{4x^2\sqrt{a}}{}$$

```
[Out] (160*x)/(27*a^2) + (8*x^3)/81 - (160*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a
*x])/(27*a^3) - (8*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(27*a) +
(8*x*ArcCosh[a*x]^2)/(3*a^2) + (4*x^3*ArcCosh[a*x]^2)/9 - (8*Sqrt[-1 + a*x]
*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(9*a^3) - (4*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x
]*ArcCosh[a*x]^3)/(9*a) + (x^3*ArcCosh[a*x]^4)/3
```

Rubi [A] time = 0.881492, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5662, 5759, 5718, 5654, 8, 30}

$$\frac{160x}{27a^2} - \frac{8\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{9a^3} + \frac{8x\cosh^{-1}(ax)^2}{3a^2} - \frac{160\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{27a^3} + \frac{1}{3}x^3\cosh^{-1}(ax)^4 - \frac{4x^2\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCosh[a*x]^4,x]
```


```
[Out] (160*x)/(27*a^2) + (8*x^3)/81 - (160*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a
*x])/(27*a^3) - (8*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(27*a) +
(8*x*ArcCosh[a*x]^2)/(3*a^2) + (4*x^3*ArcCosh[a*x]^2)/9 - (8*Sqrt[-1 + a*x]
*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(9*a^3) - (4*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x
]*ArcCosh[a*x]^3)/(9*a) + (x^3*ArcCosh[a*x]^4)/3
```


Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
```

```
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\cosh^{-1}(ax)^4\,dx = \frac{1}{3}x^3\cosh^{-1}(ax)^4 - \frac{1}{3}(4a)\int \frac{x^3\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{4x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{9a} + \frac{1}{3}x^3\cosh^{-1}(ax)^4 + \frac{4}{3}\int x^2\cosh^{-1}(ax)^2\,dx - \frac{8\int \frac{x\cosh^{-1}(a}{\sqrt{-1+ax}\sqrt{1}}}{9a}$$

$$= \frac{4}{9}x^3\cosh^{-1}(ax)^2 - \frac{8\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{9a^3} - \frac{4x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{9a} + \frac{1}{3}x^3\,c$$

$$= -\frac{8x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{27a} + \frac{8x\cosh^{-1}(ax)^2}{3a^2} + \frac{4}{9}x^3\cosh^{-1}(ax)^2 - \frac{8\sqrt{-1+ax}\sqrt{1+ax}\,c}{9a^3}$$

$$= \frac{8x^3}{81} - \frac{160\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{27a^3} - \frac{8x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{27a} + \frac{8x\cosh^{-1}(ax)^2}{3a^2} +$$

$$= \frac{160x}{27a^2} + \frac{8x^3}{81} - \frac{160\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{27a^3} - \frac{8x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{27a} + \frac{8x\cosh^{-1}}{3a^2}$$

Mathematica [A] time = 0.112116, size = 122, normalized size = 0.67

$$\frac{8ax\left(a^2x^2+60\right)+27a^3x^3\cosh^{-1}(ax)^4-36\sqrt{ax-1}\sqrt{ax+1}\left(a^2x^2+2\right)\cosh^{-1}(ax)^3+36ax\left(a^2x^2+6\right)\cosh^{-1}(ax)^2-24\sqrt{}}{81a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCosh[a*x]^4,x]
```

```
[Out] (8*a*x*(60 + a^2*x^2) - 24*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(20 + a^2*x^2)*ArcC
osh[a*x] + 36*a*x*(6 + a^2*x^2)*ArcCosh[a*x]^2 - 36*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*(2 + a^2*x^2)*ArcCosh[a*x]^3 + 27*a^3*x^3*ArcCosh[a*x]^4)/(81*a^3)
```

Maple [A] time = 0.04, size = 180, normalized size = 1.

$$\frac{1}{a^3}\left(\frac{(ax-1)(ax+1)(\mathrm{arccosh}(ax))^4\,ax}{3} + \frac{(\mathrm{arccosh}(ax))^4\,ax}{3} - \frac{4(\mathrm{arccosh}(ax))^3\,a^2x^2}{9}\sqrt{ax-1}\sqrt{ax+1} - \frac{8(\mathrm{arccosh}(a}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)^4,x)
```

```
[Out] 1/a^3*(1/3*(a*x-1)*(a*x+1)*arccosh(a*x)^4*a*x+1/3*arccosh(a*x)^4*a*x-4/9*ar
ccosh(a*x)^3*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-8/9*arccosh(a*x)^3*(a*x-1)
^(1/2)*(a*x+1)^(1/2)+4/9*arccosh(a*x)^2*(a*x-1)*(a*x+1)*a*x+28/9*arccosh(a*
x)^2*a*x-8/27*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^2*x^2-160/27*arcco
sh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+8/81*(a*x-1)*(a*x+1)*a*x+488/81*a*x)
```

Maxima [A] time = 1.2046, size = 193, normalized size = 1.06

$$\frac{1}{3}x^3\operatorname{arcosh}(ax)^4-\frac{4}{9}a\left(\frac{\sqrt{a^2x^2-1}x^2}{a^2}+\frac{2\sqrt{a^2x^2-1}}{a^4}\right)\operatorname{arcosh}(ax)^3-\frac{4}{81}\left(2a\left(\frac{3\left(\sqrt{a^2x^2-1}x^2+\frac{20\sqrt{a^2x^2-1}}{a^2}\right)\operatorname{arcosh}(ax)}{a^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccosh(a*x)^4 - 4/9*a*(sqrt(a^2*x^2 - 1)*x^2/a^2 + 2*sqrt(a^2*x^2
- 1)/a^4)*arccosh(a*x)^3 - 4/81*(2*a*(3*(sqrt(a^2*x^2 - 1)*x^2 + 20*sqrt(a^
2*x^2 - 1)/a^2)*arccosh(a*x)/a^3 - (a^2*x^3 + 60*x)/a^4) - 9*(a^2*x^3 + 6*x
)*arccosh(a*x)^2/a^3)*a
```

Fricas [A] time = 2.35788, size = 358, normalized size = 1.97

$$\frac{27a^3x^3\log\left(ax+\sqrt{a^2x^2-1}\right)^4+8a^3x^3-36\left(a^2x^2+2\right)\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)^3+36\left(a^3x^3+6ax\right)\log\left(ax+\sqrt{\right.}}{81a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/81*(27*a^3*x^3*log(a*x + sqrt(a^2*x^2 - 1))^4 + 8*a^3*x^3 - 36*(a^2*x^2 +
 2)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^3 + 36*(a^3*x^3 + 6*a*x)
*log(a*x + sqrt(a^2*x^2 - 1))^2 - 24*(a^2*x^2 + 20)*sqrt(a^2*x^2 - 1)*log(a
*x + sqrt(a^2*x^2 - 1)) + 480*a*x)/a^3
```

Sympy [A] time = 5.03241, size = 165, normalized size = 0.91

$$\begin{cases} \frac{x^3 \operatorname{acosh}^4(ax)}{3} + \frac{4x^3 \operatorname{acosh}^2(ax)}{9} + \frac{8x^3}{81} - \frac{4x^2\sqrt{a^2x^2-1} \operatorname{acosh}^3(ax)}{9a} - \frac{8x^2\sqrt{a^2x^2-1} \operatorname{acosh}(ax)}{27a} + \frac{8x \operatorname{acosh}^2(ax)}{3a^2} + \frac{160x}{27a^2} - \frac{8\sqrt{a^2x^2-1} \operatorname{acosh}^3(ax)}{9a^3} - \\ \frac{\pi^4 x^3}{48} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)**4,x)
```

```
[Out] Piecewise((x**3*acosh(a*x)**4/3 + 4*x**3*acosh(a*x)**2/9 + 8*x**3/81 - 4*x*
*2*sqrt(a**2*x**2 - 1)*acosh(a*x)**3/(9*a) - 8*x**2*sqrt(a**2*x**2 - 1)*aco
sh(a*x)/(27*a) + 8*x*acosh(a*x)**2/(3*a**2) + 160*x/(27*a**2) - 8*sqrt(a**2
*x**2 - 1)*acosh(a*x)**3/(9*a**3) - 160*sqrt(a**2*x**2 - 1)*acosh(a*x)/(27*
a**3), Ne(a, 0)), (pi**4*x**3/48, True))
```

Giac [A] time = 1.77945, size = 230, normalized size = 1.26

$$\frac{1}{3}\, x^3 \log\left(ax + \sqrt{a^2x^2-1}\right)^4 - \frac{4}{81}\, a\left(\frac{9\left(\left(a^2x^2-1\right)^{\frac{3}{2}} + 3\sqrt{a^2x^2-1}\right)\log\left(ax + \sqrt{a^2x^2-1}\right)^3}{a^4} - \frac{2\,a^2x^3 + 9\left(a^2x^3 + 6\,x\right)\log\left(a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(a*x + sqrt(a^2*x^2 - 1))^4 - 4/81*a*(9*((a^2*x^2 - 1)^(3/2) + 3
*sqrt(a^2*x^2 - 1))*log(a*x + sqrt(a^2*x^2 - 1))^3/a^4 - (2*a^2*x^3 + 9*(a^
2*x^3 + 6*x)*log(a*x + sqrt(a^2*x^2 - 1))^2 + 120*x - 6*((a^2*x^2 - 1)^(3/2
) + 21*sqrt(a^2*x^2 - 1))*log(a*x + sqrt(a^2*x^2 - 1))/a)/a^3)
```

3.36 $\int x \cosh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=120

$$-\frac{\cosh^{-1}(ax)^4}{4a^2}-\frac{3\cosh^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^4+\frac{3}{2}x^2\cosh^{-1}(ax)^2-\frac{x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{a}-\frac{3x\sqrt{ax-1}\sqrt{a}}{}$$

```
[Out] (3*x^2)/4 - (3*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(2*a) - (3*ArcC
osh[a*x]^2)/(4*a^2) + (3*x^2*ArcCosh[a*x]^2)/2 - (x*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*ArcCosh[a*x]^3)/a - ArcCosh[a*x]^4/(4*a^2) + (x^2*ArcCosh[a*x]^4)/2
```

Rubi [A] time = 0.602508, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5662, 5759, 5676, 30}

$$-\frac{\cosh^{-1}(ax)^4}{4a^2}-\frac{3\cosh^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^4+\frac{3}{2}x^2\cosh^{-1}(ax)^2-\frac{x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{a}-\frac{3x\sqrt{ax-1}\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCosh[a*x]^4,x]
```



```
[Out] (3*x^2)/4 - (3*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(2*a) - (3*ArcC
osh[a*x]^2)/(4*a^2) + (3*x^2*ArcCosh[a*x]^2)/2 - (x*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*ArcCosh[a*x]^3)/a - ArcCosh[a*x]^4/(4*a^2) + (x^2*ArcCosh[a*x]^4)/2
```


Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
```

```
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x\cosh^{-1}(ax)^4\,dx = \frac{1}{2}x^2\cosh^{-1}(ax)^4 - (2a)\int \frac{x^2\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{a} + \frac{1}{2}x^2\cosh^{-1}(ax)^4 + 3\int x\cosh^{-1}(ax)^2\,dx - \frac{\int \frac{\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{a}$$

$$= \frac{3}{2}x^2\cosh^{-1}(ax)^2 - \frac{x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{a} - \frac{\cosh^{-1}(ax)^4}{4a^2} + \frac{1}{2}x^2\cosh^{-1}(ax)^4 - (3a)\int -$$

$$= -\frac{3x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{2a} + \frac{3}{2}x^2\cosh^{-1}(ax)^2 - \frac{x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{a} - \frac{\cosh^{-1}}{4a}$$

$$= \frac{3x^2}{4} - \frac{3x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{2a} - \frac{3\cosh^{-1}(ax)^2}{4a^2} + \frac{3}{2}x^2\cosh^{-1}(ax)^2 - \frac{x\sqrt{-1+ax}\sqrt{1+ax}}{a}$$

Mathematica [A] time = 0.0700765, size = 104, normalized size = 0.87

$$\frac{3a^2x^2 + \left(2a^2x^2-1\right)\cosh^{-1}(ax)^4 + \left(6a^2x^2-3\right)\cosh^{-1}(ax)^2 - 4ax\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3 - 6ax\sqrt{ax-1}\sqrt{ax+1}\cos}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCosh[a*x]^4,x]
```

```
[Out] (3*a^2*x^2 - 6*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x] + (-3 + 6*a^2*
x^2)*ArcCosh[a*x]^2 - 4*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3 + (
-1 + 2*a^2*x^2)*ArcCosh[a*x]^4)/(4*a^2)
```

Maple [A] time = 0.033, size = 104, normalized size = 0.9

$$\frac{1}{a^2}\left(\frac{(\operatorname{arccosh}(ax))^4 a^2x^2}{2} - (\operatorname{arccosh}(ax))^3\sqrt{ax-1}\sqrt{ax+1}ax - \frac{(\operatorname{arccosh}(ax))^4}{4} + \frac{3(\operatorname{arccosh}(ax))^2 a^2x^2}{2} - \frac{3\,ax\operatorname{arcc}}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^4,x)
```

```
[Out] 1/a^2*(1/2*arccosh(a*x)^4*a^2*x^2-arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2
)*a*x-1/4*arccosh(a*x)^4+3/2*arccosh(a*x)^2*a^2*x^2-3/2*arccosh(a*x)*a*x*(a
*x-1)^(1/2)*(a*x+1)^(1/2)-3/4*arccosh(a*x)^2+3/4*a^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^4 - \int \frac{2\left(a^3x^4+\sqrt{ax+1}\sqrt{ax-1}a^2x^3-ax^2\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^3}{a^3x^3+\left(a^2x^2-1\right)\sqrt{ax+1}\sqrt{ax-1}-ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^4 - integrate(2*(a^3*x^4 + s
qrt(a*x + 1)*sqrt(a*x - 1)*a^2*x^3 - a*x^2)*log(a*x + sqrt(a*x + 1)*sqrt(a*
x - 1))^3/(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x), x)
```

Fricas [A] time = 2.60556, size = 316, normalized size = 2.63

$$-\frac{4\sqrt{a^2x^2-1}ax\log\left(ax+\sqrt{a^2x^2-1}\right)^3-\left(2a^2x^2-1\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^4-3a^2x^2+6\sqrt{a^2x^2-1}ax\log\left(ax+\sqrt{a^2x^2-}\right.}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^4,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(a^2*x^2 - 1)*a*x*log(a*x + sqrt(a^2*x^2 - 1))^3 - (2*a^2*x^2 -
 1)*log(a*x + sqrt(a^2*x^2 - 1))^4 - 3*a^2*x^2 + 6*sqrt(a^2*x^2 - 1)*a*x*lo
g(a*x + sqrt(a^2*x^2 - 1)) - 3*(2*a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))
^2)/a^2
```

Sympy [A] time = 2.61227, size = 110, normalized size = 0.92

$$\begin{cases} \frac{x^2 \operatorname{acosh}^4(ax)}{2} + \frac{3x^2 \operatorname{acosh}^2(ax)}{2} + \frac{3x^2}{4} - \frac{x\sqrt{a^2x^2-1}\operatorname{acosh}^3(ax)}{a} - \frac{3x\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{2a} - \frac{\operatorname{acosh}^4(ax)}{4a^2} - \frac{3\operatorname{acosh}^2(ax)}{4a^2} & \text{for}\ a \neq 0 \\ \frac{\pi^4x^2}{32} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**4,x)
```

```
[Out] Piecewise((x**2*acosh(a*x)**4/2 + 3*x**2*acosh(a*x)**2/2 + 3*x**2/4 - x*sqr
t(a**2*x**2 - 1)*acosh(a*x)**3/a - 3*x*sqrt(a**2*x**2 - 1)*acosh(a*x)/(2*a)
 - acosh(a*x)**4/(4*a**2) - 3*acosh(a*x)**2/(4*a**2), Ne(a, 0)), (pi**4*x**
2/32, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcosh}(ax)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x*arccosh(a*x)^4, x)
```

3.37 $\int \cosh^{-1}(ax)^4\,dx$

Optimal. Leaf size=77

$$x\cosh^{-1}(ax)^4-\frac{4\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{a}+12x\cosh^{-1}(ax)^2-\frac{24\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{a}+24x$$

```
[Out] 24*x - (24*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/a + 12*x*ArcCosh[a*x]
^2 - (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/a + x*ArcCosh[a*x]^4
```

Rubi [A] time = 0.298058, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5654, 5718, 8}

$$x\cosh^{-1}(ax)^4-\frac{4\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{a}+12x\cosh^{-1}(ax)^2-\frac{24\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{a}+24x$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^4,x]
```

```
[Out] 24*x - (24*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/a + 12*x*ArcCosh[a*x]
^2 - (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/a + x*ArcCosh[a*x]^4
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \cosh^{-1}(ax)^4\,dx &= x\cosh^{-1}(ax)^4 - (4a)\int \frac{x\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= -\frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{a} + x\cosh^{-1}(ax)^4 + 12\int \cosh^{-1}(ax)^2\,dx\\
&= 12x\cosh^{-1}(ax)^2 - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{a} + x\cosh^{-1}(ax)^4 - (24a)\int \frac{x\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx\\
&= -\frac{24\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{a} + 12x\cosh^{-1}(ax)^2 - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{a} + x\cosh^{-1}(ax)^4\\
&= 24x - \frac{24\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{a} + 12x\cosh^{-1}(ax)^2 - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{a} + x\cosh^{-1}(ax)^4
\end{aligned}
$$

Mathematica [A] time = 0.0284939, size = 77, normalized size = 1.

$$x\cosh^{-1}(ax)^4 - \frac{4\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{a} + 12x\cosh^{-1}(ax)^2 - \frac{24\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{a} + 24x$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^4,x]
```

```
[Out] 24*x - (24*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/a + 12*x*ArcCosh[a*x]
^2 - (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/a + x*ArcCosh[a*x]^4
```

Maple [A] time = 0.033, size = 71, normalized size = 0.9

$$\frac{1}{a}\Big((\mathrm{arccosh}\,(ax))^4\,ax - 4\,(\mathrm{arccosh}\,(ax))^3\sqrt{ax-1}\sqrt{ax+1} + 12\,(\mathrm{arccosh}\,(ax))^2\,ax - 24\,\mathrm{arccosh}\,(ax)\sqrt{ax-1}\sqrt{ax+1} + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^4,x)
```

```
[Out] 1/a*(arccosh(a*x)^4*a*x-4*arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)+12*arc
cosh(a*x)^2*a*x-24*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+24*a*x)
```

Maxima [A] time = 1.15713, size = 99, normalized size = 1.29

$$x \operatorname{arcosh}(ax)^4 - \frac{4\sqrt{a^2x^2-1}\operatorname{arcosh}(ax)^3}{a} + 12\left(\frac{x\operatorname{arcosh}(ax)^2}{a} + \frac{2\left(x - \frac{\sqrt{a^2x^2-1}\operatorname{arcosh}(ax)}{a}\right)}{a}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4,x, algorithm="maxima")
```

```
[Out] x*arccosh(a*x)^4 - 4*sqrt(a^2*x^2 - 1)*arccosh(a*x)^3/a + 12*(x*arccosh(a*x
)^2/a + 2*(x - sqrt(a^2*x^2 - 1)*arccosh(a*x)/a)/a)*a
```

Fricas [A] time = 2.31198, size = 262, normalized size = 3.4

$$\frac{ax\log\left(ax+\sqrt{a^2x^2-1}\right)^4 + 12\,ax\log\left(ax+\sqrt{a^2x^2-1}\right)^2 - 4\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)^3 + 24\,ax - 24\sqrt{a^2x^2-1}\log}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4,x, algorithm="fricas")
```

```
[Out] (a*x*log(a*x + sqrt(a^2*x^2 - 1))^4 + 12*a*x*log(a*x + sqrt(a^2*x^2 - 1))^2
 - 4*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^3 + 24*a*x - 24*sqrt(a^
2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1)))/a
```

Sympy [A] time = 1.18142, size = 70, normalized size = 0.91

$$\begin{cases} x\operatorname{acosh}^4(ax) + 12x\operatorname{acosh}^2(ax) + 24x - \frac{4\sqrt{a^2x^2-1}\operatorname{acosh}^3(ax)}{a} - \frac{24\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{a} & \text{for } a \neq 0 \\ \frac{\pi^4 x}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**4,x)
```

```
[Out] Piecewise((x*acosh(a*x)**4 + 12*x*acosh(a*x)**2 + 24*x - 4*sqrt(a**2*x**2 -
 1)*acosh(a*x)**3/a - 24*sqrt(a**2*x**2 - 1)*acosh(a*x)/a, Ne(a, 0)), (pi**
4*x/16, True))
```

Giac [A] time = 1.46721, size = 169, normalized size = 2.19

$$x\log\left(ax+\sqrt{a^2x^2-1}\right)^4-4\left(\frac{\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)^3}{a^2}-\frac{3\left(x\log\left(ax+\sqrt{a^2x^2-1}\right)^2+2a\left(\frac{x}{a}-\frac{\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)}{a^2}\right)\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 - 1))^4 - 4*(sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*
x^2 - 1))^3/a^2 - 3*(x*log(a*x + sqrt(a^2*x^2 - 1))^2 + 2*a*(x/a - sqrt(a^2
*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))/a^2))/a)*a
```

3.38 $\int \frac{\cosh^{-1}(ax)^4}{x}\,dx$

Optimal. Leaf size=103

$$2\cosh^{-1}(ax)^3\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)-3\cosh^{-1}(ax)^2\text{PolyLog}\left(3,-e^{2\cosh^{-1}(ax)}\right)+3\cosh^{-1}(ax)\text{PolyLog}\left(4,-e^{2\cosh^{-1}(ax)}\right)$$

```
[Out] -ArcCosh[a*x]^5/5 + ArcCosh[a*x]^4*Log[1 + E^(2*ArcCosh[a*x])] + 2*ArcCosh[
a*x]^3*PolyLog[2, -E^(2*ArcCosh[a*x])] - 3*ArcCosh[a*x]^2*PolyLog[3, -E^(2*
ArcCosh[a*x])] + 3*ArcCosh[a*x]*PolyLog[4, -E^(2*ArcCosh[a*x])] - (3*PolyLo
g[5, -E^(2*ArcCosh[a*x])])/2
```

Rubi [A] time = 0.120114, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5660, 3718, 2190, 2531, 6609, 2282, 6589}

$$2\cosh^{-1}(ax)^3\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)-3\cosh^{-1}(ax)^2\text{PolyLog}\left(3,-e^{2\cosh^{-1}(ax)}\right)+3\cosh^{-1}(ax)\text{PolyLog}\left(4,-e^{2\cosh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^4/x,x]
```


```
[Out] -ArcCosh[a*x]^5/5 + ArcCosh[a*x]^4*Log[1 + E^(2*ArcCosh[a*x])] + 2*ArcCosh[
a*x]^3*PolyLog[2, -E^(2*ArcCosh[a*x])] - 3*ArcCosh[a*x]^2*PolyLog[3, -E^(2*
ArcCosh[a*x])] + 3*ArcCosh[a*x]*PolyLog[4, -E^(2*ArcCosh[a*x])] - (3*PolyLo
g[5, -E^(2*ArcCosh[a*x])])/2
```


Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^4}{x}\,dx = \text{Subst}\left(\int x^4\tanh(x)\,dx, x, \cosh^{-1}(ax)\right)$$
$$= -\frac{1}{5}\cosh^{-1}(ax)^5 + 2\,\text{Subst}\left(\int \frac{e^{2x}x^4}{1+e^{2x}}\,dx, x, \cosh^{-1}(ax)\right)$$
$$= -\frac{1}{5}\cosh^{-1}(ax)^5 + \cosh^{-1}(ax)^4\log\left(1+e^{2\cosh^{-1}(ax)}\right) - 4\,\text{Subst}\left(\int x^3\log\left(1+e^{2x}\right)dx, x, \cosh^{-1}(a$$
$$= -\frac{1}{5}\cosh^{-1}(ax)^5 + \cosh^{-1}(ax)^4\log\left(1+e^{2\cosh^{-1}(ax)}\right) + 2\cosh^{-1}(ax)^3\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right) - 6\,\text{Subst}$$
$$= -\frac{1}{5}\cosh^{-1}(ax)^5 + \cosh^{-1}(ax)^4\log\left(1+e^{2\cosh^{-1}(ax)}\right) + 2\cosh^{-1}(ax)^3\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right) - 3\cosh^{-1}$$
$$= -\frac{1}{5}\cosh^{-1}(ax)^5 + \cosh^{-1}(ax)^4\log\left(1+e^{2\cosh^{-1}(ax)}\right) + 2\cosh^{-1}(ax)^3\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right) - 3\cosh^{-1}$$
$$= -\frac{1}{5}\cosh^{-1}(ax)^5 + \cosh^{-1}(ax)^4\log\left(1+e^{2\cosh^{-1}(ax)}\right) + 2\cosh^{-1}(ax)^3\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right) - 3\cosh^{-1}$$
$$= -\frac{1}{5}\cosh^{-1}(ax)^5 + \cosh^{-1}(ax)^4\log\left(1+e^{2\cosh^{-1}(ax)}\right) + 2\cosh^{-1}(ax)^3\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right) - 3\cosh^{-1}$$

Mathematica [A] time = 0.0292115, size = 103, normalized size = 1.

$$-2\cosh^{-1}(ax)^3\text{PolyLog}\left(2, -e^{-2\cosh^{-1}(ax)}\right) - 3\cosh^{-1}(ax)^2\text{PolyLog}\left(3, -e^{-2\cosh^{-1}(ax)}\right) - 3\cosh^{-1}(ax)\text{PolyLog}\left(4, -e^{-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^4/x,x]
```

```
[Out] ArcCosh[a*x]^5/5 + ArcCosh[a*x]^4*Log[1 + E^(-2*ArcCosh[a*x])] - 2*ArcCosh[
a*x]^3*PolyLog[2, -E^(-2*ArcCosh[a*x])] - 3*ArcCosh[a*x]^2*PolyLog[3, -E^(-
2*ArcCosh[a*x])] - 3*ArcCosh[a*x]*PolyLog[4, -E^(-2*ArcCosh[a*x])] - (3*Pol
yLog[5, -E^(-2*ArcCosh[a*x])])/2
```

Maple [A] time = 0.037, size = 165, normalized size = 1.6

$$-\frac{(\text{arccosh}\,(ax))^5}{5} + (\text{arccosh}\,(ax))^4\ln\left(1+\left(ax+\sqrt{ax-1}\sqrt{ax+1}\right)^2\right) + 2\,(\text{arccosh}\,(ax))^3\,\text{polylog}\left(2, -\left(ax+\sqrt{ax-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^4/x,x)
```

```
[Out] -1/5*arccosh(a*x)^5+arccosh(a*x)^4*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2
)+2*arccosh(a*x)^3*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)-3*arccos
h(a*x)^2*polylog(3,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)+3*arccosh(a*x)*pol
ylog(4,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)-3/2*polylog(5,-(a*x+(a*x-1)^(1
/2)*(a*x+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4/x,x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^4/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcosh}(ax)^4}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4/x,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^4/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^4(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**4/x,x)
```

```
[Out] Integral(acosh(a*x)**4/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4/x,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^4/x, x)
```

3.39 $\int \frac{\cosh^{-1}(ax)^4}{x^2} dx$

Optimal. Leaf size=150

$$-12ia\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)+12ia\cosh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)+24ia\cosh^{-1}(ax)\text{PolyLog}\left(3,-\right.$$

```
[Out] -(ArcCosh[a*x]^4/x) + 8*a*ArcCosh[a*x]^3*ArcTan[E^ArcCosh[a*x]] - (12*I)*a*
ArcCosh[a*x]^2*PolyLog[2, (-I)*E^ArcCosh[a*x]] + (12*I)*a*ArcCosh[a*x]^2*Po
lyLog[2, I*E^ArcCosh[a*x]] + (24*I)*a*ArcCosh[a*x]*PolyLog[3, (-I)*E^ArcCos
h[a*x]] - (24*I)*a*ArcCosh[a*x]*PolyLog[3, I*E^ArcCosh[a*x]] - (24*I)*a*Pol
yLog[4, (-I)*E^ArcCosh[a*x]] + (24*I)*a*PolyLog[4, I*E^ArcCosh[a*x]]
```

Rubi [A] time = 0.334837, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5662, 5761, 4180, 2531, 6609, 2282, 6589}

$$-12ia\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)+12ia\cosh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)+24ia\cosh^{-1}(ax)\text{PolyLog}\left(3,-\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^4/x^2,x]
```


```
[Out] -(ArcCosh[a*x]^4/x) + 8*a*ArcCosh[a*x]^3*ArcTan[E^ArcCosh[a*x]] - (12*I)*a*
ArcCosh[a*x]^2*PolyLog[2, (-I)*E^ArcCosh[a*x]] + (12*I)*a*ArcCosh[a*x]^2*Po
lyLog[2, I*E^ArcCosh[a*x]] + (24*I)*a*ArcCosh[a*x]*PolyLog[3, (-I)*E^ArcCos
h[a*x]] - (24*I)*a*ArcCosh[a*x]*PolyLog[3, I*E^ArcCosh[a*x]] - (24*I)*a*Pol
yLog[4, (-I)*E^ArcCosh[a*x]] + (24*I)*a*PolyLog[4, I*E^ArcCosh[a*x]]
```


Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
```

```
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^4}{x^2}\,dx = -\frac{\cosh^{-1}(ax)^4}{x} + (4a)\int \frac{\cosh^{-1}(ax)^3}{x\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$
$$= -\frac{\cosh^{-1}(ax)^4}{x} + (4a)\,\text{Subst}\left(\int x^3\,\text{sech}(x)\,dx, x, \cosh^{-1}(ax)\right)$$
$$= -\frac{\cosh^{-1}(ax)^4}{x} + 8a\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - (12ia)\,\text{Subst}\left(\int x^2\log\left(1-ie^x\right)\,dx, x, \cosh^{-1}(\right.$$
$$= -\frac{\cosh^{-1}(ax)^4}{x} + 8a\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 12ia\cosh^{-1}(ax)^2\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) + 12ia\cosh$$
$$= -\frac{\cosh^{-1}(ax)^4}{x} + 8a\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 12ia\cosh^{-1}(ax)^2\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) + 12ia\cosh$$
$$= -\frac{\cosh^{-1}(ax)^4}{x} + 8a\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 12ia\cosh^{-1}(ax)^2\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) + 12ia\cosh$$
$$= -\frac{\cosh^{-1}(ax)^4}{x} + 8a\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 12ia\cosh^{-1}(ax)^2\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) + 12ia\cosh$$

Mathematica [B] time = 0.648351, size = 478, normalized size = 3.19

$$a\left(-12i\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right) + 12\pi\cosh^{-1}(ax)\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right) - 24i\cosh^{-1}(ax)\text{PolyLog}\left(3,-\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^4/x^2,x]
```

```
[Out] a*(((-7*I)/16)*Pi^4 + (Pi^3*ArcCosh[a*x])/2 - ((3*I)/2)*Pi^2*ArcCosh[a*x]^2
 - 2*Pi*ArcCosh[a*x]^3 + I*ArcCosh[a*x]^4 - ArcCosh[a*x]^4/(a*x) + (Pi^3*Lo
g[1 + I/E^ArcCosh[a*x]])/2 - (3*I)*Pi^2*ArcCosh[a*x]*Log[1 + I/E^ArcCosh[a*
x]] - 6*Pi*ArcCosh[a*x]^2*Log[1 + I/E^ArcCosh[a*x]] + (4*I)*ArcCosh[a*x]^3*
Log[1 + I/E^ArcCosh[a*x]] + (3*I)*Pi^2*ArcCosh[a*x]*Log[1 - I*E^ArcCosh[a*x
]] + 6*Pi*ArcCosh[a*x]^2*Log[1 - I*E^ArcCosh[a*x]] - (Pi^3*Log[1 + I*E^ArcC
osh[a*x]])/2 - (4*I)*ArcCosh[a*x]^3*Log[1 + I*E^ArcCosh[a*x]] + (Pi^3*Log[T
an[(Pi + (2*I)*ArcCosh[a*x])/4]])/2 + (3*I)*(Pi - (2*I)*ArcCosh[a*x])^2*Pol
yLog[2, (-I)/E^ArcCosh[a*x]] - (12*I)*ArcCosh[a*x]^2*PolyLog[2, (-I)*E^ArcC
osh[a*x]] + (3*I)*Pi^2*PolyLog[2, I*E^ArcCosh[a*x]] + 12*Pi*ArcCosh[a*x]*Po
lyLog[2, I*E^ArcCosh[a*x]] + 12*Pi*PolyLog[3, (-I)/E^ArcCosh[a*x]] - (24*I)
*ArcCosh[a*x]*PolyLog[3, (-I)/E^ArcCosh[a*x]] + (24*I)*ArcCosh[a*x]*PolyLog
[3, (-I)*E^ArcCosh[a*x]] - 12*Pi*PolyLog[3, I*E^ArcCosh[a*x]] - (24*I)*Poly
Log[4, (-I)/E^ArcCosh[a*x]] - (24*I)*PolyLog[4, (-I)*E^ArcCosh[a*x]])
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{\left(\operatorname{arccosh}\left(ax\right)\right)^{4}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^4/x^2,x)
```

```
[Out] int(arccosh(a*x)^4/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^{4}}{x}+\int \frac{4\left(a^{3}x^{2}+\sqrt{ax+1}\sqrt{ax-1}a^{2}x-a\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^{3}}{a^{3}x^{4}-ax^{2}+\left(a^{2}x^{3}-x\right)\sqrt{ax+1}\sqrt{ax-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4/x^2,x, algorithm="maxima")
```

```
[Out] -log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^4/x + integrate(4*(a^3*x^2 + sqrt(a
*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/(
a^3*x^4 - a*x^2 + (a^2*x^3 - x)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcosh}\left(ax\right)^{4}}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^4/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^4(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**4/x**2,x)
```

```
[Out] Integral(acosh(a*x)**4/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^4}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^4/x^2, x)
```

3.40 $\int \frac{\cosh^{-1}(ax)^4}{x^3}\,dx$

Optimal. Leaf size=115

$$-6a^2\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)+3a^2\text{PolyLog}\left(3,-e^{2\cosh^{-1}(ax)}\right)+2a^2\cosh^{-1}(ax)^3-6a^2\cosh^{-1}(ax)^2\log\left(e\right.$$

```
[Out] 2*a^2*ArcCosh[a*x]^3 + (2*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/x
- ArcCosh[a*x]^4/(2*x^2) - 6*a^2*ArcCosh[a*x]^2*Log[1 + E^(2*ArcCosh[a*x])]
 - 6*a^2*ArcCosh[a*x]*PolyLog[2, -E^(2*ArcCosh[a*x])] + 3*a^2*PolyLog[3, -E
^(2*ArcCosh[a*x])]
```

Rubi [A] time = 0.358169, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {5662, 5724, 5660, 3718, 2190, 2531, 2282, 6589}

$$-6a^2\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)+3a^2\text{PolyLog}\left(3,-e^{2\cosh^{-1}(ax)}\right)+2a^2\cosh^{-1}(ax)^3-6a^2\cosh^{-1}(ax)^2\log\left(e\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^4/x^3,x]
```


```
[Out] 2*a^2*ArcCosh[a*x]^3 + (2*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/x
- ArcCosh[a*x]^4/(2*x^2) - 6*a^2*ArcCosh[a*x]^2*Log[1 + E^(2*ArcCosh[a*x])]
 - 6*a^2*ArcCosh[a*x]*PolyLog[2, -E^(2*ArcCosh[a*x])] + 3*a^2*PolyLog[3, -E
^(2*ArcCosh[a*x])]
```


Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
```

```
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^4}{x^3}\,dx = -\frac{\cosh^{-1}(ax)^4}{2x^2} + (2a)\int \frac{\cosh^{-1}(ax)^3}{x^2\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$
$$= \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{x} - \frac{\cosh^{-1}(ax)^4}{2x^2} - \left(6a^2\right)\int \frac{\cosh^{-1}(ax)^2}{x}\,dx$$
$$= \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{x} - \frac{\cosh^{-1}(ax)^4}{2x^2} - \left(6a^2\right)\text{Subst}\left(\int x^2\tanh(x)\,dx, x, \cosh^{-1}(ax)\right)$$
$$= 2a^2\cosh^{-1}(ax)^3 + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{x} - \frac{\cosh^{-1}(ax)^4}{2x^2} - \left(12a^2\right)\text{Subst}\left(\int \frac{e^{2x}x^2}{1+e^{2x}}\,dx\right.$$
$$= 2a^2\cosh^{-1}(ax)^3 + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{x} - \frac{\cosh^{-1}(ax)^4}{2x^2} - 6a^2\cosh^{-1}(ax)^2\log\left(1+e^{2}\right.$$
$$= 2a^2\cosh^{-1}(ax)^3 + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{x} - \frac{\cosh^{-1}(ax)^4}{2x^2} - 6a^2\cosh^{-1}(ax)^2\log\left(1+e^{2}\right.$$
$$= 2a^2\cosh^{-1}(ax)^3 + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{x} - \frac{\cosh^{-1}(ax)^4}{2x^2} - 6a^2\cosh^{-1}(ax)^2\log\left(1+e^{2}\right.$$
$$= 2a^2\cosh^{-1}(ax)^3 + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{x} - \frac{\cosh^{-1}(ax)^4}{2x^2} - 6a^2\cosh^{-1}(ax)^2\log\left(1+e^{2}\right.$$

Mathematica [A] time = 1.07254, size = 112, normalized size = 0.97

$$a^2\left(6\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(ax)}\right) + 3\text{PolyLog}\left(3,-e^{-2\cosh^{-1}(ax)}\right) + 2\cosh^{-1}(ax)^2\left(\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\cosh^{-1}(ax\right.}{ax}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^4/x^3,x]
```

```
[Out] -ArcCosh[a*x]^4/(2*x^2) + a^2*(2*ArcCosh[a*x]^2*(-ArcCosh[a*x] + (Sqrt[(-1
+ a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x])/(a*x) - 3*Log[1 + E^(-2*ArcCosh[a
*x])]) + 6*ArcCosh[a*x]*PolyLog[2, -E^(-2*ArcCosh[a*x])] + 3*PolyLog[3, -E^
(-2*ArcCosh[a*x])])
```

Maple [A] time = 0.071, size = 149, normalized size = 1.3

$$2a^2(\operatorname{arccosh}(ax))^3 - \frac{(\operatorname{arccosh}(ax))^4}{2x^2} - 6a^2(\operatorname{arccosh}(ax))^2\ln\left(1+\left(ax+\sqrt{ax-1}\sqrt{ax+1}\right)^2\right) - 6a^2\operatorname{arccosh}(ax)\operatorname{polylo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^4/x^3,x)
```

```
[Out] 2*a^2*arccosh(a*x)^3-1/2*arccosh(a*x)^4/x^2-6*a^2*arccosh(a*x)^2*ln(1+(a*x+
(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)-6*a^2*arccosh(a*x)*polylog(2,-(a*x+(a*x-1)^
(1/2)*(a*x+1)^(1/2))^2)+3*a^2*polylog(3,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^
2)+2*a*arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^4}{2x^2} + \int \frac{2\left(a^3x^2+\sqrt{ax+1}\sqrt{ax-1}a^2x-a\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^3}{a^3x^5-ax^3+\left(a^2x^4-x^2\right)\sqrt{ax+1}\sqrt{ax-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^4/x^2 + integrate(2*(a^3*x^2 +
sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1
))^3/(a^3*x^5 - a*x^3 + (a^2*x^4 - x^2)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcosh}(ax)^4}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4/x^3,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^4/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^4(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**4/x**3,x)
```

```
[Out] Integral(acosh(a*x)**4/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^4}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^4/x^3, x)
```

3.41 $\int \frac{\cosh^{-1}(ax)^4}{x^4} dx$

Optimal. Leaf size=268

$$-2ia^3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)+2ia^3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)+4ia^3\cosh^{-1}(ax)\text{PolyLog}\left(3,-\right.$$

```
[Out] (2*a^2*ArcCosh[a*x]^2)/x + (2*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3
)/(3*x^2) - ArcCosh[a*x]^4/(3*x^3) - 8*a^3*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*
x]] + (4*a^3*ArcCosh[a*x]^3*ArcTan[E^ArcCosh[a*x]])/3 + (4*I)*a^3*PolyLog[2
, (-I)*E^ArcCosh[a*x]] - (2*I)*a^3*ArcCosh[a*x]^2*PolyLog[2, (-I)*E^ArcCosh
[a*x]] - (4*I)*a^3*PolyLog[2, I*E^ArcCosh[a*x]] + (2*I)*a^3*ArcCosh[a*x]^2*
PolyLog[2, I*E^ArcCosh[a*x]] + (4*I)*a^3*ArcCosh[a*x]*PolyLog[3, (-I)*E^Arc
Cosh[a*x]] - (4*I)*a^3*ArcCosh[a*x]*PolyLog[3, I*E^ArcCosh[a*x]] - (4*I)*a^
3*PolyLog[4, (-I)*E^ArcCosh[a*x]] + (4*I)*a^3*PolyLog[4, I*E^ArcCosh[a*x]]
```

Rubi [A] time = 0.80247, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5662, 5748, 5761, 4180, 2531, 6609, 2282, 6589, 2279, 2391}

$$-2ia^3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)+2ia^3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)+4ia^3\cosh^{-1}(ax)\text{PolyLog}\left(3,-\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^4/x^4,x]
```


```
[Out] (2*a^2*ArcCosh[a*x]^2)/x + (2*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3
)/(3*x^2) - ArcCosh[a*x]^4/(3*x^3) - 8*a^3*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*
x]] + (4*a^3*ArcCosh[a*x]^3*ArcTan[E^ArcCosh[a*x]])/3 + (4*I)*a^3*PolyLog[2
, (-I)*E^ArcCosh[a*x]] - (2*I)*a^3*ArcCosh[a*x]^2*PolyLog[2, (-I)*E^ArcCosh
[a*x]] - (4*I)*a^3*PolyLog[2, I*E^ArcCosh[a*x]] + (2*I)*a^3*ArcCosh[a*x]^2*
PolyLog[2, I*E^ArcCosh[a*x]] + (4*I)*a^3*ArcCosh[a*x]*PolyLog[3, (-I)*E^Arc
Cosh[a*x]] - (4*I)*a^3*ArcCosh[a*x]*PolyLog[3, I*E^ArcCosh[a*x]] - (4*I)*a^
3*PolyLog[4, (-I)*E^ArcCosh[a*x]] + (4*I)*a^3*PolyLog[4, I*E^ArcCosh[a*x]]
```


Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
```

```
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^4}{x^4}\,dx = -\frac{\cosh^{-1}(ax)^4}{3x^3} + \frac{1}{3}(4a)\int \frac{\cosh^{-1}(ax)^3}{x^3\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2} - \frac{\cosh^{-1}(ax)^4}{3x^3} - \left(2a^2\right)\int \frac{\cosh^{-1}(ax)^2}{x^2}\,dx + \frac{1}{3}\left(2a^3\right)\int \frac{\cosh}{x\sqrt{-1+}}$$

$$= \frac{2a^2\cosh^{-1}(ax)^2}{x} + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2} - \frac{\cosh^{-1}(ax)^4}{3x^3} + \frac{1}{3}\left(2a^3\right)\mathrm{Subst}\left(\int x^3\,\mathbf{sech}(\right.$$

$$= \frac{2a^2\cosh^{-1}(ax)^2}{x} + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2} - \frac{\cosh^{-1}(ax)^4}{3x^3} + \frac{4}{3}a^3\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\mathrm{co}}\right.$$

$$= \frac{2a^2\cosh^{-1}(ax)^2}{x} + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2} - \frac{\cosh^{-1}(ax)^4}{3x^3} - 8a^3\cosh^{-1}(ax)\tan^{-1}\left(e^{\mathrm{cosh}}\right.$$

$$= \frac{2a^2\cosh^{-1}(ax)^2}{x} + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2} - \frac{\cosh^{-1}(ax)^4}{3x^3} - 8a^3\cosh^{-1}(ax)\tan^{-1}\left(e^{\mathrm{cosh}}\right.$$

$$= \frac{2a^2\cosh^{-1}(ax)^2}{x} + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2} - \frac{\cosh^{-1}(ax)^4}{3x^3} - 8a^3\cosh^{-1}(ax)\tan^{-1}\left(e^{\mathrm{cosh}}\right.$$

$$= \frac{2a^2\cosh^{-1}(ax)^2}{x} + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2} - \frac{\cosh^{-1}(ax)^4}{3x^3} - 8a^3\cosh^{-1}(ax)\tan^{-1}\left(e^{\mathrm{cosh}}\right.$$

Mathematica [B] time = 3.07534, size = 595, normalized size = 2.22

$$a^3\left(\frac{1}{2}i\left(-4\cosh^{-1}(ax)^2 - 4i\pi\cosh^{-1}(ax) + \pi^2 + 8\right)\mathbf{PolyLog}\left(2, -ie^{-\cosh^{-1}(ax)}\right) - \frac{1}{96}i\left(192\cosh^{-1}(ax)^2\mathbf{PolyLog}\left(2, -ie\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^4/x^4,x]
```

```
[Out] a^3*((I/2)*(8 + Pi^2 - (4*I)*Pi*ArcCosh[a*x] - 4*ArcCosh[a*x]^2)*PolyLog[2,
 (-I)/E^ArcCosh[a*x]] - (I/96)*(7*Pi^4 + (8*I)*Pi^3*ArcCosh[a*x] + 24*Pi^2*
ArcCosh[a*x]^2 + ((192*I)*ArcCosh[a*x]^2)/(a*x) - (32*I)*Pi*ArcCosh[a*x]^3
+ ((64*I)*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x]^3)/(a^2*x^2) -
16*ArcCosh[a*x]^4 - ((32*I)*ArcCosh[a*x]^4)/(a^3*x^3) - 384*ArcCosh[a*x]*Lo
g[1 - I/E^ArcCosh[a*x]] + (8*I)*Pi^3*Log[1 + I/E^ArcCosh[a*x]] + 384*ArcCos
h[a*x]*Log[1 + I/E^ArcCosh[a*x]] + 48*Pi^2*ArcCosh[a*x]*Log[1 + I/E^ArcCosh
[a*x]] - (96*I)*Pi*ArcCosh[a*x]^2*Log[1 + I/E^ArcCosh[a*x]] - 64*ArcCosh[a*
x]^3*Log[1 + I/E^ArcCosh[a*x]] - 48*Pi^2*ArcCosh[a*x]*Log[1 - I*E^ArcCosh[a
*x]] + (96*I)*Pi*ArcCosh[a*x]^2*Log[1 - I*E^ArcCosh[a*x]] - (8*I)*Pi^3*Log[
1 + I*E^ArcCosh[a*x]] + 64*ArcCosh[a*x]^3*Log[1 + I*E^ArcCosh[a*x]] + (8*I)
*Pi^3*Log[Tan[(Pi + (2*I)*ArcCosh[a*x])/4]] + 384*PolyLog[2, I/E^ArcCosh[a*
```

```
x]] + 192*ArcCosh[a*x]^2*PolyLog[2, (-I)*E^ArcCosh[a*x]] - 48*Pi^2*PolyLog[
2, I*E^ArcCosh[a*x]] + (192*I)*Pi*ArcCosh[a*x]*PolyLog[2, I*E^ArcCosh[a*x]]
 + (192*I)*Pi*PolyLog[3, (-I)/E^ArcCosh[a*x]] + 384*ArcCosh[a*x]*PolyLog[3,
 (-I)/E^ArcCosh[a*x]] - 384*ArcCosh[a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]] -
(192*I)*Pi*PolyLog[3, I*E^ArcCosh[a*x]] + 384*PolyLog[4, (-I)/E^ArcCosh[a*x
]] + 384*PolyLog[4, (-I)*E^ArcCosh[a*x]]))
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{arccosh}(ax))^4}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^4/x^4,x)
```

```
[Out] int(arccosh(a*x)^4/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^4}{3\,x^3}+\int \frac{4\left(a^3x^2+\sqrt{ax+1}\sqrt{ax-1}a^2x-a\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^3}{3\left(a^3x^6-ax^4+\left(a^2x^5-x^3\right)\sqrt{ax+1}\sqrt{ax-1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^4/x^3 + integrate(4/3*(a^3*x^2
+ sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x -
 1))^3/(a^3*x^6 - a*x^4 + (a^2*x^5 - x^3)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcosh}(ax)^4}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4/x^4,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^4/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^4(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**4/x**4,x)
```

```
[Out] Integral(acosh(a*x)**4/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^4}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^4/x^4, x)
```

3.42 $\int \frac{x^6}{\cosh^{-1}(ax)} dx$

Optimal. Leaf size=55

$$\frac{5\text{Shi}\left(\cosh^{-1}(ax)\right)}{64a^7}+\frac{9\text{Shi}\left(3\cosh^{-1}(ax)\right)}{64a^7}+\frac{5\text{Shi}\left(5\cosh^{-1}(ax)\right)}{64a^7}+\frac{\text{Shi}\left(7\cosh^{-1}(ax)\right)}{64a^7}$$

```
[Out] (5*SinhIntegral[ArcCosh[a*x]])/(64*a^7) + (9*SinhIntegral[3*ArcCosh[a*x]])/
(64*a^7) + (5*SinhIntegral[5*ArcCosh[a*x]])/(64*a^7) + SinhIntegral[7*ArcCo
sh[a*x]]/(64*a^7)
```

Rubi [A] time = 0.0960031, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5670, 5448, 3298}

$$\frac{5\text{Shi}\left(\cosh^{-1}(ax)\right)}{64a^7}+\frac{9\text{Shi}\left(3\cosh^{-1}(ax)\right)}{64a^7}+\frac{5\text{Shi}\left(5\cosh^{-1}(ax)\right)}{64a^7}+\frac{\text{Shi}\left(7\cosh^{-1}(ax)\right)}{64a^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/ArcCosh[a*x],x]
```


```
[Out] (5*SinhIntegral[ArcCosh[a*x]])/(64*a^7) + (9*SinhIntegral[3*ArcCosh[a*x]])/
(64*a^7) + (5*SinhIntegral[5*ArcCosh[a*x]])/(64*a^7) + SinhIntegral[7*ArcCo
sh[a*x]]/(64*a^7)
```


Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^6}{\cosh^{-1}(ax)}\,dx = \frac{\mathrm{Subst}\left(\int \frac{\cosh^6(x)\sinh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^7}$$
$$= \frac{\mathrm{Subst}\left(\int \left(\frac{5\sinh(x)}{64x}+\frac{9\sinh(3x)}{64x}+\frac{5\sinh(5x)}{64x}+\frac{\sinh(7x)}{64x}\right)dx, x, \cosh^{-1}(ax)\right)}{a^7}$$
$$= \frac{\mathrm{Subst}\left(\int \frac{\sinh(7x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{64a^7}+\frac{5\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{64a^7}+\frac{5\,\mathrm{Subst}\left(\int \frac{\sinh(5x)}{x}\right.}{64a}$$
$$= \frac{5\mathrm{Shi}\left(\cosh^{-1}(ax)\right)}{64a^7}+\frac{9\mathrm{Shi}\left(3\cosh^{-1}(ax)\right)}{64a^7}+\frac{5\mathrm{Shi}\left(5\cosh^{-1}(ax)\right)}{64a^7}+\frac{\mathrm{Shi}\left(7\cosh^{-1}(ax)\right)}{64a^7}$$

Mathematica [A] time = 0.0946018, size = 40, normalized size = 0.73

$$\frac{5\mathrm{Shi}\left(\cosh^{-1}(ax)\right)+9\mathrm{Shi}\left(3\cosh^{-1}(ax)\right)+5\mathrm{Shi}\left(5\cosh^{-1}(ax)\right)+\mathrm{Shi}\left(7\cosh^{-1}(ax)\right)}{64a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/ArcCosh[a*x],x]
```

```
[Out] (5*SinhIntegral[ArcCosh[a*x]] + 9*SinhIntegral[3*ArcCosh[a*x]] + 5*SinhInte
gral[5*ArcCosh[a*x]] + SinhIntegral[7*ArcCosh[a*x]])/(64*a^7)
```

Maple [A] time = 0.039, size = 40, normalized size = 0.7

$$\frac{1}{a^7}\left(\frac{5\,\mathrm{Shi}\left(\mathrm{arccosh}\left(ax\right)\right)}{64}+\frac{9\,\mathrm{Shi}\left(3\,\mathrm{arccosh}\left(ax\right)\right)}{64}+\frac{5\,\mathrm{Shi}\left(5\,\mathrm{arccosh}\left(ax\right)\right)}{64}+\frac{\mathrm{Shi}\left(7\,\mathrm{arccosh}\left(ax\right)\right)}{64}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/arccosh(a*x),x)
```

```
[Out] 1/a^7*(5/64*Shi(arccosh(a*x))+9/64*Shi(3*arccosh(a*x))+5/64*Shi(5*arccosh(a
*x))+1/64*Shi(7*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6/arccosh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{\operatorname{arcosh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^6/arccosh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/acosh(a*x),x)
```

```
[Out] Integral(x**6/acosh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^6/arccosh(a*x), x)
```

3.43 $\int \frac{x^5}{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=43

$$\frac{5\text{Shi}\left(2\cosh^{-1}(ax)\right)}{32a^6}+\frac{\text{Shi}\left(4\cosh^{-1}(ax)\right)}{8a^6}+\frac{\text{Shi}\left(6\cosh^{-1}(ax)\right)}{32a^6}$$

```
[Out] (5*SinhIntegral[2*ArcCosh[a*x]])/(32*a^6) + SinhIntegral[4*ArcCosh[a*x]]/(8
*a^6) + SinhIntegral[6*ArcCosh[a*x]]/(32*a^6)
```

Rubi [A] time = 0.0841293, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5670, 5448, 3298}

$$\frac{5\text{Shi}\left(2\cosh^{-1}(ax)\right)}{32a^6}+\frac{\text{Shi}\left(4\cosh^{-1}(ax)\right)}{8a^6}+\frac{\text{Shi}\left(6\cosh^{-1}(ax)\right)}{32a^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/ArcCosh[a*x],x]
```

```
[Out] (5*SinhIntegral[2*ArcCosh[a*x]])/(32*a^6) + SinhIntegral[4*ArcCosh[a*x]]/(8
*a^6) + SinhIntegral[6*ArcCosh[a*x]]/(32*a^6)
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\cosh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^6}\\ &= \frac{\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32x} + \frac{\sinh(4x)}{8x} + \frac{\sinh(6x)}{32x}\right)dx, x, \cosh^{-1}(ax)\right)}{a^6}\\ &= \frac{\text{Subst}\left(\int \frac{\sinh(6x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{32a^6} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{8a^6} + \frac{5\,\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,d\right.}{32a^6}\\ &= \frac{5\text{Shi}\left(2\cosh^{-1}(ax)\right)}{32a^6} + \frac{\text{Shi}\left(4\cosh^{-1}(ax)\right)}{8a^6} + \frac{\text{Shi}\left(6\cosh^{-1}(ax)\right)}{32a^6}\end{aligned}$$

Mathematica [A] time = 0.0828145, size = 33, normalized size = 0.77

$$\frac{5\text{Shi}\left(2\cosh^{-1}(ax)\right) + 4\text{Shi}\left(4\cosh^{-1}(ax)\right) + \text{Shi}\left(6\cosh^{-1}(ax)\right)}{32a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/ArcCosh[a*x],x]
```

```
[Out] (5*SinhIntegral[2*ArcCosh[a*x]] + 4*SinhIntegral[4*ArcCosh[a*x]] + SinhInte
gral[6*ArcCosh[a*x]])/(32*a^6)
```

Maple [A] time = 0.033, size = 33, normalized size = 0.8

$$\frac{1}{a^6}\left(\frac{5\,\text{Shi}\,(2\,\text{arccosh}\,(ax))}{32} + \frac{\text{Shi}\,(4\,\text{arccosh}\,(ax))}{8} + \frac{\text{Shi}\,(6\,\text{arccosh}\,(ax))}{32}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/arccosh(a*x),x)
```

```
[Out] 1/a^6*(5/32*Shi(2*arccosh(a*x))+1/8*Shi(4*arccosh(a*x))+1/32*Shi(6*arccosh(
a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5/arccosh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^5}{\operatorname{arcosh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^5/arccosh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/acosh(a*x),x)
```

```
[Out] Integral(x**5/acosh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^5/arccosh(a*x), x)
```

3.44 $\int \frac{x^4}{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=41

$$\frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{8a^5}+\frac{3\text{Shi}\left(3\cosh^{-1}(ax)\right)}{16a^5}+\frac{\text{Shi}\left(5\cosh^{-1}(ax)\right)}{16a^5}$$

```
[Out] SinhIntegral[ArcCosh[a*x]]/(8*a^5) + (3*SinhIntegral[3*ArcCosh[a*x]])/(16*a
^5) + SinhIntegral[5*ArcCosh[a*x]]/(16*a^5)
```

Rubi [A] time = 0.0801527, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5670, 5448, 3298}

$$\frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{8a^5}+\frac{3\text{Shi}\left(3\cosh^{-1}(ax)\right)}{16a^5}+\frac{\text{Shi}\left(5\cosh^{-1}(ax)\right)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCosh[a*x],x]
```

```
[Out] SinhIntegral[ArcCosh[a*x]]/(8*a^5) + (3*SinhIntegral[3*ArcCosh[a*x]])/(16*a
^5) + SinhIntegral[5*ArcCosh[a*x]]/(16*a^5)
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\cosh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^5} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{8x}+\frac{3\sinh(3x)}{16x}+\frac{\sinh(5x)}{16x}\right) dx, x, \cosh^{-1}(ax)\right)}{a^5} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{16a^5}+\frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{8a^5}+\frac{3\,\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx\right.}{16a^5} \\
&= \frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{8a^5}+\frac{3\text{Shi}\left(3\cosh^{-1}(ax)\right)}{16a^5}+\frac{\text{Shi}\left(5\cosh^{-1}(ax)\right)}{16a^5}
\end{aligned}$$

Mathematica [A] time = 0.0713324, size = 31, normalized size = 0.76

$$\frac{2\text{Shi}\left(\cosh^{-1}(ax)\right)+3\text{Shi}\left(3\cosh^{-1}(ax)\right)+\text{Shi}\left(5\cosh^{-1}(ax)\right)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcCosh[a*x],x]
```

```
[Out] (2*SinhIntegral[ArcCosh[a*x]] + 3*SinhIntegral[3*ArcCosh[a*x]] + SinhIntegr
al[5*ArcCosh[a*x]])/(16*a^5)
```

Maple [A] time = 0.026, size = 31, normalized size = 0.8

$$\frac{1}{a^5}\left(\frac{\text{Shi}\,(\text{arccosh}\,(ax))}{8}+\frac{3\,\text{Shi}\,(3\,\text{arccosh}\,(ax))}{16}+\frac{\text{Shi}\,(5\,\text{arccosh}\,(ax))}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccosh(a*x),x)
```

```
[Out] 1/a^5*(1/8*Shi(arccosh(a*x))+3/16*Shi(3*arccosh(a*x))+1/16*Shi(5*arccosh(a*
x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/arccosh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^4}{\operatorname{arcosh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^4/arccosh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acosh(a*x),x)
```

```
[Out] Integral(x**4/acosh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4/arccosh(a*x), x)
```

3.45 $\int \frac{x^3}{\cosh^{-1}(ax)} \, dx$

Optimal. Leaf size=29

$$\frac{\text{Shi}\left(2\cosh^{-1}(ax)\right)}{4a^4}+\frac{\text{Shi}\left(4\cosh^{-1}(ax)\right)}{8a^4}$$

```
[Out] SinhIntegral[2*ArcCosh[a*x]]/(4*a^4) + SinhIntegral[4*ArcCosh[a*x]]/(8*a^4)
```

Rubi [A] time = 0.0655709, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5670, 5448, 3298}

$$\frac{\text{Shi}\left(2\cosh^{-1}(ax)\right)}{4a^4}+\frac{\text{Shi}\left(4\cosh^{-1}(ax)\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCosh[a*x],x]
```

```
[Out] SinhIntegral[2*ArcCosh[a*x]]/(4*a^4) + SinhIntegral[4*ArcCosh[a*x]]/(8*a^4)
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\cosh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^4} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4x} + \frac{\sinh(4x)}{8x}\right)dx, x, \cosh^{-1}(ax)\right)}{a^4} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{8a^4} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{4a^4} \\
&= \frac{\text{Shi}\left(2\cosh^{-1}(ax)\right)}{4a^4} + \frac{\text{Shi}\left(4\cosh^{-1}(ax)\right)}{8a^4}
\end{aligned}
$$

Mathematica [A] time = 0.06114, size = 24, normalized size = 0.83

$$\frac{2\text{Shi}\left(2\cosh^{-1}(ax)\right) + \text{Shi}\left(4\cosh^{-1}(ax)\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcCosh[a*x],x]
```

```
[Out] (2*SinhIntegral[2*ArcCosh[a*x]] + SinhIntegral[4*ArcCosh[a*x]])/(8*a^4)
```

Maple [A] time = 0.027, size = 24, normalized size = 0.8

$$\frac{1}{a^4}\left(\frac{\text{Shi}\,(2\,\text{arccosh}\,(ax))}{4} + \frac{\text{Shi}\,(4\,\text{arccosh}\,(ax))}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccosh(a*x),x)
```

```
[Out] 1/a^4*(1/4*Shi(2*arccosh(a*x))+1/8*Shi(4*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arcosh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/arccosh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^3}{\operatorname{arcosh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^3/arccosh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acosh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acosh(a*x),x)
```

```
[Out] Integral(x**3/acosh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arcosh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^3/arccosh(a*x), x)
```

3.46 $\int \frac{x^2}{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{\operatorname{Shi}\left(\cosh^{-1}(ax)\right)}{4a^3}+\frac{\operatorname{Shi}\left(3\cosh^{-1}(ax)\right)}{4a^3}$$

```
[Out] SinhIntegral[ArcCosh[a*x]]/(4*a^3) + SinhIntegral[3*ArcCosh[a*x]]/(4*a^3)
```

Rubi [A] time = 0.0640484, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5670, 5448, 3298}

$$\frac{\operatorname{Shi}\left(\cosh^{-1}(ax)\right)}{4a^3}+\frac{\operatorname{Shi}\left(3\cosh^{-1}(ax)\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCosh[a*x],x]
```

```
[Out] SinhIntegral[ArcCosh[a*x]]/(4*a^3) + SinhIntegral[3*ArcCosh[a*x]]/(4*a^3)
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\cosh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^3}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{4x} + \frac{\sinh(3x)}{4x}\right)dx, x, \cosh^{-1}(ax)\right)}{a^3}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{4a^3} + \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{4a^3}\\
&= \frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{4a^3} + \frac{\text{Shi}\left(3\cosh^{-1}(ax)\right)}{4a^3}
\end{aligned}
$$

Mathematica [A] time = 0.0523676, size = 20, normalized size = 0.74

$$\frac{\text{Shi}\left(\cosh^{-1}(ax)\right) + \text{Shi}\left(3\cosh^{-1}(ax)\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcCosh[a*x],x]
```

```
[Out] (SinhIntegral[ArcCosh[a*x]] + SinhIntegral[3*ArcCosh[a*x]])/(4*a^3)
```

Maple [A] time = 0.03, size = 22, normalized size = 0.8

$$\frac{1}{a^3}\left(\frac{\text{Shi}(\text{arccosh}(ax))}{4} + \frac{\text{Shi}(3\,\text{arccosh}(ax))}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccosh(a*x),x)
```

```
[Out] 1/a^3*(1/4*Shi(arccosh(a*x))+1/4*Shi(3*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/arccosh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\operatorname{arcosh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^2/arccosh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acosh(a*x),x)
```

```
[Out] Integral(x**2/acosh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2/arccosh(a*x), x)
```

3.47 $\int \frac{x}{\cosh^{-1}(ax)} dx$

Optimal. Leaf size=14

$$\frac{\text{Shi}\left(2\cosh^{-1}(ax)\right)}{2a^2}$$

```
[Out] SinhIntegral[2*ArcCosh[a*x]]/(2*a^2)
```

Rubi [A] time = 0.0369433, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5670, 5448, 12, 3298}

$$\frac{\text{Shi}\left(2\cosh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCosh[a*x],x]
```

```
[Out] SinhIntegral[2*ArcCosh[a*x]]/(2*a^2)
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{\cosh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \cosh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{2a^2}$$
$$= \frac{\text{Shi}\left(2\cosh^{-1}(ax)\right)}{2a^2}$$

Mathematica [A] time = 0.0218729, size = 14, normalized size = 1.

$$\frac{\text{Shi}\left(2\cosh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcCosh[a*x],x]
```

```
[Out] SinhIntegral[2*ArcCosh[a*x]]/(2*a^2)
```

Maple [A] time = 0.024, size = 13, normalized size = 0.9

$$\frac{\text{Shi}\left(2\,\text{arccosh}\,(ax)\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccosh(a*x),x)
```

```
[Out] 1/2*Shi(2*arccosh(a*x))/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/arccosh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\operatorname{arcosh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x/arccosh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acosh(a*x),x)
```

```
[Out] Integral(x/acosh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x/arccosh(a*x), x)
```

3.48 $\int \frac{1}{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{a}$$

```
[Out] SinhIntegral[ArcCosh[a*x]]/a
```

Rubi [A] time = 0.0171132, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5658, 3298}

$$\frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^(-1),x]
```

```
[Out] SinhIntegral[ArcCosh[a*x]]/a
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\cosh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{a}$$

Mathematica [A] time = 0.0224391, size = 9, normalized size = 1.

$$\frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^(-1),x]
```

```
[Out] SinhIntegral[ArcCosh[a*x]]/a
```

Maple [A] time = 0.021, size = 10, normalized size = 1.1

$$\frac{\text{Shi}\left(\text{arccosh}\left(ax\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x),x)
```

```
[Out] Shi(arccosh(a*x))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\text{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/arccosh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\operatorname{arcosh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/arccosh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x),x)
```

```
[Out] Integral(1/acosh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/arccosh(a*x), x)
```

3.49 $\int \frac{1}{x \cosh^{-1}(ax)} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x \cosh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcCosh[a*x]), x]
```

Rubi [A] time = 0.0138482, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \cosh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcCosh[a*x]),x]
```

```
[Out] Defer[Int][1/(x*ArcCosh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x \cosh^{-1}(ax)} dx = \int \frac{1}{x \cosh^{-1}(ax)} dx$$

Mathematica [A] time = 0.19446, size = 0, normalized size = 0.

$$\int \frac{1}{x \cosh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcCosh[a*x]),x]
```

```
[Out] Integrate[1/(x*ArcCosh[a*x]), x]
```

Maple [A] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{x\mathrm{arccosh}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccosh(a*x),x)
```

```
[Out] int(1/x/arccosh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\mathrm{arcosh}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arccosh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x\,\mathrm{arcosh}\left(ax\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/(x*arccosh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\mathrm{acosh}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acosh(a*x),x)
```

```
[Out] Integral(1/(x*acosh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arcosh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccosh(a*x)), x)
```

3.50 $\int \frac{1}{x^2 \cosh^{-1}(ax)} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x^2 \cosh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x^2*ArcCosh[a*x]), x]
```

Rubi [A] time = 0.01444, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \cosh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*ArcCosh[a*x]),x]

[Out] Defer[Int][1/(x^2*ArcCosh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2 \cosh^{-1}(ax)} \, dx = \int \frac{1}{x^2 \cosh^{-1}(ax)} \, dx$$

Mathematica [A] time = 0.420528, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \cosh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*ArcCosh[a*x]),x]

[Out] Integrate[1/(x^2*ArcCosh[a*x]), x]
```

Maple [A] time = 0.089, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}\mathrm{arccosh}\left(a x\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccosh(a*x),x)
```

```
[Out] int(1/x^2/arccosh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \operatorname{arcosh}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*arccosh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x^{2} \operatorname{arcosh}\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*arccosh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \operatorname{acosh}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acosh(a*x),x)
```

```
[Out] Integral(1/(x**2*acosh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{arcosh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arccosh(a*x)), x)
```

3.51 $\int \frac{x^4}{\cosh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=73

$$\frac{\text{Chi}\left(\cosh^{-1}(ax)\right)}{8a^5}+\frac{9\text{Chi}\left(3\cosh^{-1}(ax)\right)}{16a^5}+\frac{5\text{Chi}\left(5\cosh^{-1}(ax)\right)}{16a^5}-\frac{x^4\sqrt{ax-1}\sqrt{ax+1}}{a\cosh^{-1}(ax)}$$

```
[Out] -((x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*ArcCosh[a*x])) + CoshIntegral[ArcCo
sh[a*x]]/(8*a^5) + (9*CoshIntegral[3*ArcCosh[a*x]])/(16*a^5) + (5*CoshInteg
ral[5*ArcCosh[a*x]])/(16*a^5)
```

Rubi [A] time = 0.0643322, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5666, 3301}

$$\frac{\text{Chi}\left(\cosh^{-1}(ax)\right)}{8a^5}+\frac{9\text{Chi}\left(3\cosh^{-1}(ax)\right)}{16a^5}+\frac{5\text{Chi}\left(5\cosh^{-1}(ax)\right)}{16a^5}-\frac{x^4\sqrt{ax-1}\sqrt{ax+1}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCosh[a*x]^2,x]
```

```
[Out] -((x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*ArcCosh[a*x])) + CoshIntegral[ArcCo
sh[a*x]]/(8*a^5) + (9*CoshIntegral[3*ArcCosh[a*x]])/(16*a^5) + (5*CoshInteg
ral[5*ArcCosh[a*x]])/(16*a^5)
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4}{\cosh^{-1}(ax)^2}\,dx = -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int\left(-\frac{\cosh(x)}{8x}-\frac{9\cosh(3x)}{16x}-\frac{5\cosh(5x)}{16x}\right)dx, x, \cosh^{-1}(ax)\right)}{a^5}$$

$$= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}dx, x, \cosh^{-1}(ax)\right)}{8a^5} + \frac{5\,\text{Subst}\left(\int \frac{\cosh(5x)}{x}dx, x, \cosh^{-1}(ax\right.}{16a^5}$$

$$= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\text{Chi}\left(\cosh^{-1}(ax)\right)}{8a^5} + \frac{9\text{Chi}\left(3\cosh^{-1}(ax)\right)}{16a^5} + \frac{5\text{Chi}\left(5\cosh^{-1}(ax)\right)}{16a^5}$$

Mathematica [A] time = 0.208964, size = 101, normalized size = 1.38

$$\frac{-16a^5x^5\sqrt{\frac{ax-1}{ax+1}} - 16a^4x^4\sqrt{\frac{ax-1}{ax+1}} + 2\cosh^{-1}(ax)\text{Chi}\left(\cosh^{-1}(ax)\right) + 9\cosh^{-1}(ax)\text{Chi}\left(3\cosh^{-1}(ax)\right) + 5\cosh^{-1}(ax)\text{Chi}\left(\right.}{16a^5\cosh^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/ArcCosh[a*x]^2,x]
```

```
[Out] (-16*a^4*x^4*Sqrt[(-1 + a*x)/(1 + a*x)] - 16*a^5*x^5*Sqrt[(-1 + a*x)/(1 + a
*x)] + 2*ArcCosh[a*x]*CoshIntegral[ArcCosh[a*x]] + 9*ArcCosh[a*x]*CoshInteg
ral[3*ArcCosh[a*x]] + 5*ArcCosh[a*x]*CoshIntegral[5*ArcCosh[a*x]])/(16*a^5*
ArcCosh[a*x])
```

Maple [A] time = 0.037, size = 83, normalized size = 1.1

$$\frac{1}{a^5}\left(-\frac{1}{8\,\text{arccosh}\,(ax)}\sqrt{ax-1}\sqrt{ax+1} + \frac{\text{Chi}\,(\text{arccosh}\,(ax))}{8} - \frac{3\,\sinh\,(3\,\text{arccosh}\,(ax))}{16\,\text{arccosh}\,(ax)} + \frac{9\,\text{Chi}\,(3\,\text{arccosh}\,(ax))}{16} - \frac{\sinh\,(}{16\,\text{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccosh(a*x)^2,x)
```

```
[Out] 1/a^5*(-1/8/arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+1/8*Chi(arccosh(a*x))-
3/16/arccosh(a*x)*sinh(3*arccosh(a*x))+9/16*Chi(3*arccosh(a*x))-1/16/arccos
h(a*x)*sinh(5*arccosh(a*x))+5/16*Chi(5*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3x^7 - ax^5 + (a^2x^6 - x^4)\sqrt{ax+1}\sqrt{ax-1}}{(a^3x^2 + \sqrt{ax+1}\sqrt{ax-1}a^2x - a)\log\left(ax + \sqrt{ax+1}\sqrt{ax-1}\right)} + \int \frac{5\,a^5x^8 - 10\,a^3x^6 + 5\,ax^4 + (5\,a^3x^6 - 3\,ax^4)(ax+1)(}{(a^5x^4 + (ax+1)(ax-1)a^3x^2 - 2\,a^3x^2 + 2\,(a^4x^3 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^7 - a*x^5 + (a^2*x^6 - x^4)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^3*x^2
+ sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x -
 1))) + integrate((5*a^5*x^8 - 10*a^3*x^6 + 5*a*x^4 + (5*a^3*x^6 - 3*a*x^4)
*(a*x + 1)*(a*x - 1) + (10*a^4*x^7 - 13*a^2*x^5 + 4*x^3)*sqrt(a*x + 1)*sqrt
(a*x - 1))/((a^5*x^4 + (a*x + 1)*(a*x - 1)*a^3*x^2 - 2*a^3*x^2 + 2*(a^4*x^3
 - a^2*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x
 - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\text{arcosh}\,(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4/arccosh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\text{acosh}^2\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acosh(a*x)**2,x)
```

```
[Out] Integral(x**4/acosh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arcosh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/arccosh(a*x)^2, x)
```

3.52 $\int \frac{x^3}{\cosh^{-1}(ax)^2} dx$

Optimal. Leaf size=61

$$\frac{\text{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^4}+\frac{\text{Chi}\left(4\cosh^{-1}(ax)\right)}{2a^4}-\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{a\cosh^{-1}(ax)}$$

```
[Out] -((x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*ArcCosh[a*x])) + CoshIntegral[2*Arc
Cosh[a*x]]/(2*a^4) + CoshIntegral[4*ArcCosh[a*x]]/(2*a^4)
```

Rubi [A] time = 0.0501213, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5666, 3301}

$$\frac{\text{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^4}+\frac{\text{Chi}\left(4\cosh^{-1}(ax)\right)}{2a^4}-\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCosh[a*x]^2,x]
```

```
[Out] -((x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*ArcCosh[a*x])) + CoshIntegral[2*Arc
Cosh[a*x]]/(2*a^4) + CoshIntegral[4*ArcCosh[a*x]]/(2*a^4)
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3}{\cosh^{-1}(ax)^2}\,dx = -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} - \frac{\operatorname{Subst}\left(\int \left(-\frac{\cosh(2x)}{2x} - \frac{\cosh(4x)}{2x}\right)dx, x, \cosh^{-1}(ax)\right)}{a^4}$$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\operatorname{Subst}\left(\int \frac{\cosh(2x)}{x}dx, x, \cosh^{-1}(ax)\right)}{2a^4} + \frac{\operatorname{Subst}\left(\int \frac{\cosh(4x)}{x}dx, x, \cosh^{-1}(ax)\right.}{2a^4}$$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\operatorname{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^4} + \frac{\operatorname{Chi}\left(4\cosh^{-1}(ax)\right)}{2a^4}$$

Mathematica [A] time = 0.230709, size = 58, normalized size = 0.95

$$\frac{-\frac{2a^3x^3\sqrt{\frac{ax-1}{ax+1}}(ax+1)}{\cosh^{-1}(ax)} + \operatorname{Chi}\left(2\cosh^{-1}(ax)\right) + \operatorname{Chi}\left(4\cosh^{-1}(ax)\right)}{2a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/ArcCosh[a*x]^2,x]
```

```
[Out] ((-2*a^3*x^3*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))/ArcCosh[a*x] + CoshInteg
ral[2*ArcCosh[a*x]] + CoshIntegral[4*ArcCosh[a*x]])/(2*a^4)
```

Maple [A] time = 0.033, size = 54, normalized size = 0.9

$$\frac{1}{a^4}\left(-\frac{\sinh(2\operatorname{arccosh}(ax))}{4\operatorname{arccosh}(ax)} + \frac{\operatorname{Chi}(2\operatorname{arccosh}(ax))}{2} - \frac{\sinh(4\operatorname{arccosh}(ax))}{8\operatorname{arccosh}(ax)} + \frac{\operatorname{Chi}(4\operatorname{arccosh}(ax))}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccosh(a*x)^2,x)
```

```
[Out] 1/a^4*(-1/4/arccosh(a*x)*sinh(2*arccosh(a*x))+1/2*Chi(2*arccosh(a*x))-1/8/a
rccosh(a*x)*sinh(4*arccosh(a*x))+1/2*Chi(4*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3x^6 - ax^4 + \left(a^2x^5 - x^3\right)\sqrt{ax+1}\sqrt{ax-1}}{\left(a^3x^2 + \sqrt{ax+1}\sqrt{ax-1}a^2x - a\right)\log\left(ax + \sqrt{ax+1}\sqrt{ax-1}\right)} + \int \frac{4a^5x^7 - 8a^3x^5 + 4ax^3 + 2\left(2a^3x^5 - ax^3\right)(ax+1)(a}{\left(a^5x^4 + (ax+1)(ax-1)a^3x^2 - 2a^3x^2 + 2\left(a^4x^3 - a^2x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^6 - a*x^4 + (a^2*x^5 - x^3)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^3*x^2
+ sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x -
 1))) + integrate((4*a^5*x^7 - 8*a^3*x^5 + 4*a*x^3 + 2*(2*a^3*x^5 - a*x^3)*
(a*x + 1)*(a*x - 1) + (8*a^4*x^6 - 10*a^2*x^4 + 3*x^2)*sqrt(a*x + 1)*sqrt(a
*x - 1))/((a^5*x^4 + (a*x + 1)*(a*x - 1)*a^3*x^2 - 2*a^3*x^2 + 2*(a^4*x^3 -
 a^2*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x -
 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\operatorname{arcosh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3/arccosh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acosh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acosh(a*x)**2,x)
```

```
[Out] Integral(x**3/acosh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arcosh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/arccosh(a*x)^2, x)
```

3.53 $\int \frac{x^2}{\cosh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=59

$$\frac{\text{Chi}\left(\cosh^{-1}(ax)\right)}{4a^3}+\frac{3\text{Chi}\left(3\cosh^{-1}(ax)\right)}{4a^3}-\frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{a\cosh^{-1}(ax)}$$

```
[Out] -((x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*ArcCosh[a*x])) + CoshIntegral[ArcCo
sh[a*x]]/(4*a^3) + (3*CoshIntegral[3*ArcCosh[a*x]])/(4*a^3)
```

Rubi [A] time = 0.0472113, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5666, 3301}

$$\frac{\text{Chi}\left(\cosh^{-1}(ax)\right)}{4a^3}+\frac{3\text{Chi}\left(3\cosh^{-1}(ax)\right)}{4a^3}-\frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCosh[a*x]^2,x]
```


```
[Out] -((x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*ArcCosh[a*x])) + CoshIntegral[ArcCo
sh[a*x]]/(4*a^3) + (3*CoshIntegral[3*ArcCosh[a*x]])/(4*a^3)
```


Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2}{\cosh^{-1}(ax)^2} dx = -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4x} - \frac{3\cosh(3x)}{4x}\right) dx, x, \cosh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{4a^3} + \frac{3\,\text{Subst}\left(\int \frac{\cosh(3x)}{x} dx, x, \cosh^{-1}(ax\right.}{4a^3}$$
$$= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\text{Chi}\left(\cosh^{-1}(ax)\right)}{4a^3} + \frac{3\text{Chi}\left(3\cosh^{-1}(ax)\right)}{4a^3}$$

Mathematica [A] time = 0.226049, size = 58, normalized size = 0.98

$$\frac{-\frac{4a^2x^2\sqrt{\frac{ax-1}{ax+1}}(ax+1)}{\cosh^{-1}(ax)} + \text{Chi}\left(\cosh^{-1}(ax)\right) + 3\text{Chi}\left(3\cosh^{-1}(ax)\right)}{4a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/ArcCosh[a*x]^2,x]
```

```
[Out] ((-4*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))/ArcCosh[a*x] + CoshInteg
ral[ArcCosh[a*x]] + 3*CoshIntegral[3*ArcCosh[a*x]])/(4*a^3)
```

Maple [A] time = 0.03, size = 59, normalized size = 1.

$$\frac{1}{a^3}\left(-\frac{1}{4\operatorname{arccosh}(ax)}\sqrt{ax-1}\sqrt{ax+1} + \frac{\text{Chi}(\operatorname{arccosh}(ax))}{4} - \frac{\sinh(3\operatorname{arccosh}(ax))}{4\operatorname{arccosh}(ax)} + \frac{3\,\text{Chi}(3\operatorname{arccosh}(ax))}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccosh(a*x)^2,x)
```

```
[Out] 1/a^3*(-1/4/arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+1/4*Chi(arccosh(a*x))-
1/4/arccosh(a*x)*sinh(3*arccosh(a*x))+3/4*Chi(3*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3x^5 - ax^3 + \left(a^2x^4 - x^2\right)\sqrt{ax+1}\sqrt{ax-1}}{\left(a^3x^2 + \sqrt{ax+1}\sqrt{ax-1}a^2x - a\right)\log\left(ax + \sqrt{ax+1}\sqrt{ax-1}\right)} + \int \frac{3\,a^5x^6 - 6\,a^3x^4 + \left(3\,a^3x^4 - ax^2\right)(ax+1)(ax-1) +}{\left(a^5x^4 + (ax+1)(ax-1)a^3x^2 - 2\,a^3x^2 + 2\left(a^4x^3 - a^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^5 - a*x^3 + (a^2*x^4 - x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^3*x^2
+ sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x -
 1))) + integrate((3*a^5*x^6 - 6*a^3*x^4 + (3*a^3*x^4 - a*x^2)*(a*x + 1)*(a
*x - 1) + 3*a*x^2 + (6*a^4*x^5 - 7*a^2*x^3 + 2*x)*sqrt(a*x + 1)*sqrt(a*x -
1))/((a^5*x^4 + (a*x + 1)*(a*x - 1)*a^3*x^2 - 2*a^3*x^2 + 2*(a^4*x^3 - a^2*
x)*sqrt(a*x + 1)*sqrt(a*x - 1) + a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)))
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\operatorname{arcosh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/arccosh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{acosh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acosh(a*x)**2,x)
```

```
[Out] Integral(x**2/acosh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arcosh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/arccosh(a*x)^2, x)
```

3.54 $\int \frac{x}{\cosh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=42

$$\frac{\text{Chi}\left(2\cosh^{-1}(ax)\right)}{a^2}-\frac{x\sqrt{ax-1}\sqrt{ax+1}}{a\cosh^{-1}(ax)}$$

```
[Out] -((x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*ArcCosh[a*x])) + CoshIntegral[2*ArcCo
sh[a*x]]/a^2
```

Rubi [A] time = 0.0251702, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5666, 3301}

$$\frac{\text{Chi}\left(2\cosh^{-1}(ax)\right)}{a^2}-\frac{x\sqrt{ax-1}\sqrt{ax+1}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCosh[a*x]^2,x]
```

```
[Out] -((x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*ArcCosh[a*x])) + CoshIntegral[2*ArcCo
sh[a*x]]/a^2
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{\cosh^{-1}(ax)^2}\,dx = -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^2}$$
$$= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\text{Chi}\left(2\cosh^{-1}(ax)\right)}{a^2}$$

Mathematica [A] time = 0.240567, size = 44, normalized size = 1.05

$$\frac{\text{Chi}\left(2\cosh^{-1}(ax)\right) - \frac{ax\sqrt{\frac{ax-1}{ax+1}}(ax+1)}{\cosh^{-1}(ax)}}{a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/ArcCosh[a*x]^2,x]
```

```
[Out] (-((a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))/ArcCosh[a*x]) + CoshIntegral[
2*ArcCosh[a*x]])/a^2
```

Maple [A] time = 0.026, size = 28, normalized size = 0.7

$$\frac{1}{a^2}\left(-\frac{\sinh(2\,\text{arccosh}(ax))}{2\,\text{arccosh}(ax)} + \text{Chi}(2\,\text{arccosh}(ax))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccosh(a*x)^2,x)
```

```
[Out] 1/a^2*(-1/2/arccosh(a*x)*sinh(2*arccosh(a*x))+Chi(2*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3x^4 - ax^2 + \left(a^2x^3 - x\right)\sqrt{ax+1}\sqrt{ax-1}}{\left(a^3x^2 + \sqrt{ax+1}\sqrt{ax-1}a^2x - a\right)\log\left(ax + \sqrt{ax+1}\sqrt{ax-1}\right)} + \int \frac{2\,a^5x^5 + 2\,(ax+1)(ax-1)a^3x^3 - 4\,a^3x^3 + \left(4 \cdots\right.}{\left(a^5x^4 + (ax+1)(ax-1)a^3x^2 - 2\,a^3x^2 + 2\left(a^4x^3 - a^2x \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^4 - a*x^2 + (a^2*x^3 - x)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^3*x^2 +
sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1
))) + integrate((2*a^5*x^5 + 2*(a*x + 1)*(a*x - 1)*a^3*x^3 - 4*a^3*x^3 + (4
*a^4*x^4 - 4*a^2*x^2 + 1)*sqrt(a*x + 1)*sqrt(a*x - 1) + 2*a*x)/((a^5*x^4 +
(a*x + 1)*(a*x - 1)*a^3*x^2 - 2*a^3*x^2 + 2*(a^4*x^3 - a^2*x)*sqrt(a*x + 1)
*sqrt(a*x - 1) + a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\operatorname{arcosh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x/arccosh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{acosh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acosh(a*x)**2,x)
```

```
[Out] Integral(x/acosh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arcosh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/arccosh(a*x)^2, x)
```

3.55 $\int \frac{1}{\cosh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=39

$$\frac{\text{Chi}\left(\cosh^{-1}(ax)\right)}{a}-\frac{\sqrt{ax-1}\sqrt{ax+1}}{a\cosh^{-1}(ax)}$$

```
[Out] -((Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*ArcCosh[a*x])) + CoshIntegral[ArcCosh[a
*x]]/a
```

Rubi [A] time = 0.184479, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5656, 5781, 3301}

$$\frac{\text{Chi}\left(\cosh^{-1}(ax)\right)}{a}-\frac{\sqrt{ax-1}\sqrt{ax+1}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^(-2),x]
```

```
[Out] -((Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*ArcCosh[a*x])) + CoshIntegral[ArcCosh[a
*x]]/a
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cosh^{-1}(ax)^2}\,dx &= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + a\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx\\ &= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a}\\ &= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\textbf{Chi}\left(\cosh^{-1}(ax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.100184, size = 60, normalized size = 1.54

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\cosh^{-1}(ax)\textbf{Chi}\left(\cosh^{-1}(ax)\right) - ax + 1}{a\sqrt{\frac{ax-1}{ax+1}}\cosh^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^(-2),x]
```

```
[Out] (1 - a*x + Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*CoshIntegral[ArcCosh[a*x
]])/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x])
```

Maple [A] time = 0.026, size = 33, normalized size = 0.9

$$\frac{1}{a}\left(-\frac{1}{\operatorname{arccosh}(ax)}\sqrt{ax-1}\sqrt{ax+1} + \operatorname{Chi}(\operatorname{arccosh}(ax))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x)^2,x)
```

```
[Out] 1/a*(-1/arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+Chi(arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3x^3 + \left(a^2x^2 - 1\right)\sqrt{ax+1}\sqrt{ax-1} - ax}{\left(a^3x^2 + \sqrt{ax+1}\sqrt{ax-1}a^2x - a\right)\log\left(ax + \sqrt{ax+1}\sqrt{ax-1}\right)} + \int \frac{a^4x^4 - 2\,a^2x^2 + \left(a^2x^2 + 1\right)(ax+1)(ax -}{\left(a^4x^4 + (ax+1)(ax-1)a^2x^2 - 2\,a^2x^2 + 2\left(a^3x^3 - a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x)/((a^3*x^2 + sq
rt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))
) + integrate((a^4*x^4 - 2*a^2*x^2 + (a^2*x^2 + 1)*(a*x + 1)*(a*x - 1) + (2
*a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + 1)/((a^4*x^4 + (a*x + 1)*(a*x
 - 1)*a^2*x^2 - 2*a^2*x^2 + 2*(a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*x - 1) +
 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\operatorname{arcosh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^(-2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acosh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)**2,x)
```

```
[Out] Integral(acosh(a*x)**(-2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arcosh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^(-2), x)
```

3.56 $\int \frac{1}{x \cosh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x \cosh^{-1}(ax)^2}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcCosh[a*x]^2), x]
```

Rubi [A] time = 0.0127509, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \cosh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcCosh[a*x]^2),x]
```

```
[Out] Defer[Int][1/(x*ArcCosh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{x \cosh^{-1}(ax)^2} \, dx = \int \frac{1}{x \cosh^{-1}(ax)^2} \, dx$$

Mathematica [A] time = 4.3499, size = 0, normalized size = 0.

$$\int \frac{1}{x \cosh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcCosh[a*x]^2),x]
```

```
[Out] Integrate[1/(x*ArcCosh[a*x]^2), x]
```

Maple [A] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\operatorname{arccosh}\left(ax\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccosh(a*x)^2,x)
```

```
[Out] int(1/x/arccosh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{3}x^{3} + (a^{2}x^{2} - 1)\sqrt{ax+1}\sqrt{ax-1} - ax}{(a^{3}x^{3} + \sqrt{ax+1}\sqrt{ax-1}a^{2}x^{2} - ax)\log(ax + \sqrt{ax+1}\sqrt{ax-1})} + \int \frac{2\,(ax+1)(ax-1)ax + }{(a^{5}x^{6} + (ax+1)(ax-1)a^{3}x^{4} - 2\,a^{3}x^{4} + ax^{2} + 2\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x)/((a^3*x^3 + sq
rt(a*x + 1)*sqrt(a*x - 1)*a^2*x^2 - a*x)*log(a*x + sqrt(a*x + 1)*sqrt(a*x -
 1))) + integrate((2*(a*x + 1)*(a*x - 1)*a*x + (2*a^2*x^2 - 1)*sqrt(a*x + 1
)*sqrt(a*x - 1))/((a^5*x^6 + (a*x + 1)*(a*x - 1)*a^3*x^4 - 2*a^3*x^4 + a*x^
2 + 2*(a^4*x^5 - a^2*x^3)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x +
 1)*sqrt(a*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\operatorname{arcosh}\left(ax\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arccosh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{acosh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acosh(a*x)**2,x)
```

```
[Out] Integral(1/(x*acosh(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arcosh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccosh(a*x)^2), x)
```

3.57 $\int \frac{1}{x^2 \cosh^{-1}(ax)^2} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x^2 \cosh^{-1}(ax)^2}, x\right)$$

```
[Out] Unintegrable[1/(x^2*ArcCosh[a*x]^2), x]
```

Rubi [A] time = 0.0140318, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*ArcCosh[a*x]^2),x]
```

```
[Out] Defer[Int][1/(x^2*ArcCosh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^2} dx = \int \frac{1}{x^2 \cosh^{-1}(ax)^2} dx$$

Mathematica [A] time = 7.91706, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*ArcCosh[a*x]^2),x]
```

```
[Out] Integrate[1/(x^2*ArcCosh[a*x]^2), x]
```

Maple [A] time = 0.091, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (\operatorname{arccosh}(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccosh(a*x)^2,x)
```

```
[Out] int(1/x^2/arccosh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3x^3 + (a^2x^2-1)\sqrt{ax+1}\sqrt{ax-1} - ax}{(a^3x^4 + \sqrt{ax+1}\sqrt{ax-1}a^2x^3 - ax^2)\log\left(ax + \sqrt{ax+1}\sqrt{ax-1}\right)} - \int \frac{a^5x^5 - 2a^3x^3 + (a^3x^3 - 3ax)(ax+1)(a}{(a^5x^7 + (ax+1)(ax-1)a^3x^5 - 2a^3x^5 + ax^3 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x)/((a^3*x^4 + sq
rt(a*x + 1)*sqrt(a*x - 1)*a^2*x^3 - a*x^2)*log(a*x + sqrt(a*x + 1)*sqrt(a*x
 - 1))) - integrate((a^5*x^5 - 2*a^3*x^3 + (a^3*x^3 - 3*a*x)*(a*x + 1)*(a*x
 - 1) + (2*a^4*x^4 - 5*a^2*x^2 + 2)*sqrt(a*x + 1)*sqrt(a*x - 1) + a*x)/((a^
5*x^7 + (a*x + 1)*(a*x - 1)*a^3*x^5 - 2*a^3*x^5 + a*x^3 + 2*(a^4*x^6 - a^2*
x^4)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))),
x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^2 \operatorname{arcosh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*arccosh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{acosh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acosh(a*x)**2,x)
```

```
[Out] Integral(1/(x**2*acosh(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{arcosh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arccosh(a*x)^2), x)
```

3.58 $\int \frac{x^4}{\cosh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=102

$$\frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{16a^5}+\frac{27\text{Shi}\left(3\cosh^{-1}(ax)\right)}{32a^5}+\frac{25\text{Shi}\left(5\cosh^{-1}(ax)\right)}{32a^5}+\frac{2x^3}{a^2\cosh^{-1}(ax)}-\frac{5x^5}{2\cosh^{-1}(ax)}-\frac{x^4\sqrt{ax-1}\sqrt{ax+}}{2a\cosh^{-1}(ax)^2}$$

```
[Out] -(x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*ArcCosh[a*x]^2) + (2*x^3)/(a^2*Arc
Cosh[a*x]) - (5*x^5)/(2*ArcCosh[a*x]) + SinhIntegral[ArcCosh[a*x]]/(16*a^5)
 + (27*SinhIntegral[3*ArcCosh[a*x]])/(32*a^5) + (25*SinhIntegral[5*ArcCosh[
a*x]])/(32*a^5)
```

Rubi [A] time = 0.644703, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5668, 5775, 5670, 5448, 3298}

$$\frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{16a^5}+\frac{27\text{Shi}\left(3\cosh^{-1}(ax)\right)}{32a^5}+\frac{25\text{Shi}\left(5\cosh^{-1}(ax)\right)}{32a^5}+\frac{2x^3}{a^2\cosh^{-1}(ax)}-\frac{5x^5}{2\cosh^{-1}(ax)}-\frac{x^4\sqrt{ax-1}\sqrt{ax+}}{2a\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCosh[a*x]^3,x]
```

```
[Out] -(x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*ArcCosh[a*x]^2) + (2*x^3)/(a^2*Arc
Cosh[a*x]) - (5*x^5)/(2*ArcCosh[a*x]) + SinhIntegral[ArcCosh[a*x]]/(16*a^5)
 + (27*SinhIntegral[3*ArcCosh[a*x]])/(32*a^5) + (25*SinhIntegral[5*ArcCosh[
a*x]])/(32*a^5)
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
```

```
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4}{\cosh^{-1}(ax)^3}\,dx = -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} - \frac{2\int \frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}\,dx}{a} + \frac{1}{2}(5a)\int \frac{x^5}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}$$

$$= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{2x^3}{a^2\cosh^{-1}(ax)} - \frac{5x^5}{2\cosh^{-1}(ax)} + \frac{25}{2}\int \frac{x^4}{\cosh^{-1}(ax)}\,dx - \frac{6\int \frac{x^2}{\cosh^{-1}(ax)}\,dx}{a^2}$$

$$= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{2x^3}{a^2\cosh^{-1}(ax)} - \frac{5x^5}{2\cosh^{-1}(ax)} - \frac{6\,\mathrm{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{x}\,dx, x, \cosh^{-1}(\right.}{a^5}$$

$$= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{2x^3}{a^2\cosh^{-1}(ax)} - \frac{5x^5}{2\cosh^{-1}(ax)} - \frac{6\,\mathrm{Subst}\left(\int \left(\frac{\sinh(x)}{4x} + \frac{\sinh(3x)}{4x}\right)dx, x, \cos\right.}{a^5}$$

$$= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{2x^3}{a^2\cosh^{-1}(ax)} - \frac{5x^5}{2\cosh^{-1}(ax)} + \frac{25\,\mathrm{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{32a^5}$$

$$= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{2x^3}{a^2\cosh^{-1}(ax)} - \frac{5x^5}{2\cosh^{-1}(ax)} + \frac{\mathbf{Shi}\left(\cosh^{-1}(ax)\right)}{16a^5} + \frac{27\mathbf{Shi}\left(3\cosh^{-1}(a\right.}{32a^5}$$

Mathematica [A] time = 0.134561, size = 107, normalized size = 1.05

$$\frac{-16a^4x^4\sqrt{ax-1}\sqrt{ax+1} - 80a^5x^5\cosh^{-1}(ax) + 64a^3x^3\cosh^{-1}(ax) + 2\cosh^{-1}(ax)^2\mathbf{Shi}\left(\cosh^{-1}(ax)\right) + 27\cosh^{-1}(ax)^2}{32a^5\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcCosh[a*x]^3,x]
```

```
[Out] (-16*a^4*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x] + 64*a^3*x^3*ArcCosh[a*x] - 80*a^
5*x^5*ArcCosh[a*x] + 2*ArcCosh[a*x]^2*SinhIntegral[ArcCosh[a*x]] + 27*ArcCo
sh[a*x]^2*SinhIntegral[3*ArcCosh[a*x]] + 25*ArcCosh[a*x]^2*SinhIntegral[5*A
rcCosh[a*x]])/(32*a^5*ArcCosh[a*x]^2)
```

Maple [A] time = 0.042, size = 123, normalized size = 1.2

$$\frac{1}{a^5}\left(-\frac{1}{16\,(\mathrm{arccosh}\,(ax))^2}\sqrt{ax-1}\sqrt{ax+1} - \frac{ax}{16\,\mathrm{arccosh}\,(ax)} + \frac{\mathrm{Shi}\,(\mathrm{arccosh}\,(ax))}{16} - \frac{3\,\sinh\,(3\,\mathrm{arccosh}\,(ax))}{32\,(\mathrm{arccosh}\,(ax))^2} - \frac{9\,\cosh}{32\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccosh(a*x)^3,x)
```

```
[Out] 1/a^5*(-1/16/arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-1/16*a*x/arccosh(a*
x)+1/16*Shi(arccosh(a*x))-3/32/arccosh(a*x)^2*sinh(3*arccosh(a*x))-9/32/arc
cosh(a*x)*cosh(3*arccosh(a*x))+27/32*Shi(3*arccosh(a*x))-1/32/arccosh(a*x)^
2*sinh(5*arccosh(a*x))-5/32/arccosh(a*x)*cosh(5*arccosh(a*x))+25/32*Shi(5*a
rccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^11 - 3*a^6*x^9 + 3*a^4*x^7 - a^2*x^5 + (a^5*x^8 - a^3*x^6)*(a*x
 + 1)^(3/2)*(a*x - 1)^(3/2) + (3*a^6*x^9 - 5*a^4*x^7 + 2*a^2*x^5)*(a*x + 1)
*(a*x - 1) + (3*a^7*x^10 - 7*a^5*x^8 + 5*a^3*x^6 - a*x^4)*sqrt(a*x + 1)*sqr
t(a*x - 1) + (5*a^8*x^11 - 15*a^6*x^9 + 15*a^4*x^7 - 5*a^2*x^5 + (5*a^5*x^8
 - 8*a^3*x^6 + 3*a*x^4)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + (15*a^6*x^9 - 31*
a^4*x^7 + 20*a^2*x^5 - 4*x^3)*(a*x + 1)*(a*x - 1) + (15*a^7*x^10 - 38*a^5*x
^8 + 32*a^3*x^6 - 9*a*x^4)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x
+ 1)*sqrt(a*x - 1)))/((a^8*x^6 + (a*x + 1)^(3/2)*(a*x - 1)^(3/2)*a^5*x^3 -
3*a^6*x^4 + 3*a^4*x^2 + 3*(a^6*x^4 - a^4*x^2)*(a*x + 1)*(a*x - 1) + 3*(a^7*
x^5 - 2*a^5*x^3 + a^3*x)*sqrt(a*x + 1)*sqrt(a*x - 1) - a^2)*log(a*x + sqrt(
a*x + 1)*sqrt(a*x - 1))^2) + integrate(1/2*(25*a^10*x^12 - 100*a^8*x^10 + 1
50*a^6*x^8 - 100*a^4*x^6 + 25*a^2*x^4 + (25*a^6*x^8 - 24*a^4*x^6 + 3*a^2*x^
4)*(a*x + 1)^2*(a*x - 1)^2 + (100*a^7*x^9 - 172*a^5*x^7 + 87*a^3*x^5 - 12*a
*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 3*(50*a^8*x^10 - 124*a^6*x^8 + 105*
a^4*x^6 - 35*a^2*x^4 + 4*x^2)*(a*x + 1)*(a*x - 1) + (100*a^9*x^11 - 324*a^7
*x^9 + 381*a^5*x^7 - 193*a^3*x^5 + 36*a*x^3)*sqrt(a*x + 1)*sqrt(a*x - 1))/(
(a^10*x^8 + (a*x + 1)^2*(a*x - 1)^2*a^6*x^4 - 4*a^8*x^6 + 6*a^6*x^4 - 4*a^4
*x^2 + 4*(a^7*x^5 - a^5*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 6*(a^8*x^6 -
 2*a^6*x^4 + a^4*x^2)*(a*x + 1)*(a*x - 1) + 4*(a^9*x^7 - 3*a^7*x^5 + 3*a^5*
x^3 - a^3*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + a^2)*log(a*x + sqrt(a*x + 1)*sqr
t(a*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\text{arcosh}\,(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^4/arccosh(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\text{acosh}^3\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acosh(a*x)**3,x)
```

```
[Out] Integral(x**4/acosh(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\text{arcosh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^4/arccosh(a*x)^3, x)
```

3.59 $\int \frac{x^3}{\cosh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=87

$$\frac{\text{Shi}\left(2\cosh^{-1}(ax)\right)}{2a^4}+\frac{\text{Shi}\left(4\cosh^{-1}(ax)\right)}{a^4}+\frac{3x^2}{2a^2\cosh^{-1}(ax)}-\frac{2x^4}{\cosh^{-1}(ax)}-\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{2a\cosh^{-1}(ax)^2}$$

```
[Out] -(x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*ArcCosh[a*x]^2) + (3*x^2)/(2*a^2*A
rcCosh[a*x]) - (2*x^4)/ArcCosh[a*x] + SinhIntegral[2*ArcCosh[a*x]]/(2*a^4)
+ SinhIntegral[4*ArcCosh[a*x]]/a^4
```

Rubi [A] time = 0.597955, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5668, 5775, 5670, 5448, 3298, 12}

$$\frac{\text{Shi}\left(2\cosh^{-1}(ax)\right)}{2a^4}+\frac{\text{Shi}\left(4\cosh^{-1}(ax)\right)}{a^4}+\frac{3x^2}{2a^2\cosh^{-1}(ax)}-\frac{2x^4}{\cosh^{-1}(ax)}-\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{2a\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCosh[a*x]^3,x]
```

```
[Out] -(x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*ArcCosh[a*x]^2) + (3*x^2)/(2*a^2*A
rcCosh[a*x]) - (2*x^4)/ArcCosh[a*x] + SinhIntegral[2*ArcCosh[a*x]]/(2*a^4)
+ SinhIntegral[4*ArcCosh[a*x]]/a^4
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
```

```
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{x^3}{\cosh^{-1}(ax)^3}\,dx = -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} - \frac{3\int \frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}\,dx}{2a} + (2a)\int \frac{x^4}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}\,dx$$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{3x^2}{2a^2\cosh^{-1}(ax)} - \frac{2x^4}{\cosh^{-1}(ax)} + 8\int \frac{x^3}{\cosh^{-1}(ax)}\,dx - \frac{3\int \frac{x}{\cosh^{-1}(ax)}\,dx}{a^2}$$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{3x^2}{2a^2\cosh^{-1}(ax)} - \frac{2x^4}{\cosh^{-1}(ax)} - \frac{3\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right.}{a^4}$$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{3x^2}{2a^2\cosh^{-1}(ax)} - \frac{2x^4}{\cosh^{-1}(ax)} - \frac{3\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \cosh^{-1}(ax)\right)}{a^4} + $$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{3x^2}{2a^2\cosh^{-1}(ax)} - \frac{2x^4}{\cosh^{-1}(ax)} + \frac{\mathrm{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^4} - $$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{3x^2}{2a^2\cosh^{-1}(ax)} - \frac{2x^4}{\cosh^{-1}(ax)} + \frac{\mathrm{Shi}\left(2\cosh^{-1}(ax)\right)}{2a^4} + \frac{\mathrm{Shi}\left(4\cosh^{-1}(ax)\right)}{a^4}$$

Mathematica [A] time = 0.166134, size = 75, normalized size = 0.86

$$\frac{-\frac{a^2x^2\left(\left(4a^2x^2-3\right)\cosh^{-1}(ax)+ax\sqrt{ax-1}\sqrt{ax+1}\right)}{\cosh^{-1}(ax)^2} + \mathrm{Shi}\left(2\cosh^{-1}(ax)\right) + 2\mathrm{Shi}\left(4\cosh^{-1}(ax)\right)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcCosh[a*x]^3,x]
```

```
[Out] (-((a^2*x^2*(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x] + (-3 + 4*a^2*x^2)*ArcCosh[a*
x]))/ArcCosh[a*x]^2) + SinhIntegral[2*ArcCosh[a*x]] + 2*SinhIntegral[4*ArcC
osh[a*x]])/(2*a^4)
```

Maple [A] time = 0.036, size = 82, normalized size = 0.9

$$\frac{1}{a^4}\left(-\frac{\sinh(2\operatorname{arccosh}(ax))}{8\left(\operatorname{arccosh}(ax)\right)^2} - \frac{\cosh(2\operatorname{arccosh}(ax))}{4\operatorname{arccosh}(ax)} + \frac{\mathrm{Shi}(2\operatorname{arccosh}(ax))}{2} - \frac{\sinh(4\operatorname{arccosh}(ax))}{16\left(\operatorname{arccosh}(ax)\right)^2} - \frac{\cosh(4\operatorname{arccosh}(}{4\operatorname{arccosh}(ax}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccosh(a*x)^3,x)
```

```
[Out] 1/a^4*(-1/8/arccosh(a*x)^2*sinh(2*arccosh(a*x))-1/4/arccosh(a*x)*cosh(2*arc
cosh(a*x))+1/2*Shi(2*arccosh(a*x))-1/16/arccosh(a*x)^2*sinh(4*arccosh(a*x))
-1/4/arccosh(a*x)*cosh(4*arccosh(a*x))+Shi(4*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^10 - 3*a^6*x^8 + 3*a^4*x^6 - a^2*x^4 + (a^5*x^7 - a^3*x^5)*(a*x
 + 1)^(3/2)*(a*x - 1)^(3/2) + (3*a^6*x^8 - 5*a^4*x^6 + 2*a^2*x^4)*(a*x + 1)
*(a*x - 1) + (3*a^7*x^9 - 7*a^5*x^7 + 5*a^3*x^5 - a*x^3)*sqrt(a*x + 1)*sqrt
(a*x - 1) + (4*a^8*x^10 - 12*a^6*x^8 + 12*a^4*x^6 - 4*a^2*x^4 + 2*(2*a^5*x^
7 - 3*a^3*x^5 + a*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 3*(4*a^6*x^8 - 8*a
^4*x^6 + 5*a^2*x^4 - x^2)*(a*x + 1)*(a*x - 1) + (12*a^7*x^9 - 30*a^5*x^7 +
25*a^3*x^5 - 7*a*x^3)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*
sqrt(a*x - 1)))/((a^8*x^6 + (a*x + 1)^(3/2)*(a*x - 1)^(3/2)*a^5*x^3 - 3*a^6
*x^4 + 3*a^4*x^2 + 3*(a^6*x^4 - a^4*x^2)*(a*x + 1)*(a*x - 1) + 3*(a^7*x^5 -
 2*a^5*x^3 + a^3*x)*sqrt(a*x + 1)*sqrt(a*x - 1) - a^2)*log(a*x + sqrt(a*x +
 1)*sqrt(a*x - 1))^2) + integrate(1/2*(16*a^10*x^11 - 64*a^8*x^9 + 96*a^6*x
^7 - 64*a^4*x^5 + 4*(4*a^6*x^7 - 3*a^4*x^5)*(a*x + 1)^2*(a*x - 1)^2 + 16*a^
2*x^3 + (64*a^7*x^8 - 100*a^5*x^6 + 42*a^3*x^4 - 3*a*x^2)*(a*x + 1)^(3/2)*(
a*x - 1)^(3/2) + 6*(16*a^8*x^9 - 38*a^6*x^7 + 30*a^4*x^5 - 9*a^2*x^3 + x)*(
a*x + 1)*(a*x - 1) + (64*a^9*x^10 - 204*a^7*x^8 + 234*a^5*x^6 - 115*a^3*x^4
 + 21*a*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^10*x^8 + (a*x + 1)^2*(a*x - 1
)^2*a^6*x^4 - 4*a^8*x^6 + 6*a^6*x^4 - 4*a^4*x^2 + 4*(a^7*x^5 - a^5*x^3)*(a*
x + 1)^(3/2)*(a*x - 1)^(3/2) + 6*(a^8*x^6 - 2*a^6*x^4 + a^4*x^2)*(a*x + 1)*
(a*x - 1) + 4*(a^9*x^7 - 3*a^7*x^5 + 3*a^5*x^3 - a^3*x)*sqrt(a*x + 1)*sqrt(
a*x - 1) + a^2)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\operatorname{arcosh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3/arccosh(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acosh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acosh(a*x)**3,x)
```

```
[Out] Integral(x**3/acosh(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arcosh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3/arccosh(a*x)^3, x)
```

3.60 $\int \frac{x^2}{\cosh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=85

$$\frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{8a^3}+\frac{9\text{Shi}\left(3\cosh^{-1}(ax)\right)}{8a^3}+\frac{x}{a^2\cosh^{-1}(ax)}-\frac{3x^3}{2\cosh^{-1}(ax)}-\frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{2a\cosh^{-1}(ax)^2}$$

```
[Out] -(x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*ArcCosh[a*x]^2) + x/(a^2*ArcCosh[a
*x]) - (3*x^3)/(2*ArcCosh[a*x]) + SinhIntegral[ArcCosh[a*x]]/(8*a^3) + (9*S
inhIntegral[3*ArcCosh[a*x]])/(8*a^3)
```

Rubi [A] time = 0.504109, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5668, 5775, 5670, 5448, 3298, 5658}

$$\frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{8a^3}+\frac{9\text{Shi}\left(3\cosh^{-1}(ax)\right)}{8a^3}+\frac{x}{a^2\cosh^{-1}(ax)}-\frac{3x^3}{2\cosh^{-1}(ax)}-\frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{2a\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCosh[a*x]^3,x]
```

```
[Out] -(x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*ArcCosh[a*x]^2) + x/(a^2*ArcCosh[a
*x]) - (3*x^3)/(2*ArcCosh[a*x]) + SinhIntegral[ArcCosh[a*x]]/(8*a^3) + (9*S
inhIntegral[3*ArcCosh[a*x]])/(8*a^3)
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
```

```
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rubi steps

$$\int \frac{x^2}{\cosh^{-1}(ax)^3}\,dx = -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} - \frac{\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}\,dx}{a} + \frac{1}{2}(3a)\int \frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}\,dx$$

$$= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{x}{a^2\cosh^{-1}(ax)} - \frac{3x^3}{2\cosh^{-1}(ax)} + \frac{9}{2}\int \frac{x^2}{\cosh^{-1}(ax)}\,dx - \frac{\int \frac{1}{\cosh^{-1}(ax)}\,dx}{a^2}$$

$$= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{x}{a^2\cosh^{-1}(ax)} - \frac{3x^3}{2\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^3} + \frac{9}{}$$

$$= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{x}{a^2\cosh^{-1}(ax)} - \frac{3x^3}{2\cosh^{-1}(ax)} - \frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{a^3} + \frac{9\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4x}\right.\right.}{}$$

$$= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{x}{a^2\cosh^{-1}(ax)} - \frac{3x^3}{2\cosh^{-1}(ax)} - \frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{a^3} + \frac{9\,\text{Subst}\left(\int \frac{\sinh(x)}{x}\right.}{8a}$$

$$= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{x}{a^2\cosh^{-1}(ax)} - \frac{3x^3}{2\cosh^{-1}(ax)} + \frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{8a^3} + \frac{9\text{Shi}\left(3\cosh^{-1}(ax\right.}{8a^3}$$

Mathematica [A] time = 0.14575, size = 69, normalized size = 0.81

$$\frac{-\frac{4ax\left(\left(3a^2x^2-2\right)\cosh^{-1}(ax)+ax\sqrt{ax-1}\sqrt{ax+1}\right)}{\cosh^{-1}(ax)^2} + \text{Shi}\left(\cosh^{-1}(ax)\right) + 9\text{Shi}\left(3\cosh^{-1}(ax)\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcCosh[a*x]^3,x]
```

```
[Out] ((-4*a*x*(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x] + (-2 + 3*a^2*x^2)*ArcCosh[a*x])
)/ArcCosh[a*x]^2 + SinhIntegral[ArcCosh[a*x]] + 9*SinhIntegral[3*ArcCosh[a*
x]])/(8*a^3)
```

Maple [A] time = 0.029, size = 84, normalized size = 1.

$$\frac{1}{a^3}\left(-\frac{1}{8\,(\text{arccosh}\,(ax))^2}\sqrt{ax-1}\sqrt{ax+1} - \frac{ax}{8\,\text{arccosh}\,(ax)} + \frac{\text{Shi}\,(\text{arccosh}\,(ax))}{8} - \frac{\sinh\,(3\,\text{arccosh}\,(ax))}{8\,(\text{arccosh}\,(ax))^2} - \frac{3\,\cosh\,(3\,\text{ar}}{8\,\text{arcco}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccosh(a*x)^3,x)
```

```
[Out] 1/a^3*(-1/8/arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-1/8*a*x/arccosh(a*x)
+1/8*Shi(arccosh(a*x))-1/8/arccosh(a*x)^2*sinh(3*arccosh(a*x))-3/8/arccosh(
a*x)*cosh(3*arccosh(a*x))+9/8*Shi(3*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^9 - 3*a^6*x^7 + 3*a^4*x^5 - a^2*x^3 + (a^5*x^6 - a^3*x^4)*(a*x
+ 1)^(3/2)*(a*x - 1)^(3/2) + (3*a^6*x^7 - 5*a^4*x^5 + 2*a^2*x^3)*(a*x + 1)*
(a*x - 1) + (3*a^7*x^8 - 7*a^5*x^6 + 5*a^3*x^4 - a*x^2)*sqrt(a*x + 1)*sqrt(
a*x - 1) + (3*a^8*x^9 - 9*a^6*x^7 + 9*a^4*x^5 - 3*a^2*x^3 + (3*a^5*x^6 - 4*
a^3*x^4 + a*x^2)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + (9*a^6*x^7 - 17*a^4*x^5
+ 10*a^2*x^3 - 2*x)*(a*x + 1)*(a*x - 1) + (9*a^7*x^8 - 22*a^5*x^6 + 18*a^3*
x^4 - 5*a*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*
x - 1)))/((a^8*x^6 + (a*x + 1)^(3/2)*(a*x - 1)^(3/2)*a^5*x^3 - 3*a^6*x^4 +
3*a^4*x^2 + 3*(a^6*x^4 - a^4*x^2)*(a*x + 1)*(a*x - 1) + 3*(a^7*x^5 - 2*a^5*
x^3 + a^3*x)*sqrt(a*x + 1)*sqrt(a*x - 1) - a^2)*log(a*x + sqrt(a*x + 1)*sqr
t(a*x - 1))^2) + integrate(1/2*(9*a^10*x^10 - 36*a^8*x^8 + 54*a^6*x^6 - 36*
a^4*x^4 + (9*a^6*x^6 - 4*a^4*x^4 - a^2*x^2)*(a*x + 1)^2*(a*x - 1)^2 + (36*a
^7*x^7 - 48*a^5*x^5 + 13*a^3*x^3 + 2*a*x)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) +
 9*a^2*x^2 + (54*a^8*x^8 - 120*a^6*x^6 + 83*a^4*x^4 - 19*a^2*x^2 + 2)*(a*x
+ 1)*(a*x - 1) + (36*a^9*x^9 - 112*a^7*x^7 + 123*a^5*x^5 - 57*a^3*x^3 + 10*
a*x)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^10*x^8 + (a*x + 1)^2*(a*x - 1)^2*a^6*
x^4 - 4*a^8*x^6 + 6*a^6*x^4 - 4*a^4*x^2 + 4*(a^7*x^5 - a^5*x^3)*(a*x + 1)^(
3/2)*(a*x - 1)^(3/2) + 6*(a^8*x^6 - 2*a^6*x^4 + a^4*x^2)*(a*x + 1)*(a*x - 1
) + 4*(a^9*x^7 - 3*a^7*x^5 + 3*a^5*x^3 - a^3*x)*sqrt(a*x + 1)*sqrt(a*x - 1)
 + a^2)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\operatorname{arcosh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2/arccosh(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{acosh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acosh(a*x)**3,x)
```

```
[Out] Integral(x**2/acosh(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arcosh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/arccosh(a*x)^3, x)
```

3.61 $\int \frac{x}{\cosh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=68

$$\frac{\text{Shi}\left(2\cosh^{-1}(ax)\right)}{a^2}+\frac{1}{2a^2\cosh^{-1}(ax)}-\frac{x^2}{\cosh^{-1}(ax)}-\frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a\cosh^{-1}(ax)^2}$$

```
[Out] -(x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*ArcCosh[a*x]^2) + 1/(2*a^2*ArcCosh[a
*x]) - x^2/ArcCosh[a*x] + SinhIntegral[2*ArcCosh[a*x]]/a^2
```

Rubi [A] time = 0.392987, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5668, 5775, 5670, 5448, 12, 3298, 5676}

$$\frac{\text{Shi}\left(2\cosh^{-1}(ax)\right)}{a^2}+\frac{1}{2a^2\cosh^{-1}(ax)}-\frac{x^2}{\cosh^{-1}(ax)}-\frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCosh[a*x]^3,x]
```

```
[Out] -(x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*ArcCosh[a*x]^2) + 1/(2*a^2*ArcCosh[a
*x]) - x^2/ArcCosh[a*x] + SinhIntegral[2*ArcCosh[a*x]]/a^2
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
```

```
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{x}{\cosh^{-1}(ax)^3}\,dx = -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} - \frac{\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}\,dx}{2a} + a\int \frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}\,dx$$
$$= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{1}{2a^2\cosh^{-1}(ax)} - \frac{x^2}{\cosh^{-1}(ax)} + 2\int \frac{x}{\cosh^{-1}(ax)}\,dx$$
$$= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{1}{2a^2\cosh^{-1}(ax)} - \frac{x^2}{\cosh^{-1}(ax)} + \frac{2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^2}$$
$$= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{1}{2a^2\cosh^{-1}(ax)} - \frac{x^2}{\cosh^{-1}(ax)} + \frac{2\,\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \cosh^{-1}(ax)\right)}{a^2}$$
$$= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{1}{2a^2\cosh^{-1}(ax)} - \frac{x^2}{\cosh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^2}$$
$$= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{1}{2a^2\cosh^{-1}(ax)} - \frac{x^2}{\cosh^{-1}(ax)} + \frac{\text{Shi}\left(2\cosh^{-1}(ax)\right)}{a^2}$$

Mathematica [A] time = 0.0493593, size = 67, normalized size = 0.99

$$\frac{\text{Shi}\left(2\cosh^{-1}(ax)\right)}{a^2} + \frac{1-2a^2x^2}{2a^2\cosh^{-1}(ax)} - \frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcCosh[a*x]^3,x]
```

```
[Out] -(x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*ArcCosh[a*x]^2) + (1 - 2*a^2*x^2)/(2
*a^2*ArcCosh[a*x]) + SinhIntegral[2*ArcCosh[a*x]]/a^2
```

Maple [A] time = 0.026, size = 43, normalized size = 0.6

$$\frac{1}{a^2}\left(-\frac{\sinh(2\,\text{arccosh}(ax))}{4\,(\text{arccosh}(ax))^2} - \frac{\cosh(2\,\text{arccosh}(ax))}{2\,\text{arccosh}(ax)} + \text{Shi}(2\,\text{arccosh}(ax))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccosh(a*x)^3,x)
```

```
[Out] 1/a^2*(-1/4/arccosh(a*x)^2*sinh(2*arccosh(a*x))-1/2/arccosh(a*x)*cosh(2*arc
cosh(a*x))+Shi(2*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^8 - 3*a^6*x^6 + 3*a^4*x^4 + (a^5*x^5 - a^3*x^3)*(a*x + 1)^(3/2)
*(a*x - 1)^(3/2) - a^2*x^2 + (3*a^6*x^6 - 5*a^4*x^4 + 2*a^2*x^2)*(a*x + 1)*
(a*x - 1) + (3*a^7*x^7 - 7*a^5*x^5 + 5*a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*
x - 1) + (2*a^8*x^8 - 6*a^6*x^6 + 6*a^4*x^4 + 2*(a^5*x^5 - a^3*x^3)*(a*x +
1)^(3/2)*(a*x - 1)^(3/2) - 2*a^2*x^2 + (6*a^6*x^6 - 10*a^4*x^4 + 5*a^2*x^2
- 1)*(a*x + 1)*(a*x - 1) + (6*a^7*x^7 - 14*a^5*x^5 + 11*a^3*x^3 - 3*a*x)*sq
rt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)))/((a^8*x^
6 + (a*x + 1)^(3/2)*(a*x - 1)^(3/2)*a^5*x^3 - 3*a^6*x^4 + 3*a^4*x^2 + 3*(a^
6*x^4 - a^4*x^2)*(a*x + 1)*(a*x - 1) + 3*(a^7*x^5 - 2*a^5*x^3 + a^3*x)*sqrt
(a*x + 1)*sqrt(a*x - 1) - a^2)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2) +
integrate(1/2*(4*a^9*x^9 + 4*(a*x + 1)^2*(a*x - 1)^2*a^5*x^5 - 16*a^7*x^7 +
 24*a^5*x^5 - 16*a^3*x^3 + (16*a^6*x^6 - 16*a^4*x^4 + 3)*(a*x + 1)^(3/2)*(a
*x - 1)^(3/2) + 24*(a^7*x^7 - 2*a^5*x^5 + a^3*x^3)*(a*x + 1)*(a*x - 1) + (1
6*a^8*x^8 - 48*a^6*x^6 + 48*a^4*x^4 - 19*a^2*x^2 + 3)*sqrt(a*x + 1)*sqrt(a*
x - 1) + 4*a*x)/((a^9*x^8 + (a*x + 1)^2*(a*x - 1)^2*a^5*x^4 - 4*a^7*x^6 + 6
*a^5*x^4 - 4*a^3*x^2 + 4*(a^6*x^5 - a^4*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2
) + 6*(a^7*x^6 - 2*a^5*x^4 + a^3*x^2)*(a*x + 1)*(a*x - 1) + 4*(a^8*x^7 - 3*
a^6*x^5 + 3*a^4*x^3 - a^2*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + a)*log(a*x + sqr
t(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\operatorname{arcosh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x/arccosh(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{acosh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acosh(a*x)**3,x)
```

```
[Out] Integral(x/acosh(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arcosh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/arccosh(a*x)^3, x)
```

3.62 $\int \frac{1}{\cosh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=55

$$\frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{2a}-\frac{x}{2\cosh^{-1}(ax)}-\frac{\sqrt{ax-1}\sqrt{ax+1}}{2a\cosh^{-1}(ax)^2}$$

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*ArcCosh[a*x]^2) - x/(2*ArcCosh[a*x]) +
 SinhIntegral[ArcCosh[a*x]]/(2*a)
```

Rubi [A] time = 0.187885, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5656, 5775, 5658, 3298}

$$\frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{2a}-\frac{x}{2\cosh^{-1}(ax)}-\frac{\sqrt{ax-1}\sqrt{ax+1}}{2a\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^(-3),x]
```

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*ArcCosh[a*x]^2) - x/(2*ArcCosh[a*x]) +
 SinhIntegral[ArcCosh[a*x]]/(2*a)
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\cosh^{-1}(ax)^3}\,dx &= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2}+\frac{1}{2}a\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}\,dx\\
&= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2}-\frac{x}{2\cosh^{-1}(ax)}+\frac{1}{2}\int \frac{1}{\cosh^{-1}(ax)}\,dx\\
&= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2}-\frac{x}{2\cosh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\cosh^{-1}(ax)\right)}{2a}\\
&= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2}-\frac{x}{2\cosh^{-1}(ax)}+\frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.0419904, size = 55, normalized size = 1.

$$\frac{\text{Shi}\left(\cosh^{-1}(ax)\right)}{2a}-\frac{x}{2\cosh^{-1}(ax)}-\frac{\sqrt{ax-1}\sqrt{ax+1}}{2a\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^(-3),x]
```

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*ArcCosh[a*x]^2) - x/(2*ArcCosh[a*x]) +
 SinhIntegral[ArcCosh[a*x]]/(2*a)
```

Maple [A] time = 0.027, size = 45, normalized size = 0.8

$$\frac{1}{a}\left(-\frac{1}{2\left(\operatorname{arccosh}\left(ax\right)\right)^{2}}\sqrt{ax-1}\sqrt{ax+1}-\frac{ax}{2\operatorname{arccosh}\left(ax\right)}+\frac{\operatorname{Shi}\left(\operatorname{arccosh}\left(ax\right)\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x)^3,x)
```

```
[Out] 1/a*(-1/2/arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-1/2*a*x/arccosh(a*x)+1
/2*Shi(arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^7*x^7 - 3*a^5*x^5 + 3*a^3*x^3 + (a^4*x^4 - a^2*x^2)*(a*x + 1)^(3/2)
*(a*x - 1)^(3/2) + (3*a^5*x^5 - 5*a^3*x^3 + 2*a*x)*(a*x + 1)*(a*x - 1) + (3
*a^6*x^6 - 7*a^4*x^4 + 5*a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x + (
a^7*x^7 - 3*a^5*x^5 + 3*a^3*x^3 + (a^4*x^4 - 1)*(a*x + 1)^(3/2)*(a*x - 1)^(
3/2) + 3*(a^5*x^5 - a^3*x^3)*(a*x + 1)*(a*x - 1) + (3*a^6*x^6 - 6*a^4*x^4 +
 4*a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x)*log(a*x + sqrt(a*x + 1)*
sqrt(a*x - 1)))/((a^7*x^6 + (a*x + 1)^(3/2)*(a*x - 1)^(3/2)*a^4*x^3 - 3*a^5
*x^4 + 3*a^3*x^2 + 3*(a^5*x^4 - a^3*x^2)*(a*x + 1)*(a*x - 1) + 3*(a^6*x^5 -
 2*a^4*x^3 + a^2*x)*sqrt(a*x + 1)*sqrt(a*x - 1) - a)*log(a*x + sqrt(a*x + 1
)*sqrt(a*x - 1))^2) + integrate(1/2*(a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 + (a^4
*x^4 + 3)*(a*x + 1)^2*(a*x - 1)^2 + (4*a^5*x^5 - 4*a^3*x^3 + 3*a*x)*(a*x +
1)^(3/2)*(a*x - 1)^(3/2) - 4*a^2*x^2 + 3*(2*a^6*x^6 - 4*a^4*x^4 + a^2*x^2 +
 1)*(a*x + 1)*(a*x - 1) + (4*a^7*x^7 - 12*a^5*x^5 + 9*a^3*x^3 - a*x)*sqrt(a
*x + 1)*sqrt(a*x - 1) + 1)/((a^8*x^8 + (a*x + 1)^2*(a*x - 1)^2*a^4*x^4 - 4*
a^6*x^6 + 6*a^4*x^4 + 4*(a^5*x^5 - a^3*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2)
 - 4*a^2*x^2 + 6*(a^6*x^6 - 2*a^4*x^4 + a^2*x^2)*(a*x + 1)*(a*x - 1) + 4*(a
^7*x^7 - 3*a^5*x^5 + 3*a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + 1)*log(
a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\operatorname{arcosh}(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^(-3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acosh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)**3,x)
```

```
[Out] Integral(acosh(a*x)**(-3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arcosh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^(-3), x)
```

3.63 $\int \frac{1}{x\cosh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x\cosh^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[1/(x*ArcCosh[a*x]^3), x]
```

Rubi [A] time = 0.0136231, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\cosh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcCosh[a*x]^3),x]
```

```
[Out] Defer[Int][1/(x*ArcCosh[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{x\cosh^{-1}(ax)^3}\,dx = \int \frac{1}{x\cosh^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 0.658753, size = 0, normalized size = 0.

$$\int \frac{1}{x\cosh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcCosh[a*x]^3),x]
```

```
[Out] Integrate[1/(x*ArcCosh[a*x]^3), x]
```

Maple [A] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\operatorname{arccosh}\left(ax\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccosh(a*x)^3,x)
```

```
[Out] int(1/x/arccosh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^8 - 3*a^6*x^6 + 3*a^4*x^4 + (a^5*x^5 - a^3*x^3)*(a*x + 1)^(3/2)
*(a*x - 1)^(3/2) - a^2*x^2 + (3*a^6*x^6 - 5*a^4*x^4 + 2*a^2*x^2)*(a*x + 1)*
(a*x - 1) + (3*a^7*x^7 - 7*a^5*x^5 + 5*a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*
x - 1) + (2*(a^3*x^3 - a*x)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + (4*a^4*x^4 -
5*a^2*x^2 + 1)*(a*x + 1)*(a*x - 1) + (2*a^5*x^5 - 3*a^3*x^3 + a*x)*sqrt(a*x
 + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)))/((a^8*x^8 + (a
*x + 1)^(3/2)*(a*x - 1)^(3/2)*a^5*x^5 - 3*a^6*x^6 + 3*a^4*x^4 - a^2*x^2 + 3
*(a^6*x^6 - a^4*x^4)*(a*x + 1)*(a*x - 1) + 3*(a^7*x^7 - 2*a^5*x^5 + a^3*x^3
)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2) -
integrate(1/2*(4*(a^4*x^4 - 2*a^2*x^2)*(a*x + 1)^2*(a*x - 1)^2 + (12*a^5*x^
5 - 22*a^3*x^3 + 7*a*x)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 2*(6*a^6*x^6 - 10
*a^4*x^4 + 5*a^2*x^2 - 1)*(a*x + 1)*(a*x - 1) + (4*a^7*x^7 - 6*a^5*x^5 + 3*
a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^10*x^11 + (a*x + 1)^2*(a*x
- 1)^2*a^6*x^7 - 4*a^8*x^9 + 6*a^6*x^7 - 4*a^4*x^5 + a^2*x^3 + 4*(a^7*x^8 -
 a^5*x^6)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 6*(a^8*x^9 - 2*a^6*x^7 + a^4*x^
5)*(a*x + 1)*(a*x - 1) + 4*(a^9*x^10 - 3*a^7*x^8 + 3*a^5*x^6 - a^3*x^4)*sqr
t(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x \operatorname{arcosh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arccosh(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{acosh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acosh(a*x)**3,x)
```

```
[Out] Integral(1/(x*acosh(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arcosh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccosh(a*x)^3), x)
```

3.64 $\int \frac{1}{x^2 \cosh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x^2 \cosh^{-1}(ax)^3}, x\right)$$

```
[Out] Unintegrable[1/(x^2*ArcCosh[a*x]^3), x]
```

Rubi [A] time = 0.0149753, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*ArcCosh[a*x]^3),x]
```

```
[Out] Defer[Int][1/(x^2*ArcCosh[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^3} \, dx = \int \frac{1}{x^2 \cosh^{-1}(ax)^3} \, dx$$

Mathematica [A] time = 2.19435, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*ArcCosh[a*x]^3),x]
```

```
[Out] Integrate[1/(x^2*ArcCosh[a*x]^3), x]
```

Maple [A] time = 0.089, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(\operatorname{arccosh}\left(ax\right)\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccosh(a*x)^3,x)
```

```
[Out] int(1/x^2/arccosh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^8 - 3*a^6*x^6 + 3*a^4*x^4 + (a^5*x^5 - a^3*x^3)*(a*x + 1)^(3/2)
*(a*x - 1)^(3/2) - a^2*x^2 + (3*a^6*x^6 - 5*a^4*x^4 + 2*a^2*x^2)*(a*x + 1)*
(a*x - 1) + (3*a^7*x^7 - 7*a^5*x^5 + 5*a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*
x - 1) - (a^8*x^8 - 3*a^6*x^6 + 3*a^4*x^4 + (a^5*x^5 - 4*a^3*x^3 + 3*a*x)*(
a*x + 1)^(3/2)*(a*x - 1)^(3/2) - a^2*x^2 + (3*a^6*x^6 - 11*a^4*x^4 + 10*a^2
*x^2 - 2)*(a*x + 1)*(a*x - 1) + (3*a^7*x^7 - 10*a^5*x^5 + 10*a^3*x^3 - 3*a*
x)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)))/((a
^8*x^9 + (a*x + 1)^(3/2)*(a*x - 1)^(3/2)*a^5*x^6 - 3*a^6*x^7 + 3*a^4*x^5 -
a^2*x^3 + 3*(a^6*x^7 - a^4*x^5)*(a*x + 1)*(a*x - 1) + 3*(a^7*x^8 - 2*a^5*x^
6 + a^3*x^4)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x
- 1))^2) + integrate(1/2*(a^10*x^10 - 4*a^8*x^8 + 6*a^6*x^6 - 4*a^4*x^4 + (
a^6*x^6 - 12*a^4*x^4 + 15*a^2*x^2)*(a*x + 1)^2*(a*x - 1)^2 + (4*a^7*x^7 - 4
0*a^5*x^5 + 57*a^3*x^3 - 18*a*x)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + a^2*x^2
+ 3*(2*a^8*x^8 - 16*a^6*x^6 + 25*a^4*x^4 - 13*a^2*x^2 + 2)*(a*x + 1)*(a*x -
 1) + (4*a^9*x^9 - 24*a^7*x^7 + 39*a^5*x^5 - 25*a^3*x^3 + 6*a*x)*sqrt(a*x +
 1)*sqrt(a*x - 1))/((a^10*x^12 + (a*x + 1)^2*(a*x - 1)^2*a^6*x^8 - 4*a^8*x^
10 + 6*a^6*x^8 - 4*a^4*x^6 + a^2*x^4 + 4*(a^7*x^9 - a^5*x^7)*(a*x + 1)^(3/2
)*(a*x - 1)^(3/2) + 6*(a^8*x^10 - 2*a^6*x^8 + a^4*x^6)*(a*x + 1)*(a*x - 1)
+ 4*(a^9*x^11 - 3*a^7*x^9 + 3*a^5*x^7 - a^3*x^5)*sqrt(a*x + 1)*sqrt(a*x - 1
))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^2 \operatorname{arcosh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*arccosh(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{acosh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acosh(a*x)**3,x)
```

```
[Out] Integral(1/(x**2*acosh(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{arcosh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arccosh(a*x)^3), x)
```

3.65 $\int \frac{x^4}{\cosh^{-1}(ax)^4}\,dx$

Optimal. Leaf size=170

$$\frac{\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{48a^5}+\frac{27\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{32a^5}+\frac{125\mathrm{Chi}\left(5\cosh^{-1}(ax)\right)}{96a^5}+\frac{2x^3}{3a^2\cosh^{-1}(ax)^2}+\frac{2x^2\sqrt{ax-1}\sqrt{ax+1}}{a^3\cosh^{-1}(ax)}-\frac{}{6\cos}$$

```
[Out] -(x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^3) + (2*x^3)/(3*a^2*A
rcCosh[a*x]^2) - (5*x^5)/(6*ArcCosh[a*x]^2) + (2*x^2*Sqrt[-1 + a*x]*Sqrt[1
+ a*x])/(a^3*ArcCosh[a*x]) - (25*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(6*a*Arc
Cosh[a*x]) + CoshIntegral[ArcCosh[a*x]]/(48*a^5) + (27*CoshIntegral[3*ArcCo
sh[a*x]])/(32*a^5) + (125*CoshIntegral[5*ArcCosh[a*x]])/(96*a^5)
```

Rubi [A] time = 0.617029, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5668, 5775, 5666, 3301}

$$\frac{\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{48a^5}+\frac{27\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{32a^5}+\frac{125\mathrm{Chi}\left(5\cosh^{-1}(ax)\right)}{96a^5}+\frac{2x^3}{3a^2\cosh^{-1}(ax)^2}+\frac{2x^2\sqrt{ax-1}\sqrt{ax+1}}{a^3\cosh^{-1}(ax)}-\frac{}{6\cos}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCosh[a*x]^4,x]
```


```
[Out] -(x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^3) + (2*x^3)/(3*a^2*A
rcCosh[a*x]^2) - (5*x^5)/(6*ArcCosh[a*x]^2) + (2*x^2*Sqrt[-1 + a*x]*Sqrt[1
+ a*x])/(a^3*ArcCosh[a*x]) - (25*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(6*a*Arc
Cosh[a*x]) + CoshIntegral[ArcCosh[a*x]]/(48*a^5) + (27*CoshIntegral[3*ArcCo
sh[a*x]])/(32*a^5) + (125*CoshIntegral[5*ArcCosh[a*x]])/(96*a^5)
```


Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4}{\cosh^{-1}(ax)^4}\,dx = -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} - \frac{4\int \frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}\,dx}{3a} + \frac{1}{3}(5a)\int \frac{x^5}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}\,dx$$

$$= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{2x^3}{3a^2\cosh^{-1}(ax)^2} - \frac{5x^5}{6\cosh^{-1}(ax)^2} + \frac{25}{6}\int \frac{x^4}{\cosh^{-1}(ax)^2}\,dx - \frac{2\int \frac{x^2}{\cosh^{-1}(ax)^2}}{a^2}$$

$$= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{2x^3}{3a^2\cosh^{-1}(ax)^2} - \frac{5x^5}{6\cosh^{-1}(ax)^2} + \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{a^3\cosh^{-1}(ax)} - \frac{25x^4\sqrt{-1+a}}{6a\cosh}$$

$$= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{2x^3}{3a^2\cosh^{-1}(ax)^2} - \frac{5x^5}{6\cosh^{-1}(ax)^2} + \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{a^3\cosh^{-1}(ax)} - \frac{25x^4\sqrt{-1+a}}{6a\cosh}$$

$$= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{2x^3}{3a^2\cosh^{-1}(ax)^2} - \frac{5x^5}{6\cosh^{-1}(ax)^2} + \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{a^3\cosh^{-1}(ax)} - \frac{25x^4\sqrt{-1+a}}{6a\cosh}$$

Mathematica [A] time = 0.608565, size = 126, normalized size = 0.74

$$\frac{-\frac{16a^2x^2\left(2a^2x^2\sqrt{ax-1}\sqrt{ax+1}+ax\left(5a^2x^2-4\right)\cosh^{-1}(ax)+\sqrt{ax-1}\sqrt{ax+1}\left(25a^2x^2-12\right)\cosh^{-1}(ax)^2\right)}{\cosh^{-1}(ax)^3} + 2\text{Chi}\left(\cosh^{-1}(ax)\right) + 81\text{Chi}\left(3\cosh^{-1}(ax)\right)}{96a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcCosh[a*x]^4,x]
```

```
[Out] ((-16*a^2*x^2*(2*a^2*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x] + a*x*(-4 + 5*a^2*x^2
)*ArcCosh[a*x] + Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-12 + 25*a^2*x^2)*ArcCosh[a*
x]^2))/ArcCosh[a*x]^3 + 2*CoshIntegral[ArcCosh[a*x]] + 81*CoshIntegral[3*Ar
cCosh[a*x]] + 125*CoshIntegral[5*ArcCosh[a*x]])/(96*a^5)
```

Maple [A] time = 0.043, size = 175, normalized size = 1.

$$\frac{1}{a^5}\left(-\frac{1}{24\left(\operatorname{arccosh}(ax)\right)^3}\sqrt{ax-1}\sqrt{ax+1}-\frac{ax}{48\left(\operatorname{arccosh}(ax)\right)^2}-\frac{1}{48\operatorname{arccosh}(ax)}\sqrt{ax-1}\sqrt{ax+1}+\frac{\operatorname{Chi}(\operatorname{arccosh}(ax}{48}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccosh(a*x)^4,x)
```

```
[Out] 1/a^5*(-1/24/arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)-1/48*a*x/arccosh(a*
x)^2-1/48/arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+1/48*Chi(arccosh(a*x))-1
/16/arccosh(a*x)^3*sinh(3*arccosh(a*x))-3/32/arccosh(a*x)^2*cosh(3*arccosh(
a*x))-9/32/arccosh(a*x)*sinh(3*arccosh(a*x))+27/32*Chi(3*arccosh(a*x))-1/48
/arccosh(a*x)^3*sinh(5*arccosh(a*x))-5/96/arccosh(a*x)^2*cosh(5*arccosh(a*x
))-25/96/arccosh(a*x)*sinh(5*arccosh(a*x))+125/96*Chi(5*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^15 - 10*a^11*x^13 + 20*a^9*x^11 - 20*a^7*x^9 + 10*a^5*x^7 -
2*a^3*x^5 + 2*(a^8*x^10 - a^6*x^8)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + 2*(5*a
^9*x^11 - 9*a^7*x^9 + 4*a^5*x^7)*(a*x + 1)^2*(a*x - 1)^2 + 4*(5*a^10*x^12 -
 13*a^8*x^10 + 11*a^6*x^8 - 3*a^4*x^6)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 4*
(5*a^11*x^13 - 17*a^9*x^11 + 21*a^7*x^9 - 11*a^5*x^7 + 2*a^3*x^5)*(a*x + 1)
*(a*x - 1) + (25*a^13*x^15 - 125*a^11*x^13 + 250*a^9*x^11 - 250*a^7*x^9 + 1
```

```
25*a^5*x^7 - 25*a^3*x^5 + (25*a^8*x^10 - 49*a^6*x^8 + 27*a^4*x^6 - 3*a^2*x^
4)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + (125*a^9*x^11 - 321*a^7*x^9 + 286*a^5*
x^7 - 102*a^3*x^5 + 12*a*x^3)*(a*x + 1)^2*(a*x - 1)^2 + (250*a^10*x^12 - 79
4*a^8*x^10 + 946*a^6*x^8 - 519*a^4*x^6 + 129*a^2*x^4 - 12*x^2)*(a*x + 1)^(3
/2)*(a*x - 1)^(3/2) + 2*(125*a^11*x^13 - 473*a^9*x^11 + 696*a^7*x^9 - 497*a
^5*x^7 + 173*a^3*x^5 - 24*a*x^3)*(a*x + 1)*(a*x - 1) + (125*a^12*x^14 - 549
*a^10*x^12 + 955*a^8*x^10 - 824*a^6*x^8 + 354*a^4*x^6 - 61*a^2*x^4)*sqrt(a*
x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2 + 2*(5*a^12*
x^14 - 21*a^10*x^12 + 34*a^8*x^10 - 26*a^6*x^8 + 9*a^4*x^6 - a^2*x^4)*sqrt(
a*x + 1)*sqrt(a*x - 1) + (5*a^13*x^15 - 25*a^11*x^13 + 50*a^9*x^11 - 50*a^7
*x^9 + 25*a^5*x^7 - 5*a^3*x^5 + (5*a^8*x^10 - 8*a^6*x^8 + 3*a^4*x^6)*(a*x +
 1)^(5/2)*(a*x - 1)^(5/2) + (25*a^9*x^11 - 57*a^7*x^9 + 42*a^5*x^7 - 10*a^3
*x^5)*(a*x + 1)^2*(a*x - 1)^2 + (50*a^10*x^12 - 148*a^8*x^10 + 158*a^6*x^8
- 71*a^4*x^6 + 11*a^2*x^4)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 2*(25*a^11*x^1
3 - 91*a^9*x^11 + 126*a^7*x^9 - 81*a^5*x^7 + 23*a^3*x^5 - 2*a*x^3)*(a*x + 1
)*(a*x - 1) + (25*a^12*x^14 - 108*a^10*x^12 + 183*a^8*x^10 - 151*a^6*x^8 +
60*a^4*x^6 - 9*a^2*x^4)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1
)*sqrt(a*x - 1)))/((a^13*x^10 - 5*a^11*x^8 + (a*x + 1)^(5/2)*(a*x - 1)^(5/2
)*a^8*x^5 + 10*a^9*x^6 - 10*a^7*x^4 + 5*a^5*x^2 + 5*(a^9*x^6 - a^7*x^4)*(a*
x + 1)^2*(a*x - 1)^2 + 10*(a^10*x^7 - 2*a^8*x^5 + a^6*x^3)*(a*x + 1)^(3/2)*
(a*x - 1)^(3/2) + 10*(a^11*x^8 - 3*a^9*x^6 + 3*a^7*x^4 - a^5*x^2)*(a*x + 1)
*(a*x - 1) - a^3 + 5*(a^12*x^9 - 4*a^10*x^7 + 6*a^8*x^5 - 4*a^6*x^3 + a^4*x
)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3) +
integrate(1/6*(125*a^15*x^16 - 750*a^13*x^14 + 1875*a^11*x^12 - 2500*a^9*x^
10 + 1875*a^7*x^8 - 750*a^5*x^6 + (125*a^9*x^10 - 147*a^7*x^8 + 27*a^5*x^6
+ 3*a^3*x^4)*(a*x + 1)^3*(a*x - 1)^3 + 125*a^3*x^4 + (750*a^10*x^11 - 1485*
a^8*x^9 + 901*a^6*x^7 - 147*a^4*x^5 - 12*a^2*x^3)*(a*x + 1)^(5/2)*(a*x - 1)
^(5/2) + (1875*a^11*x^12 - 5220*a^9*x^10 + 5209*a^7*x^8 - 2185*a^5*x^6 + 32
1*a^3*x^4)*(a*x + 1)^2*(a*x - 1)^2 + (2500*a^12*x^13 - 8970*a^10*x^11 + 123
66*a^8*x^9 - 8143*a^6*x^7 + 2583*a^4*x^5 - 360*a^2*x^3 + 24*x)*(a*x + 1)^(3
/2)*(a*x - 1)^(3/2) + (1875*a^13*x^14 - 8235*a^11*x^12 + 14449*a^9*x^10 - 1
2834*a^7*x^8 + 6030*a^5*x^6 - 1429*a^3*x^4 + 144*a*x^2)*(a*x + 1)*(a*x - 1)
 + (750*a^14*x^15 - 3897*a^12*x^13 + 8293*a^10*x^11 - 9226*a^8*x^9 + 5655*a
^6*x^7 - 1819*a^4*x^5 + 244*a^2*x^3)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^15*x^
12 - 6*a^13*x^10 + (a*x + 1)^3*(a*x - 1)^3*a^9*x^6 + 15*a^11*x^8 - 20*a^9*x
^6 + 15*a^7*x^4 - 6*a^5*x^2 + 6*(a^10*x^7 - a^8*x^5)*(a*x + 1)^(5/2)*(a*x -
 1)^(5/2) + 15*(a^11*x^8 - 2*a^9*x^6 + a^7*x^4)*(a*x + 1)^2*(a*x - 1)^2 + 2
0*(a^12*x^9 - 3*a^10*x^7 + 3*a^8*x^5 - a^6*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(
3/2) + 15*(a^13*x^10 - 4*a^11*x^8 + 6*a^9*x^6 - 4*a^7*x^4 + a^5*x^2)*(a*x +
 1)*(a*x - 1) + a^3 + 6*(a^14*x^11 - 5*a^12*x^9 + 10*a^10*x^7 - 10*a^8*x^5
+ 5*a^6*x^3 - a^4*x)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*s
qrt(a*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\operatorname{arcosh}(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^4/arccosh(a*x)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{acosh}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acosh(a*x)**4,x)
```

```
[Out] Integral(x**4/acosh(a*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arcosh}(ax)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^4/arccosh(a*x)^4, x)
```

3.66 $\int \frac{x^3}{\cosh^{-1}(ax)^4}\,dx$

Optimal. Leaf size=155

$$\frac{\text{Chi}\left(2\cosh^{-1}(ax)\right)}{3a^4}+\frac{4\text{Chi}\left(4\cosh^{-1}(ax)\right)}{3a^4}+\frac{x^2}{2a^2\cosh^{-1}(ax)^2}+\frac{x\sqrt{ax-1}\sqrt{ax+1}}{a^3\cosh^{-1}(ax)}-\frac{2x^4}{3\cosh^{-1}(ax)^2}-\frac{8x^3\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)}$$

```
[Out] -(x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^3) + x^2/(2*a^2*ArcCo
sh[a*x]^2) - (2*x^4)/(3*ArcCosh[a*x]^2) + (x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/
(a^3*ArcCosh[a*x]) - (8*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]
) + CoshIntegral[2*ArcCosh[a*x]]/(3*a^4) + (4*CoshIntegral[4*ArcCosh[a*x]])
/(3*a^4)
```

Rubi [A] time = 0.587555, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5668, 5775, 5666, 3301}

$$\frac{\text{Chi}\left(2\cosh^{-1}(ax)\right)}{3a^4}+\frac{4\text{Chi}\left(4\cosh^{-1}(ax)\right)}{3a^4}+\frac{x^2}{2a^2\cosh^{-1}(ax)^2}+\frac{x\sqrt{ax-1}\sqrt{ax+1}}{a^3\cosh^{-1}(ax)}-\frac{2x^4}{3\cosh^{-1}(ax)^2}-\frac{8x^3\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCosh[a*x]^4,x]
```

```
[Out] -(x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^3) + x^2/(2*a^2*ArcCo
sh[a*x]^2) - (2*x^4)/(3*ArcCosh[a*x]^2) + (x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/
(a^3*ArcCosh[a*x]) - (8*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]
) + CoshIntegral[2*ArcCosh[a*x]]/(3*a^4) + (4*CoshIntegral[4*ArcCosh[a*x]])
/(3*a^4)
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3}{\cosh^{-1}(ax)^4}\,dx = -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} - \frac{\int \frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}\,dx}{a} + \frac{1}{3}(4a)\int \frac{x^4}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}\,dx$$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{x^2}{2a^2\cosh^{-1}(ax)^2} - \frac{2x^4}{3\cosh^{-1}(ax)^2} + \frac{8}{3}\int \frac{x^3}{\cosh^{-1}(ax)^2}\,dx - \frac{\int \frac{x}{\cosh^{-1}(ax)^2}}{a^2}$$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{x^2}{2a^2\cosh^{-1}(ax)^2} - \frac{2x^4}{3\cosh^{-1}(ax)^2} + \frac{x\sqrt{-1+ax}\sqrt{1+ax}}{a^3\cosh^{-1}(ax)} - \frac{8x^3\sqrt{-1+ax}}{3a\cosh^{-1}}$$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{x^2}{2a^2\cosh^{-1}(ax)^2} - \frac{2x^4}{3\cosh^{-1}(ax)^2} + \frac{x\sqrt{-1+ax}\sqrt{1+ax}}{a^3\cosh^{-1}(ax)} - \frac{8x^3\sqrt{-1+ax}}{3a\cosh^{-1}}$$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{x^2}{2a^2\cosh^{-1}(ax)^2} - \frac{2x^4}{3\cosh^{-1}(ax)^2} + \frac{x\sqrt{-1+ax}\sqrt{1+ax}}{a^3\cosh^{-1}(ax)} - \frac{8x^3\sqrt{-1+ax}}{3a\cosh^{-1}}$$

Mathematica [A] time = 0.373256, size = 188, normalized size = 1.21

$$\frac{\sqrt{ax-1}\left(ax\sqrt{\frac{ax-1}{ax+1}}\left(-2a^4x^4+2a^2x^2-ax\sqrt{ax-1}\sqrt{ax+1}\left(4a^2x^2-3\right)\cosh^{-1}(ax)-2\left(8a^4x^4-11a^2x^2+3\right)\cosh^{-1}(ax)^2\right)\right.}{6a^4\left(\frac{ax-1}{ax+1}\right)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/ArcCosh[a*x]^4,x]
```

```
[Out] (Sqrt[-1 + a*x]*(a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*(2*a^2*x^2 - 2*a^4*x^4 - a*
x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-3 + 4*a^2*x^2)*ArcCosh[a*x] - 2*(3 - 11*a^
2*x^2 + 8*a^4*x^4)*ArcCosh[a*x]^2) + 2*(-1 + a*x)*ArcCosh[a*x]^3*CoshIntegr
al[2*ArcCosh[a*x]] + 8*(-1 + a*x)*ArcCosh[a*x]^3*CoshIntegral[4*ArcCosh[a*x
]]))/(6*a^4*((-1 + a*x)/(1 + a*x))^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x]^3)
```

Maple [A] time = 0.043, size = 114, normalized size = 0.7

$$\frac{1}{a^4}\left(-\frac{\sinh(2\operatorname{arccosh}(ax))}{12(\operatorname{arccosh}(ax))^3}-\frac{\cosh(2\operatorname{arccosh}(ax))}{12(\operatorname{arccosh}(ax))^2}-\frac{\sinh(2\operatorname{arccosh}(ax))}{6\operatorname{arccosh}(ax)}+\frac{\operatorname{Chi}(2\operatorname{arccosh}(ax))}{3}-\frac{\sinh(4\operatorname{arccosh}(}{24(\operatorname{arccosh}(a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccosh(a*x)^4,x)
```

```
[Out] 1/a^4*(-1/12/arccosh(a*x)^3*sinh(2*arccosh(a*x))-1/12/arccosh(a*x)^2*cosh(2
*arccosh(a*x))-1/6/arccosh(a*x)*sinh(2*arccosh(a*x))+1/3*Chi(2*arccosh(a*x)
)-1/24/arccosh(a*x)^3*sinh(4*arccosh(a*x))-1/12/arccosh(a*x)^2*cosh(4*arcco
sh(a*x))-1/3/arccosh(a*x)*sinh(4*arccosh(a*x))+4/3*Chi(4*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^14 - 10*a^11*x^12 + 20*a^9*x^10 - 20*a^7*x^8 + 10*a^5*x^6 -
2*a^3*x^4 + 2*(a^8*x^9 - a^6*x^7)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + 2*(5*a^
9*x^10 - 9*a^7*x^8 + 4*a^5*x^6)*(a*x + 1)^2*(a*x - 1)^2 + 4*(5*a^10*x^11 -
13*a^8*x^9 + 11*a^6*x^7 - 3*a^4*x^5)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 4*(5
*a^11*x^12 - 17*a^9*x^10 + 21*a^7*x^8 - 11*a^5*x^6 + 2*a^3*x^4)*(a*x + 1)*(
a*x - 1) + (16*a^13*x^14 - 80*a^11*x^12 + 160*a^9*x^10 - 160*a^7*x^8 + 80*a
^5*x^6 - 16*a^3*x^4 + 4*(4*a^8*x^9 - 7*a^6*x^7 + 3*a^4*x^5)*(a*x + 1)^(5/2)
*(a*x - 1)^(5/2) + (80*a^9*x^10 - 192*a^7*x^8 + 154*a^5*x^6 - 45*a^3*x^4 +
3*a*x^2)*(a*x + 1)^2*(a*x - 1)^2 + (160*a^10*x^11 - 488*a^8*x^9 + 550*a^6*x
^7 - 279*a^4*x^5 + 63*a^2*x^3 - 6*x)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + (160
*a^11*x^12 - 592*a^9*x^10 + 846*a^7*x^8 - 583*a^5*x^6 + 196*a^3*x^4 - 27*a*
x^2)*(a*x + 1)*(a*x - 1) + (80*a^12*x^13 - 348*a^10*x^11 + 598*a^8*x^9 - 50
9*a^6*x^7 + 216*a^4*x^5 - 37*a^2*x^3)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x
+ sqrt(a*x + 1)*sqrt(a*x - 1))^2 + 2*(5*a^12*x^13 - 21*a^10*x^11 + 34*a^8*x
^9 - 26*a^6*x^7 + 9*a^4*x^5 - a^2*x^3)*sqrt(a*x + 1)*sqrt(a*x - 1) + (4*a^1
3*x^14 - 20*a^11*x^12 + 40*a^9*x^10 - 40*a^7*x^8 + 20*a^5*x^6 - 4*a^3*x^4 +
 2*(2*a^8*x^9 - 3*a^6*x^7 + a^4*x^5)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + (20*
a^9*x^10 - 44*a^7*x^8 + 31*a^5*x^6 - 7*a^3*x^4)*(a*x + 1)^2*(a*x - 1)^2 + (
40*a^10*x^11 - 116*a^8*x^9 + 121*a^6*x^7 - 53*a^4*x^5 + 8*a^2*x^3)*(a*x + 1
)^(3/2)*(a*x - 1)^(3/2) + (40*a^11*x^12 - 144*a^9*x^10 + 197*a^7*x^8 - 125*
a^5*x^6 + 35*a^3*x^4 - 3*a*x^2)*(a*x + 1)*(a*x - 1) + (20*a^12*x^13 - 86*a^
10*x^11 + 145*a^8*x^9 - 119*a^6*x^7 + 47*a^4*x^5 - 7*a^2*x^3)*sqrt(a*x + 1)
*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)))/((a^13*x^10 - 5*a^1
1*x^8 + (a*x + 1)^(5/2)*(a*x - 1)^(5/2)*a^8*x^5 + 10*a^9*x^6 - 10*a^7*x^4 +
 5*a^5*x^2 + 5*(a^9*x^6 - a^7*x^4)*(a*x + 1)^2*(a*x - 1)^2 + 10*(a^10*x^7 -
 2*a^8*x^5 + a^6*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 10*(a^11*x^8 - 3*a^
9*x^6 + 3*a^7*x^4 - a^5*x^2)*(a*x + 1)*(a*x - 1) - a^3 + 5*(a^12*x^9 - 4*a^
10*x^7 + 6*a^8*x^5 - 4*a^6*x^3 + a^4*x)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*
x + sqrt(a*x + 1)*sqrt(a*x - 1))^3) + integrate(1/6*(64*a^15*x^15 - 384*a^1
3*x^13 + 960*a^11*x^11 - 1280*a^9*x^9 + 960*a^7*x^7 - 384*a^5*x^5 + 8*(8*a^
9*x^9 - 7*a^7*x^7)*(a*x + 1)^3*(a*x - 1)^3 + (384*a^10*x^10 - 664*a^8*x^8 +
 308*a^6*x^6 - 12*a^4*x^4 - 9*a^2*x^2)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + 64
*a^3*x^3 + 2*(480*a^11*x^11 - 1240*a^9*x^9 + 1096*a^7*x^7 - 360*a^5*x^5 + 1
5*a^3*x^3 + 9*a*x)*(a*x + 1)^2*(a*x - 1)^2 + 2*(640*a^12*x^12 - 2200*a^10*x
^10 + 2844*a^8*x^8 - 1684*a^6*x^6 + 433*a^4*x^4 - 36*a^2*x^2 + 3)*(a*x + 1)
^(3/2)*(a*x - 1)^(3/2) + 2*(480*a^13*x^13 - 2060*a^11*x^11 + 3496*a^9*x^9 -
 2952*a^7*x^7 + 1283*a^5*x^5 - 274*a^3*x^3 + 27*a*x)*(a*x + 1)*(a*x - 1) +
(384*a^14*x^14 - 1976*a^12*x^12 + 4148*a^10*x^10 - 4524*a^8*x^8 + 2699*a^6*
x^6 - 842*a^4*x^4 + 111*a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^15*x^12 -
 6*a^13*x^10 + (a*x + 1)^3*(a*x - 1)^3*a^9*x^6 + 15*a^11*x^8 - 20*a^9*x^6 +
 15*a^7*x^4 - 6*a^5*x^2 + 6*(a^10*x^7 - a^8*x^5)*(a*x + 1)^(5/2)*(a*x - 1)^
(5/2) + 15*(a^11*x^8 - 2*a^9*x^6 + a^7*x^4)*(a*x + 1)^2*(a*x - 1)^2 + 20*(a
^12*x^9 - 3*a^10*x^7 + 3*a^8*x^5 - a^6*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2)
 + 15*(a^13*x^10 - 4*a^11*x^8 + 6*a^9*x^6 - 4*a^7*x^4 + a^5*x^2)*(a*x + 1)*
(a*x - 1) + a^3 + 6*(a^14*x^11 - 5*a^12*x^9 + 10*a^10*x^7 - 10*a^8*x^5 + 5*
```

```
a^6*x^3 - a^4*x)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(
a*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\text{arcosh}\,(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^4,x, algorithm="fricas")

[Out] integral(x^3/arccosh(a*x)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\text{acosh}^4\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acosh(a*x)**4,x)

[Out] Integral(x**3/acosh(a*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\text{arcosh}\,(ax)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^4,x, algorithm="giac")

[Out] integrate(x^3/arccosh(a*x)^4, x)
```

3.67 $\int \frac{x^2}{\cosh^{-1}(ax)^4} dx$

Optimal. Leaf size=153

$$\frac{\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{24a^3}+\frac{9\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{8a^3}+\frac{x}{3a^2\cosh^{-1}(ax)^2}+\frac{\sqrt{ax-1}\sqrt{ax+1}}{3a^3\cosh^{-1}(ax)}-\frac{x^3}{2\cosh^{-1}(ax)^2}-\frac{3x^2\sqrt{ax-1}\sqrt{ax+}}{2a\cosh^{-1}(ax)}$$

```
[Out] -(x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^3) + x/(3*a^2*ArcCosh
[a*x]^2) - x^3/(2*ArcCosh[a*x]^2) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a^3*A
rcCosh[a*x]) - (3*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*ArcCosh[a*x]) + Co
shIntegral[ArcCosh[a*x]]/(24*a^3) + (9*CoshIntegral[3*ArcCosh[a*x]])/(8*a^3
)
```

Rubi [A] time = 0.675705, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5668, 5775, 5666, 3301, 5656, 5781}

$$\frac{\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{24a^3}+\frac{9\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{8a^3}+\frac{x}{3a^2\cosh^{-1}(ax)^2}+\frac{\sqrt{ax-1}\sqrt{ax+1}}{3a^3\cosh^{-1}(ax)}-\frac{x^3}{2\cosh^{-1}(ax)^2}-\frac{3x^2\sqrt{ax-1}\sqrt{ax+}}{2a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCosh[a*x]^4,x]
```

```
[Out] -(x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^3) + x/(3*a^2*ArcCosh
[a*x]^2) - x^3/(2*ArcCosh[a*x]^2) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a^3*A
rcCosh[a*x]) - (3*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*ArcCosh[a*x]) + Co
shIntegral[ArcCosh[a*x]]/(24*a^3) + (9*CoshIntegral[3*ArcCosh[a*x]])/(8*a^3
)
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{x^2}{\cosh^{-1}(ax)^4}\,dx = -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} - \frac{2\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}\,dx}{3a} + a\int \frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}\,dx$$

$$= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{x}{3a^2\cosh^{-1}(ax)^2} - \frac{x^3}{2\cosh^{-1}(ax)^2} + \frac{3}{2}\int \frac{x^2}{\cosh^{-1}(ax)^2}\,dx - \frac{\int \frac{1}{\cosh^{-1}(ax)^2}\ldots}{3a^2}$$

$$= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{x}{3a^2\cosh^{-1}(ax)^2} - \frac{x^3}{2\cosh^{-1}(ax)^2} + \frac{\sqrt{-1+ax}\sqrt{1+ax}}{3a^3\cosh^{-1}(ax)} - \frac{3x^2\sqrt{-1+ax}\ldots}{2a\cosh^{-1}(\ldots}$$

$$= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{x}{3a^2\cosh^{-1}(ax)^2} - \frac{x^3}{2\cosh^{-1}(ax)^2} + \frac{\sqrt{-1+ax}\sqrt{1+ax}}{3a^3\cosh^{-1}(ax)} - \frac{3x^2\sqrt{-1+ax}\ldots}{2a\cosh^{-1}(\ldots}$$

$$= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{x}{3a^2\cosh^{-1}(ax)^2} - \frac{x^3}{2\cosh^{-1}(ax)^2} + \frac{\sqrt{-1+ax}\sqrt{1+ax}}{3a^3\cosh^{-1}(ax)} - \frac{3x^2\sqrt{-1+ax}\ldots}{2a\cosh^{-1}(\ldots}$$

Mathematica [A] time = 0.364899, size = 183, normalized size = 1.2

$$\frac{\sqrt{ax-1}\left(-4\sqrt{\frac{ax-1}{ax+1}}\left(2a^2x^2\left(a^2x^2-1\right)+ax\sqrt{ax-1}\sqrt{ax+1}\left(3a^2x^2-2\right)\cosh^{-1}(ax)+\left(9a^4x^4-11a^2x^2+2\right)\cosh^{-1}(ax)^2\right)\ldots}{24a^3\left(\frac{ax-1}{ax+1}\right)^{3/2}(ax+1)^{3/2}\cosh^{-1}(a\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/ArcCosh[a*x]^4,x]
```

```
[Out] (Sqrt[-1 + a*x]*(-4*Sqrt[(-1 + a*x)/(1 + a*x)]*(2*a^2*x^2*(-1 + a^2*x^2) +
a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-2 + 3*a^2*x^2)*ArcCosh[a*x] + (2 - 11*a^
2*x^2 + 9*a^4*x^4)*ArcCosh[a*x]^2) + (-1 + a*x)*ArcCosh[a*x]^3*CoshIntegral
[ArcCosh[a*x]] + 27*(-1 + a*x)*ArcCosh[a*x]^3*CoshIntegral[3*ArcCosh[a*x]])
)/(24*a^3*((-1 + a*x)/(1 + a*x))^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x]^3)
```

Maple [A] time = 0.035, size = 121, normalized size = 0.8

$$\frac{1}{a^3}\left(-\frac{1}{12\,(\operatorname{arccosh}(ax))^3}\sqrt{ax-1}\sqrt{ax+1} - \frac{ax}{24\,(\operatorname{arccosh}(ax))^2} - \frac{1}{24\operatorname{arccosh}(ax)}\sqrt{ax-1}\sqrt{ax+1} + \frac{\operatorname{Chi}(\operatorname{arccosh}(ax\ldots}{24}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccosh(a*x)^4,x)
```

```
[Out] 1/a^3*(-1/12/arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)-1/24*a*x/arccosh(a*
x)^2-1/24/arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+1/24*Chi(arccosh(a*x))-1
/12/arccosh(a*x)^3*sinh(3*arccosh(a*x))-1/8/arccosh(a*x)^2*cosh(3*arccosh(a
*x))-3/8/arccosh(a*x)*sinh(3*arccosh(a*x))+9/8*Chi(3*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^13 - 10*a^11*x^11 + 20*a^9*x^9 - 20*a^7*x^7 + 10*a^5*x^5 + 2
*(a^8*x^8 - a^6*x^6)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) - 2*a^3*x^3 + 2*(5*a^9
*x^9 - 9*a^7*x^7 + 4*a^5*x^5)*(a*x + 1)^2*(a*x - 1)^2 + 4*(5*a^10*x^10 - 13
*a^8*x^8 + 11*a^6*x^6 - 3*a^4*x^4)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 4*(5*a
^11*x^11 - 17*a^9*x^9 + 21*a^7*x^7 - 11*a^5*x^5 + 2*a^3*x^3)*(a*x + 1)*(a*x
 - 1) + (9*a^13*x^13 - 45*a^11*x^11 + 90*a^9*x^9 - 90*a^7*x^7 + 45*a^5*x^5
+ (9*a^8*x^8 - 13*a^6*x^6 + 3*a^4*x^4 + a^2*x^2)*(a*x + 1)^(5/2)*(a*x - 1)^
(5/2) - 9*a^3*x^3 + (45*a^9*x^9 - 97*a^7*x^7 + 64*a^5*x^5 - 10*a^3*x^3 - 2*
a*x)*(a*x + 1)^2*(a*x - 1)^2 + (90*a^10*x^10 - 258*a^8*x^8 + 264*a^6*x^6 -
113*a^4*x^4 + 19*a^2*x^2 - 2)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 2*(45*a^11*
x^11 - 161*a^9*x^9 + 219*a^7*x^7 - 141*a^5*x^5 + 44*a^3*x^3 - 6*a*x)*(a*x +
 1)*(a*x - 1) + (45*a^12*x^12 - 193*a^10*x^10 + 325*a^8*x^8 - 270*a^6*x^6 +
 112*a^4*x^4 - 19*a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x
+ 1)*sqrt(a*x - 1))^2 + 2*(5*a^12*x^12 - 21*a^10*x^10 + 34*a^8*x^8 - 26*a^6
*x^6 + 9*a^4*x^4 - a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1) + (3*a^13*x^13 - 15
*a^11*x^11 + 30*a^9*x^9 - 30*a^7*x^7 + 15*a^5*x^5 + (3*a^8*x^8 - 4*a^6*x^6
+ a^4*x^4)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) - 3*a^3*x^3 + (15*a^9*x^9 - 31*a
^7*x^7 + 20*a^5*x^5 - 4*a^3*x^3)*(a*x + 1)^2*(a*x - 1)^2 + (30*a^10*x^10 -
84*a^8*x^8 + 84*a^6*x^6 - 35*a^4*x^4 + 5*a^2*x^2)*(a*x + 1)^(3/2)*(a*x - 1)
^(3/2) + 2*(15*a^11*x^11 - 53*a^9*x^9 + 71*a^7*x^7 - 44*a^5*x^5 + 12*a^3*x^
3 - a*x)*(a*x + 1)*(a*x - 1) + (15*a^12*x^12 - 64*a^10*x^10 + 107*a^8*x^8 -
 87*a^6*x^6 + 34*a^4*x^4 - 5*a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x
+ sqrt(a*x + 1)*sqrt(a*x - 1)))/((a^13*x^10 - 5*a^11*x^8 + (a*x + 1)^(5/2)*
(a*x - 1)^(5/2)*a^8*x^5 + 10*a^9*x^6 - 10*a^7*x^4 + 5*a^5*x^2 + 5*(a^9*x^6
- a^7*x^4)*(a*x + 1)^2*(a*x - 1)^2 + 10*(a^10*x^7 - 2*a^8*x^5 + a^6*x^3)*(a
*x + 1)^(3/2)*(a*x - 1)^(3/2) + 10*(a^11*x^8 - 3*a^9*x^6 + 3*a^7*x^4 - a^5*
x^2)*(a*x + 1)*(a*x - 1) - a^3 + 5*(a^12*x^9 - 4*a^10*x^7 + 6*a^8*x^5 - 4*a
```

```
^6*x^3 + a^4*x)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a
*x - 1))^3) + integrate(1/6*(27*a^14*x^14 - 162*a^12*x^12 + 405*a^10*x^10 -
 540*a^8*x^8 + 405*a^6*x^6 - 162*a^4*x^4 + (27*a^8*x^8 - 13*a^6*x^6 - 3*a^4
*x^4 - 3*a^2*x^2)*(a*x + 1)^3*(a*x - 1)^3 + (162*a^9*x^9 - 227*a^7*x^7 + 63
*a^5*x^5 + 3*a^3*x^3 + 6*a*x)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + (405*a^10*x
^10 - 940*a^8*x^8 + 687*a^6*x^6 - 143*a^4*x^4 - 21*a^2*x^2 + 12)*(a*x + 1)^
2*(a*x - 1)^2 + (540*a^11*x^11 - 1750*a^9*x^9 + 2058*a^7*x^7 - 1017*a^5*x^5
 + 145*a^3*x^3 + 24*a*x)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 27*a^2*x^2 + (40
5*a^12*x^12 - 1685*a^10*x^10 + 2727*a^8*x^8 - 2118*a^6*x^6 + 782*a^4*x^4 -
123*a^2*x^2 + 12)*(a*x + 1)*(a*x - 1) + (162*a^13*x^13 - 823*a^11*x^11 + 16
95*a^9*x^9 - 1790*a^7*x^7 + 1015*a^5*x^5 - 297*a^3*x^3 + 38*a*x)*sqrt(a*x +
 1)*sqrt(a*x - 1))/((a^14*x^12 - 6*a^12*x^10 + (a*x + 1)^3*(a*x - 1)^3*a^8*
x^6 + 15*a^10*x^8 - 20*a^8*x^6 + 15*a^6*x^4 + 6*(a^9*x^7 - a^7*x^5)*(a*x +
1)^(5/2)*(a*x - 1)^(5/2) - 6*a^4*x^2 + 15*(a^10*x^8 - 2*a^8*x^6 + a^6*x^4)*
(a*x + 1)^2*(a*x - 1)^2 + 20*(a^11*x^9 - 3*a^9*x^7 + 3*a^7*x^5 - a^5*x^3)*(
a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 15*(a^12*x^10 - 4*a^10*x^8 + 6*a^8*x^6 - 4
*a^6*x^4 + a^4*x^2)*(a*x + 1)*(a*x - 1) + 6*(a^13*x^11 - 5*a^11*x^9 + 10*a^
9*x^7 - 10*a^7*x^5 + 5*a^5*x^3 - a^3*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + a^2)*
log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\operatorname{arcosh}(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^2/arccosh(a*x)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{acosh}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acosh(a*x)**4,x)
```

```
[Out] Integral(x**2/acosh(a*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arcosh}(ax)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^2/arccosh(a*x)^4, x)
```

3.68 $\int \frac{x}{\cosh^{-1}(ax)^4}\,dx$

Optimal. Leaf size=105

$$\frac{2\text{Chi}\left(2\cosh^{-1}(ax)\right)}{3a^2}+\frac{1}{6a^2\cosh^{-1}(ax)^2}-\frac{x^2}{3\cosh^{-1}(ax)^2}-\frac{2x\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)}-\frac{x\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)^3}$$

```
[Out] -(x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^3) + 1/(6*a^2*ArcCosh[a
*x]^2) - x^2/(3*ArcCosh[a*x]^2) - (2*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*A
rcCosh[a*x]) + (2*CoshIntegral[2*ArcCosh[a*x]])/(3*a^2)
```

Rubi [A] time = 0.396025, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5668, 5775, 5666, 3301, 5676}

$$\frac{2\text{Chi}\left(2\cosh^{-1}(ax)\right)}{3a^2}+\frac{1}{6a^2\cosh^{-1}(ax)^2}-\frac{x^2}{3\cosh^{-1}(ax)^2}-\frac{2x\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)}-\frac{x\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCosh[a*x]^4,x]
```

```
[Out] -(x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^3) + 1/(6*a^2*ArcCosh[a
*x]^2) - x^2/(3*ArcCosh[a*x]^2) - (2*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*A
rcCosh[a*x]) + (2*CoshIntegral[2*ArcCosh[a*x]])/(3*a^2)
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
```

```
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{x}{\cosh^{-1}(ax)^4}\,dx = -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} - \frac{\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}\,dx}{3a} + \frac{1}{3}(2a)\int \frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}\,dx$$

$$= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{1}{6a^2\cosh^{-1}(ax)^2} - \frac{x^2}{3\cosh^{-1}(ax)^2} + \frac{2}{3}\int \frac{x}{\cosh^{-1}(ax)^2}\,dx$$

$$= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{1}{6a^2\cosh^{-1}(ax)^2} - \frac{x^2}{3\cosh^{-1}(ax)^2} - \frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)} + \frac{2\,\text{Subst}\left(\int \frac{\cosh}{}\right.}{}$$

$$= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{1}{6a^2\cosh^{-1}(ax)^2} - \frac{x^2}{3\cosh^{-1}(ax)^2} - \frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)} + \frac{2\text{Chi}\left(2\cosh^{-1}\right.}{3a^2}$$

Mathematica [A] time = 0.29365, size = 131, normalized size = 1.25

$$\frac{\frac{-2a^3x^3+\left(4ax-4a^3x^3\right)\cosh^{-1}(ax)^2-\sqrt{ax-1}\sqrt{ax+1}\left(2a^2x^2-1\right)\cosh^{-1}(ax)+2ax}{\cosh^{-1}(ax)^3}+4\sqrt{\frac{ax-1}{ax+1}}(ax+1)\text{Chi}\left(2\cosh^{-1}(ax)\right)}{6a^2\sqrt{ax-1}\sqrt{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/ArcCosh[a*x]^4,x]
```

```
[Out] ((2*a*x - 2*a^3*x^3 - Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-1 + 2*a^2*x^2)*ArcCosh
[a*x] + (4*a*x - 4*a^3*x^3)*ArcCosh[a*x]^2)/ArcCosh[a*x]^3 + 4*Sqrt[(-1 + a
*x)/(1 + a*x)]*(1 + a*x)*CoshIntegral[2*ArcCosh[a*x]])/(6*a^2*Sqrt[-1 + a*x
]*Sqrt[1 + a*x])
```

Maple [A] time = 0.026, size = 60, normalized size = 0.6

$$\frac{1}{a^2}\left(-\frac{\sinh(2\operatorname{arccosh}(ax))}{6\,(\operatorname{arccosh}(ax))^3}-\frac{\cosh(2\operatorname{arccosh}(ax))}{6\,(\operatorname{arccosh}(ax))^2}-\frac{\sinh(2\operatorname{arccosh}(ax))}{3\operatorname{arccosh}(ax)}+\frac{2\operatorname{Chi}(2\operatorname{arccosh}(ax))}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccosh(a*x)^4,x)
```

```
[Out] 1/a^2*(-1/6/arccosh(a*x)^3*sinh(2*arccosh(a*x))-1/6/arccosh(a*x)^2*cosh(2*a
rccosh(a*x))-1/3/arccosh(a*x)*sinh(2*arccosh(a*x))+2/3*Chi(2*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^12*x^12 - 10*a^10*x^10 + 20*a^8*x^8 - 20*a^6*x^6 + 10*a^4*x^4 + 2
*(a^7*x^7 - a^5*x^5)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + 2*(5*a^8*x^8 - 9*a^6
*x^6 + 4*a^4*x^4)*(a*x + 1)^2*(a*x - 1)^2 + 4*(5*a^9*x^9 - 13*a^7*x^7 + 11*
a^5*x^5 - 3*a^3*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) - 2*a^2*x^2 + 4*(5*a^1
```

```
0*x^10 - 17*a^8*x^8 + 21*a^6*x^6 - 11*a^4*x^4 + 2*a^2*x^2)*(a*x + 1)*(a*x -
 1) + (4*a^12*x^12 - 20*a^10*x^10 + 40*a^8*x^8 - 40*a^6*x^6 + 20*a^4*x^4 +
4*(a^7*x^7 - a^5*x^5)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + (20*a^8*x^8 - 36*a^
6*x^6 + 16*a^4*x^4 + 3*a^2*x^2 - 3)*(a*x + 1)^2*(a*x - 1)^2 + (40*a^9*x^9 -
 104*a^7*x^7 + 88*a^5*x^5 - 21*a^3*x^3 - 3*a*x)*(a*x + 1)^(3/2)*(a*x - 1)^(
3/2) - 4*a^2*x^2 + (40*a^10*x^10 - 136*a^8*x^8 + 168*a^6*x^6 - 91*a^4*x^4 +
 22*a^2*x^2 - 3)*(a*x + 1)*(a*x - 1) + (20*a^11*x^11 - 84*a^9*x^9 + 136*a^7
*x^7 - 107*a^5*x^5 + 42*a^3*x^3 - 7*a*x)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a
*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2 + 2*(5*a^11*x^11 - 21*a^9*x^9 + 34*a^7*
x^7 - 26*a^5*x^5 + 9*a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + (2*a^12*x
^12 - 10*a^10*x^10 + 20*a^8*x^8 - 20*a^6*x^6 + 10*a^4*x^4 + 2*(a^7*x^7 - a^
5*x^5)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + (10*a^8*x^8 - 18*a^6*x^6 + 9*a^4*x
^4 - a^2*x^2)*(a*x + 1)^2*(a*x - 1)^2 + (20*a^9*x^9 - 52*a^7*x^7 + 47*a^5*x
^5 - 17*a^3*x^3 + 2*a*x)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) - 2*a^2*x^2 + (20*
a^10*x^10 - 68*a^8*x^8 + 87*a^6*x^6 - 51*a^4*x^4 + 13*a^2*x^2 - 1)*(a*x + 1
)*(a*x - 1) + (10*a^11*x^11 - 42*a^9*x^9 + 69*a^7*x^7 - 55*a^5*x^5 + 21*a^3
*x^3 - 3*a*x)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x
 - 1)))/((a^12*x^10 - 5*a^10*x^8 + (a*x + 1)^(5/2)*(a*x - 1)^(5/2)*a^7*x^5
+ 10*a^8*x^6 - 10*a^6*x^4 + 5*a^4*x^2 + 5*(a^8*x^6 - a^6*x^4)*(a*x + 1)^2*(
a*x - 1)^2 + 10*(a^9*x^7 - 2*a^7*x^5 + a^5*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(
3/2) + 10*(a^10*x^8 - 3*a^8*x^6 + 3*a^6*x^4 - a^4*x^2)*(a*x + 1)*(a*x - 1)
+ 5*(a^11*x^9 - 4*a^9*x^7 + 6*a^7*x^5 - 4*a^5*x^3 + a^3*x)*sqrt(a*x + 1)*sq
rt(a*x - 1) - a^2)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3) + integrate(1/
6*(8*a^13*x^13 - 48*a^11*x^11 + 8*(a*x + 1)^3*(a*x - 1)^3*a^7*x^7 + 120*a^9
*x^9 - 160*a^7*x^7 + 120*a^5*x^5 + (48*a^8*x^8 - 48*a^6*x^6 + 4*a^4*x^4 - 1
2*a^2*x^2 + 15)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) - 48*a^3*x^3 + 8*(15*a^9*x^
9 - 30*a^7*x^7 + 17*a^5*x^5 - 5*a^3*x^3 + 3*a*x)*(a*x + 1)^2*(a*x - 1)^2 +
2*(80*a^10*x^10 - 240*a^8*x^8 + 252*a^6*x^6 - 104*a^4*x^4 + 3*a^2*x^2 + 9)*
(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 8*(15*a^11*x^11 - 60*a^9*x^9 + 92*a^7*x^7
 - 63*a^5*x^5 + 15*a^3*x^3 + a*x)*(a*x + 1)*(a*x - 1) + (48*a^12*x^12 - 240
*a^10*x^10 + 484*a^8*x^8 - 484*a^6*x^6 + 243*a^4*x^4 - 58*a^2*x^2 + 7)*sqrt
(a*x + 1)*sqrt(a*x - 1) + 8*a*x)/((a^13*x^12 - 6*a^11*x^10 + (a*x + 1)^3*(a
*x - 1)^3*a^7*x^6 + 15*a^9*x^8 - 20*a^7*x^6 + 15*a^5*x^4 + 6*(a^8*x^7 - a^6
*x^5)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + 15*(a^9*x^8 - 2*a^7*x^6 + a^5*x^4)*
(a*x + 1)^2*(a*x - 1)^2 - 6*a^3*x^2 + 20*(a^10*x^9 - 3*a^8*x^7 + 3*a^6*x^5
- a^4*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 15*(a^11*x^10 - 4*a^9*x^8 + 6*
a^7*x^6 - 4*a^5*x^4 + a^3*x^2)*(a*x + 1)*(a*x - 1) + 6*(a^12*x^11 - 5*a^10*
x^9 + 10*a^8*x^7 - 10*a^6*x^5 + 5*a^4*x^3 - a^2*x)*sqrt(a*x + 1)*sqrt(a*x -
 1) + a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\operatorname{arcosh}\left(ax\right)^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x/arccosh(a*x)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{acosh}^{4}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acosh(a*x)**4,x)
```

```
[Out] Integral(x/acosh(a*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arcosh}\left(ax\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x/arccosh(a*x)^4, x)
```

3.69 $\int \frac{1}{\cosh^{-1}(ax)^4} dx$

Optimal. Leaf size=86

$$\frac{\text{Chi}\left(\cosh^{-1}(ax)\right)}{6a} - \frac{x}{6\cosh^{-1}(ax)^2} - \frac{\sqrt{ax-1}\sqrt{ax+1}}{6a\cosh^{-1}(ax)} - \frac{\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)^3}$$

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^3) - x/(6*ArcCosh[a*x]^2)
 - (Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(6*a*ArcCosh[a*x]) + CoshIntegral[ArcCosh
[a*x]]/(6*a)
```

Rubi [A] time = 0.377686, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5656, 5775, 5781, 3301}

$$\frac{\text{Chi}\left(\cosh^{-1}(ax)\right)}{6a} - \frac{x}{6\cosh^{-1}(ax)^2} - \frac{\sqrt{ax-1}\sqrt{ax+1}}{6a\cosh^{-1}(ax)} - \frac{\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^(-4),x]
```

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^3) - x/(6*ArcCosh[a*x]^2)
 - (Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(6*a*ArcCosh[a*x]) + CoshIntegral[ArcCosh
[a*x]]/(6*a)
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
```

```
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\cosh^{-1}(ax)^4}\,dx &= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3}+\frac{1}{3}a\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}\,dx \\
&= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3}-\frac{x}{6\cosh^{-1}(ax)^2}+\frac{1}{6}\int \frac{1}{\cosh^{-1}(ax)^2}\,dx \\
&= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3}-\frac{x}{6\cosh^{-1}(ax)^2}-\frac{\sqrt{-1+ax}\sqrt{1+ax}}{6a\cosh^{-1}(ax)}+\frac{1}{6}a\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)} \\
&= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3}-\frac{x}{6\cosh^{-1}(ax)^2}-\frac{\sqrt{-1+ax}\sqrt{1+ax}}{6a\cosh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx,x,\cosh^{-1}(ax)\right)}{6a} \\
&= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3}-\frac{x}{6\cosh^{-1}(ax)^2}-\frac{\sqrt{-1+ax}\sqrt{1+ax}}{6a\cosh^{-1}(ax)}+\frac{\text{Chi}\left(\cosh^{-1}(ax)\right)}{6a}
\end{aligned}
$$

Mathematica [A] time = 0.179326, size = 116, normalized size = 1.35

$$
\frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{ax+1}\cosh^{-1}(ax)^3\text{Chi}\left(\cosh^{-1}(ax)\right)-2(ax-1)\sqrt{ax+1}-(ax-1)\sqrt{ax+1}\cosh^{-1}(ax)^2-ax\sqrt{ax-1}\cosh^{-1}(a}{6a\sqrt{ax-1}\cosh^{-1}(ax)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^(-4),x]
```

```
[Out] (-2*(-1 + a*x)*Sqrt[1 + a*x] - a*x*Sqrt[-1 + a*x]*ArcCosh[a*x] - (-1 + a*x)
*Sqrt[1 + a*x]*ArcCosh[a*x]^2 + Sqrt[(-1 + a*x)/(1 + a*x)]*Sqrt[1 + a*x]*Ar
cCosh[a*x]^3*CoshIntegral[ArcCosh[a*x]])/(6*a*Sqrt[-1 + a*x]*ArcCosh[a*x]^3
)
```

Maple [A] time = 0.027, size = 67, normalized size = 0.8

$$\frac{1}{a}\left(-\frac{1}{3\left(\operatorname{arccosh}(ax)\right)^{3}}\sqrt{ax-1}\sqrt{ax+1}-\frac{ax}{6\left(\operatorname{arccosh}(ax)\right)^{2}}-\frac{1}{6\operatorname{arccosh}(ax)}\sqrt{ax-1}\sqrt{ax+1}+\frac{\operatorname{Chi}\left(\operatorname{arccosh}(ax)\right)}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x)^4,x)
```

```
[Out] 1/a*(-1/3/arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)-1/6*a*x/arccosh(a*x)^2
-1/6/arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+1/6*Chi(arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^11*x^11 - 10*a^9*x^9 + 20*a^7*x^7 - 20*a^5*x^5 + 2*(a^6*x^6 - a^4
*x^4)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + 10*a^3*x^3 + 2*(5*a^7*x^7 - 9*a^5*x
^5 + 4*a^3*x^3)*(a*x + 1)^2*(a*x - 1)^2 + 4*(5*a^8*x^8 - 13*a^6*x^6 + 11*a^
4*x^4 - 3*a^2*x^2)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 4*(5*a^9*x^9 - 17*a^7*
x^7 + 21*a^5*x^5 - 11*a^3*x^3 + 2*a*x)*(a*x + 1)*(a*x - 1) + (a^11*x^11 - 5
*a^9*x^9 + 10*a^7*x^7 - 10*a^5*x^5 + (a^6*x^6 - a^4*x^4 + 3*a^2*x^2 - 3)*(a
*x + 1)^(5/2)*(a*x - 1)^(5/2) + 5*a^3*x^3 + (5*a^7*x^7 - 9*a^5*x^5 + 10*a^3
*x^3 - 6*a*x)*(a*x + 1)^2*(a*x - 1)^2 + (10*a^8*x^8 - 26*a^6*x^6 + 22*a^4*x
^4 - 3*a^2*x^2 - 3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 2*(5*a^9*x^9 - 17*a^7
*x^7 + 18*a^5*x^5 - 5*a^3*x^3 - a*x)*(a*x + 1)*(a*x - 1) + (5*a^10*x^10 - 2
1*a^8*x^8 + 31*a^6*x^6 - 20*a^4*x^4 + 6*a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x
 - 1) - a*x)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2 + 2*(5*a^10*x^10 - 21
*a^8*x^8 + 34*a^6*x^6 - 26*a^4*x^4 + 9*a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x
- 1) - 2*a*x + (a^11*x^11 - 5*a^9*x^9 + 10*a^7*x^7 - 10*a^5*x^5 + (a^6*x^6
- a^2*x^2)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + 5*a^3*x^3 + (5*a^7*x^7 - 5*a^5
```

```
*x^5 - 2*a^3*x^3 + 2*a*x)*(a*x + 1)^2*(a*x - 1)^2 + (10*a^8*x^8 - 20*a^6*x^
6 + 10*a^4*x^4 + a^2*x^2 - 1)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 2*(5*a^9*x^
9 - 15*a^7*x^7 + 16*a^5*x^5 - 7*a^3*x^3 + a*x)*(a*x + 1)*(a*x - 1) + (5*a^1
0*x^10 - 20*a^8*x^8 + 31*a^6*x^6 - 23*a^4*x^4 + 8*a^2*x^2 - 1)*sqrt(a*x + 1
)*sqrt(a*x - 1) - a*x)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)))/((a^11*x^10
- 5*a^9*x^8 + (a*x + 1)^(5/2)*(a*x - 1)^(5/2)*a^6*x^5 + 10*a^7*x^6 - 10*a^5
*x^4 + 5*(a^7*x^6 - a^5*x^4)*(a*x + 1)^2*(a*x - 1)^2 + 5*a^3*x^2 + 10*(a^8*
x^7 - 2*a^6*x^5 + a^4*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 10*(a^9*x^8 -
3*a^7*x^6 + 3*a^5*x^4 - a^3*x^2)*(a*x + 1)*(a*x - 1) + 5*(a^10*x^9 - 4*a^8*
x^7 + 6*a^6*x^5 - 4*a^4*x^3 + a^2*x)*sqrt(a*x + 1)*sqrt(a*x - 1) - a)*log(a
*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3) + integrate(1/6*(a^12*x^12 - 6*a^10*x^
10 + 15*a^8*x^8 - 20*a^6*x^6 + 15*a^4*x^4 + (a^6*x^6 + a^4*x^4 - 9*a^2*x^2
+ 15)*(a*x + 1)^3*(a*x - 1)^3 + (6*a^7*x^7 - a^5*x^5 - 31*a^3*x^3 + 33*a*x)
*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + (15*a^8*x^8 - 20*a^6*x^6 - 19*a^4*x^4 +
3*a^2*x^2 + 21)*(a*x + 1)^2*(a*x - 1)^2 + (20*a^9*x^9 - 50*a^7*x^7 + 54*a^5
*x^5 - 59*a^3*x^3 + 35*a*x)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) - 6*a^2*x^2 + (
15*a^10*x^10 - 55*a^8*x^8 + 101*a^6*x^6 - 90*a^4*x^4 + 22*a^2*x^2 + 7)*(a*x
 + 1)*(a*x - 1) + (6*a^11*x^11 - 29*a^9*x^9 + 65*a^7*x^7 - 66*a^5*x^5 + 23*
a^3*x^3 + a*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + 1)/((a^12*x^12 - 6*a^10*x^10 +
 (a*x + 1)^3*(a*x - 1)^3*a^6*x^6 + 15*a^8*x^8 - 20*a^6*x^6 + 15*a^4*x^4 + 6
*(a^7*x^7 - a^5*x^5)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + 15*(a^8*x^8 - 2*a^6*
x^6 + a^4*x^4)*(a*x + 1)^2*(a*x - 1)^2 + 20*(a^9*x^9 - 3*a^7*x^7 + 3*a^5*x^
5 - a^3*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) - 6*a^2*x^2 + 15*(a^10*x^10 -
4*a^8*x^8 + 6*a^6*x^6 - 4*a^4*x^4 + a^2*x^2)*(a*x + 1)*(a*x - 1) + 6*(a^11*
x^11 - 5*a^9*x^9 + 10*a^7*x^7 - 10*a^5*x^5 + 5*a^3*x^3 - a*x)*sqrt(a*x + 1)
*sqrt(a*x - 1) + 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\operatorname{arcosh}(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^(-4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acosh}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)**4,x)
```

```
[Out] Integral(acosh(a*x)**(-4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arcosh}(ax)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^(-4), x)
```

3.70 $\int \frac{1}{x\cosh^{-1}(ax)^4}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x\cosh^{-1}(ax)^4},x\right)$$

```
[Out] Unintegrable[1/(x*ArcCosh[a*x]^4), x]
```

Rubi [A] time = 0.0136799, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\cosh^{-1}(ax)^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcCosh[a*x]^4),x]
```

```
[Out] Defer[Int][1/(x*ArcCosh[a*x]^4), x]
```

Rubi steps

$$\int \frac{1}{x\cosh^{-1}(ax)^4}\,dx = \int \frac{1}{x\cosh^{-1}(ax)^4}\,dx$$

Mathematica [A] time = 9.82626, size = 0, normalized size = 0.

$$\int \frac{1}{x\cosh^{-1}(ax)^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcCosh[a*x]^4),x]
```

```
[Out] Integrate[1/(x*ArcCosh[a*x]^4), x]
```

Maple [A] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\operatorname{arccosh}\left(ax\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccosh(a*x)^4,x)
```

```
[Out] int(1/x/arccosh(a*x)^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^13 - 10*a^11*x^11 + 20*a^9*x^9 - 20*a^7*x^7 + 10*a^5*x^5 + 2
*(a^8*x^8 - a^6*x^6)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) - 2*a^3*x^3 + 2*(5*a^9
*x^9 - 9*a^7*x^7 + 4*a^5*x^5)*(a*x + 1)^2*(a*x - 1)^2 + 4*(5*a^10*x^10 - 13
*a^8*x^8 + 11*a^6*x^6 - 3*a^4*x^4)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 4*(5*a
^11*x^11 - 17*a^9*x^9 + 21*a^7*x^7 - 11*a^5*x^5 + 2*a^3*x^3)*(a*x + 1)*(a*x
 - 1) - (4*(a^6*x^6 - 3*a^4*x^4 + 2*a^2*x^2)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2
) + (16*a^7*x^7 - 46*a^5*x^5 + 37*a^3*x^3 - 7*a*x)*(a*x + 1)^2*(a*x - 1)^2
+ (24*a^8*x^8 - 66*a^6*x^6 + 59*a^4*x^4 - 19*a^2*x^2 + 2)*(a*x + 1)^(3/2)*(
a*x - 1)^(3/2) + (16*a^9*x^9 - 42*a^7*x^7 + 39*a^5*x^5 - 16*a^3*x^3 + 3*a*x
)*(a*x + 1)*(a*x - 1) + (4*a^10*x^10 - 10*a^8*x^8 + 9*a^6*x^6 - 4*a^4*x^4 +
 a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1
))^2 + 2*(5*a^12*x^12 - 21*a^10*x^10 + 34*a^8*x^8 - 26*a^6*x^6 + 9*a^4*x^4
- a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1) + (2*(a^6*x^6 - a^4*x^4)*(a*x + 1)^(
5/2)*(a*x - 1)^(5/2) + (8*a^7*x^7 - 13*a^5*x^5 + 5*a^3*x^3)*(a*x + 1)^2*(a*
x - 1)^2 + (12*a^8*x^8 - 27*a^6*x^6 + 19*a^4*x^4 - 4*a^2*x^2)*(a*x + 1)^(3/
2)*(a*x - 1)^(3/2) + (8*a^9*x^9 - 23*a^7*x^7 + 23*a^5*x^5 - 9*a^3*x^3 + a*x
)*(a*x + 1)*(a*x - 1) + (2*a^10*x^10 - 7*a^8*x^8 + 9*a^6*x^6 - 5*a^4*x^4 +
a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)
))/((a^13*x^13 - 5*a^11*x^11 + (a*x + 1)^(5/2)*(a*x - 1)^(5/2)*a^8*x^8 + 10
*a^9*x^9 - 10*a^7*x^7 + 5*a^5*x^5 - a^3*x^3 + 5*(a^9*x^9 - a^7*x^7)*(a*x +
1)^2*(a*x - 1)^2 + 10*(a^10*x^10 - 2*a^8*x^8 + a^6*x^6)*(a*x + 1)^(3/2)*(a*
```

```
x - 1)^(3/2) + 10*(a^11*x^11 - 3*a^9*x^9 + 3*a^7*x^7 - a^5*x^5)*(a*x + 1)*(
a*x - 1) + 5*(a^12*x^12 - 4*a^10*x^10 + 6*a^8*x^8 - 4*a^6*x^6 + a^4*x^4)*sq
rt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3) + inte
grate(1/6*(8*(a^7*x^7 - 6*a^5*x^5 + 6*a^3*x^3)*(a*x + 1)^3*(a*x - 1)^3 + (4
0*a^8*x^8 - 204*a^6*x^6 + 228*a^4*x^4 - 57*a^2*x^2)*(a*x + 1)^(5/2)*(a*x -
1)^(5/2) + 2*(40*a^9*x^9 - 168*a^7*x^7 + 200*a^5*x^5 - 87*a^3*x^3 + 15*a*x)
*(a*x + 1)^2*(a*x - 1)^2 + 2*(40*a^10*x^10 - 132*a^8*x^8 + 156*a^6*x^6 - 91
*a^4*x^4 + 30*a^2*x^2 - 3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 2*(20*a^11*x^1
1 - 48*a^9*x^9 + 48*a^7*x^7 - 35*a^5*x^5 + 18*a^3*x^3 - 3*a*x)*(a*x + 1)*(a
*x - 1) + (8*a^12*x^12 - 12*a^10*x^10 + 4*a^8*x^8 - 5*a^6*x^6 + 6*a^4*x^4 -
 a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^15*x^16 - 6*a^13*x^14 + (a*x + 1
)^3*(a*x - 1)^3*a^9*x^10 + 15*a^11*x^12 - 20*a^9*x^10 + 15*a^7*x^8 - 6*a^5*
x^6 + a^3*x^4 + 6*(a^10*x^11 - a^8*x^9)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + 1
5*(a^11*x^12 - 2*a^9*x^10 + a^7*x^8)*(a*x + 1)^2*(a*x - 1)^2 + 20*(a^12*x^1
3 - 3*a^10*x^11 + 3*a^8*x^9 - a^6*x^7)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 15
*(a^13*x^14 - 4*a^11*x^12 + 6*a^9*x^10 - 4*a^7*x^8 + a^5*x^6)*(a*x + 1)*(a*
x - 1) + 6*(a^14*x^15 - 5*a^12*x^13 + 10*a^10*x^11 - 10*a^8*x^9 + 5*a^6*x^7
 - a^4*x^5)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x -
 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\,\text{arcosh}\,(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arccosh(a*x)^4), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\text{acosh}^4\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acosh(a*x)**4,x)
```

```
[Out] Integral(1/(x*acosh(a*x)**4), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arcosh}(ax)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccosh(a*x)^4), x)
```

3.71 $\int \frac{1}{x^2 \cosh^{-1}(ax)^4} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x^2 \cosh^{-1}(ax)^4}, x\right)$$

```
[Out] Unintegrable[1/(x^2*ArcCosh[a*x]^4), x]
```

Rubi [A] time = 0.014538, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^4} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*ArcCosh[a*x]^4),x]
```

```
[Out] Defer[Int][1/(x^2*ArcCosh[a*x]^4), x]
```

Rubi steps

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^4} \, dx = \int \frac{1}{x^2 \cosh^{-1}(ax)^4} \, dx$$

Mathematica [A] time = 11.9407, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^4} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*ArcCosh[a*x]^4),x]
```

```
[Out] Integrate[1/(x^2*ArcCosh[a*x]^4), x]
```

Maple [A] time = 0.093, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(\operatorname{arccosh}(ax)\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccosh(a*x)^4,x)
```

```
[Out] int(1/x^2/arccosh(a*x)^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^13 - 10*a^11*x^11 + 20*a^9*x^9 - 20*a^7*x^7 + 10*a^5*x^5 + 2
*(a^8*x^8 - a^6*x^6)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) - 2*a^3*x^3 + 2*(5*a^9
*x^9 - 9*a^7*x^7 + 4*a^5*x^5)*(a*x + 1)^2*(a*x - 1)^2 + 4*(5*a^10*x^10 - 13
*a^8*x^8 + 11*a^6*x^6 - 3*a^4*x^4)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 4*(5*a
^11*x^11 - 17*a^9*x^9 + 21*a^7*x^7 - 11*a^5*x^5 + 2*a^3*x^3)*(a*x + 1)*(a*x
 - 1) + (a^13*x^13 - 5*a^11*x^11 + 10*a^9*x^9 - 10*a^7*x^7 + 5*a^5*x^5 + (a
^8*x^8 - 13*a^6*x^6 + 27*a^4*x^4 - 15*a^2*x^2)*(a*x + 1)^(5/2)*(a*x - 1)^(5
/2) - a^3*x^3 + (5*a^9*x^9 - 57*a^7*x^7 + 124*a^5*x^5 - 90*a^3*x^3 + 18*a*x
)*(a*x + 1)^2*(a*x - 1)^2 + (10*a^10*x^10 - 98*a^8*x^8 + 220*a^6*x^6 - 189*
a^4*x^4 + 63*a^2*x^2 - 6)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 2*(5*a^11*x^11
- 41*a^9*x^9 + 93*a^7*x^7 - 89*a^5*x^5 + 38*a^3*x^3 - 6*a*x)*(a*x + 1)*(a*x
 - 1) + (5*a^12*x^12 - 33*a^10*x^10 + 73*a^8*x^8 - 74*a^6*x^6 + 36*a^4*x^4
- 7*a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x
- 1))^2 + 2*(5*a^12*x^12 - 21*a^10*x^10 + 34*a^8*x^8 - 26*a^6*x^6 + 9*a^4*x
^4 - a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1) - (a^13*x^13 - 5*a^11*x^11 + 10*a
^9*x^9 - 10*a^7*x^7 + 5*a^5*x^5 + (a^8*x^8 - 4*a^6*x^6 + 3*a^4*x^4)*(a*x +
1)^(5/2)*(a*x - 1)^(5/2) - a^3*x^3 + (5*a^9*x^9 - 21*a^7*x^7 + 24*a^5*x^5 -
 8*a^3*x^3)*(a*x + 1)^2*(a*x - 1)^2 + (10*a^10*x^10 - 44*a^8*x^8 + 64*a^6*x
^6 - 37*a^4*x^4 + 7*a^2*x^2)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 2*(5*a^11*x^
11 - 23*a^9*x^9 + 39*a^7*x^7 - 30*a^5*x^5 + 10*a^3*x^3 - a*x)*(a*x + 1)*(a*
x - 1) + (5*a^12*x^12 - 24*a^10*x^10 + 45*a^8*x^8 - 41*a^6*x^6 + 18*a^4*x^4
```

```
 - 3*a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x
 - 1)))/((a^13*x^14 - 5*a^11*x^12 + (a*x + 1)^(5/2)*(a*x - 1)^(5/2)*a^8*x^9
 + 10*a^9*x^10 - 10*a^7*x^8 + 5*a^5*x^6 - a^3*x^4 + 5*(a^9*x^10 - a^7*x^8)*
(a*x + 1)^2*(a*x - 1)^2 + 10*(a^10*x^11 - 2*a^8*x^9 + a^6*x^7)*(a*x + 1)^(3
/2)*(a*x - 1)^(3/2) + 10*(a^11*x^12 - 3*a^9*x^10 + 3*a^7*x^8 - a^5*x^6)*(a*
x + 1)*(a*x - 1) + 5*(a^12*x^13 - 4*a^10*x^11 + 6*a^8*x^9 - 4*a^6*x^7 + a^4
*x^5)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3
) - integrate(1/6*(a^15*x^15 - 6*a^13*x^13 + 15*a^11*x^11 - 20*a^9*x^9 + 15
*a^7*x^7 - 6*a^5*x^5 + (a^9*x^9 - 39*a^7*x^7 + 135*a^5*x^5 - 105*a^3*x^3)*(
a*x + 1)^3*(a*x - 1)^3 + (6*a^10*x^10 - 201*a^8*x^8 + 677*a^6*x^6 - 663*a^4
*x^4 + 174*a^2*x^2)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + a^3*x^3 + (15*a^11*x^
11 - 420*a^9*x^9 + 1373*a^7*x^7 - 1565*a^5*x^5 + 705*a^3*x^3 - 108*a*x)*(a*
x + 1)^2*(a*x - 1)^2 + (20*a^12*x^12 - 450*a^10*x^10 + 1422*a^8*x^8 - 1787*
a^6*x^6 + 1059*a^4*x^4 - 288*a^2*x^2 + 24)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2)
+ (15*a^13*x^13 - 255*a^11*x^11 + 773*a^9*x^9 - 1026*a^7*x^7 + 714*a^5*x^5
- 257*a^3*x^3 + 36*a*x)*(a*x + 1)*(a*x - 1) + (6*a^14*x^14 - 69*a^12*x^12 +
 197*a^10*x^10 - 266*a^8*x^8 + 201*a^6*x^6 - 83*a^4*x^4 + 14*a^2*x^2)*sqrt(
a*x + 1)*sqrt(a*x - 1))/((a^15*x^17 - 6*a^13*x^15 + (a*x + 1)^3*(a*x - 1)^3
*a^9*x^11 + 15*a^11*x^13 - 20*a^9*x^11 + 15*a^7*x^9 - 6*a^5*x^7 + a^3*x^5 +
 6*(a^10*x^12 - a^8*x^10)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + 15*(a^11*x^13 -
 2*a^9*x^11 + a^7*x^9)*(a*x + 1)^2*(a*x - 1)^2 + 20*(a^12*x^14 - 3*a^10*x^1
2 + 3*a^8*x^10 - a^6*x^8)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 15*(a^13*x^15 -
 4*a^11*x^13 + 6*a^9*x^11 - 4*a^7*x^9 + a^5*x^7)*(a*x + 1)*(a*x - 1) + 6*(a
^14*x^16 - 5*a^12*x^14 + 10*a^10*x^12 - 10*a^8*x^10 + 5*a^6*x^8 - a^4*x^6)*
sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x^2\,\mathrm{arcosh}\,(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*arccosh(a*x)^4), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\,\mathrm{acosh}^4\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acosh(a*x)**4,x)
```

```
[Out] Integral(1/(x**2*acosh(a*x)**4), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{arcosh}(ax)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arccosh(a*x)^4), x)
```

3.72 $\int x^4\sqrt{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=182

$$-\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^5}-\frac{\sqrt{\frac{\pi}{3}}\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{64a^5}-\frac{\sqrt{\frac{\pi}{5}}\text{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{320a^5}-\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^5}-\sqrt{\frac{\pi}{3}}\text{E}$$

```
[Out] (x^5*Sqrt[ArcCosh[a*x]])/5 - (Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(32*a^5) -
(Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(64*a^5) - (Sqrt[Pi/5]*Erf[Sqr
t[5]*Sqrt[ArcCosh[a*x]]])/(320*a^5) - (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(
32*a^5) - (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(64*a^5) - (Sqrt[Pi
/5]*Erfi[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(320*a^5)
```

Rubi [A] time = 0.475392, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^5}-\frac{\sqrt{\frac{\pi}{3}}\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{64a^5}-\frac{\sqrt{\frac{\pi}{5}}\text{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{320a^5}-\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^5}-\sqrt{\frac{\pi}{3}}\text{E}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[ArcCosh[a*x]],x]
```


```
[Out] (x^5*Sqrt[ArcCosh[a*x]])/5 - (Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(32*a^5) -
(Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(64*a^5) - (Sqrt[Pi/5]*Erf[Sqr
t[5]*Sqrt[ArcCosh[a*x]]])/(320*a^5) - (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(
32*a^5) - (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(64*a^5) - (Sqrt[Pi
/5]*Erfi[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(320*a^5)
```


Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^4\sqrt{\cosh^{-1}(ax)}\,dx = \frac{1}{5}x^5\sqrt{\cosh^{-1}(ax)} - \frac{1}{10}a\int \frac{x^5}{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}\,dx$$

$$= \frac{1}{5}x^5\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh^5(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{10a^5}$$

$$= \frac{1}{5}x^5\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{5\cosh(x)}{8\sqrt{x}} + \frac{5\cosh(3x)}{16\sqrt{x}} + \frac{\cosh(5x)}{16\sqrt{x}}\right)dx, x, \cosh^{-1}(ax)\right)}{10a^5}$$

$$= \frac{1}{5}x^5\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{160a^5} - \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{32a^5}$$

$$= \frac{1}{5}x^5\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{320a^5} - \frac{\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{320a^5} - \text{Subs}$$

$$= \frac{1}{5}x^5\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int e^{-5x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{160a^5} - \frac{\text{Subst}\left(\int e^{5x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{160a^5} -$$

$$= \frac{1}{5}x^5\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{\pi}\,\mathbf{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^5} - \frac{\sqrt{\frac{\pi}{3}}\,\mathbf{erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{64a^5} - \frac{\sqrt{\frac{\pi}{5}}\,\mathbf{erf}\left(\sqrt{5}\sqrt{\cosh^{-1}\right.}}{320a^5}$$

Mathematica [A] time = 0.0947883, size = 162, normalized size = 0.89

$$\frac{3\sqrt{5}\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\frac{3}{2}, -5\cosh^{-1}(ax)\right) + 25\sqrt{3}\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\frac{3}{2}, -3\cosh^{-1}(ax)\right) + 150\sqrt{\cosh^{-1}(ax)}\text{Ga}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (3*Sqrt[5]*Sqrt[ArcCosh[a*x]]*Gamma[3/2, -5*ArcCosh[a*x]] + 25*Sqrt[3]*Sqrt
[ArcCosh[a*x]]*Gamma[3/2, -3*ArcCosh[a*x]] + 150*Sqrt[ArcCosh[a*x]]*Gamma[3
/2, -ArcCosh[a*x]] + 150*Sqrt[-ArcCosh[a*x]]*Gamma[3/2, ArcCosh[a*x]] + 25*
Sqrt[3]*Sqrt[-ArcCosh[a*x]]*Gamma[3/2, 3*ArcCosh[a*x]] + 3*Sqrt[5]*Sqrt[-Ar
cCosh[a*x]]*Gamma[3/2, 5*ArcCosh[a*x]])/(2400*a^5*Sqrt[-ArcCosh[a*x]])
```

Maple [F] time = 0.183, size = 0, normalized size = 0.

$$\int x^4\sqrt{\operatorname{arccosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)^(1/2),x)
```

```
[Out] int(x^4*arccosh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \sqrt{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \sqrt{\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.73 $\int x^3\sqrt{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=139

$$-\frac{\sqrt{\pi}\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a^4}-\frac{\sqrt{\frac{\pi}{2}}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{32a^4}-\frac{\sqrt{\pi}\text{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a^4}-\frac{\sqrt{\frac{\pi}{2}}\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{32a^4}-\frac{3\sqrt{c}}{}$$

```
[Out] (-3*Sqrt[ArcCosh[a*x]])/(32*a^4) + (x^4*Sqrt[ArcCosh[a*x]])/4 - (Sqrt[Pi]*E
rf[2*Sqrt[ArcCosh[a*x]]])/(256*a^4) - (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcCosh[
a*x]]])/(32*a^4) - (Sqrt[Pi]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(256*a^4) - (Sqrt[
Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(32*a^4)
```

Rubi [A] time = 0.403064, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a^4}-\frac{\sqrt{\frac{\pi}{2}}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{32a^4}-\frac{\sqrt{\pi}\text{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a^4}-\frac{\sqrt{\frac{\pi}{2}}\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{32a^4}-\frac{3\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[ArcCosh[a*x]],x]
```


```
[Out] (-3*Sqrt[ArcCosh[a*x]])/(32*a^4) + (x^4*Sqrt[ArcCosh[a*x]])/4 - (Sqrt[Pi]*E
rf[2*Sqrt[ArcCosh[a*x]]])/(256*a^4) - (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcCosh[
a*x]]])/(32*a^4) - (Sqrt[Pi]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(256*a^4) - (Sqrt[
Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(32*a^4)
```


Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
```

```
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^3\sqrt{\cosh^{-1}(ax)}\,dx = \frac{1}{4}x^4\sqrt{\cosh^{-1}(ax)} - \frac{1}{8}a\int \frac{x^4}{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}\,dx$$

$$= \frac{1}{4}x^4\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{8a^4}$$

$$= \frac{1}{4}x^4\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} + \frac{\cosh(2x)}{2\sqrt{x}} + \frac{\cosh(4x)}{8\sqrt{x}}\right)dx, x, \cosh^{-1}(ax)\right)}{8a^4}$$

$$= -\frac{3\sqrt{\cosh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{64a^4} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}} \cdots\right.}{16\cdots}$$

$$= -\frac{3\sqrt{\cosh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{128a^4} - \frac{\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \cdots\right.}{128a^4}$$

$$= -\frac{3\sqrt{\cosh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{64a^4} - \frac{\text{Subst}\left(\int e^{4x^2}\,dx, \cdots\right.}{64\cdots}$$

$$= -\frac{3\sqrt{\cosh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a^4} - \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{32a^4} - \cdots$$

Mathematica [A] time = 0.0839095, size = 101, normalized size = 0.73

$$\frac{\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\frac{3}{2}, -4\cosh^{-1}(ax)\right) + 4\sqrt{2}\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\frac{3}{2}, -2\cosh^{-1}(ax)\right) + \sqrt{-\cosh^{-1}(ax)}\left(4\sqrt{2}\text{Gamm}\cdots\right.}{128a^4\sqrt{-\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (Sqrt[ArcCosh[a*x]]*Gamma[3/2, -4*ArcCosh[a*x]] + 4*Sqrt[2]*Sqrt[ArcCosh[a*
x]]*Gamma[3/2, -2*ArcCosh[a*x]] + Sqrt[-ArcCosh[a*x]]*(4*Sqrt[2]*Gamma[3/2,
 2*ArcCosh[a*x]] + Gamma[3/2, 4*ArcCosh[a*x]]))/(128*a^4*Sqrt[-ArcCosh[a*x]
])
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int x^3\sqrt{\text{arccosh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)^(1/2),x)
```

```
[Out] int(x^3*arccosh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{\operatorname{arcosh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{\operatorname{acosh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.74 $\int x^2\sqrt{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=120

$$-\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^3}-\frac{\sqrt{\frac{\pi}{3}}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{48a^3}-\frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^3}-\frac{\sqrt{\frac{\pi}{3}}\operatorname{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{48a^3}+\frac{1}{3}x^3$$

```
[Out] (x^3*Sqrt[ArcCosh[a*x]])/3 - (Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(16*a^3) -
(Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(48*a^3) - (Sqrt[Pi]*Erfi[Sqrt
[ArcCosh[a*x]]])/(16*a^3) - (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(
48*a^3)
```

Rubi [A] time = 0.397389, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^3}-\frac{\sqrt{\frac{\pi}{3}}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{48a^3}-\frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^3}-\frac{\sqrt{\frac{\pi}{3}}\operatorname{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{48a^3}+\frac{1}{3}x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (x^3*Sqrt[ArcCosh[a*x]])/3 - (Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(16*a^3) -
(Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(48*a^3) - (Sqrt[Pi]*Erfi[Sqrt
[ArcCosh[a*x]]])/(16*a^3) - (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(
48*a^3)
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
```

```
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^2\sqrt{\cosh^{-1}(ax)}\,dx = \frac{1}{3}x^3\sqrt{\cosh^{-1}(ax)} - \frac{1}{6}a\int \frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}\,dx$$

$$= \frac{1}{3}x^3\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh^3(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{6a^3}$$

$$= \frac{1}{3}x^3\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4\sqrt{x}} + \frac{\cosh(3x)}{4\sqrt{x}}\right)dx, x, \cosh^{-1}(ax)\right)}{6a^3}$$

$$= \frac{1}{3}x^3\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{24a^3} - \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{8a^3}$$

$$= \frac{1}{3}x^3\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{48a^3} - \frac{\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{48a^3} - \text{Subs}$$

$$= \frac{1}{3}x^3\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{24a^3} - \frac{\text{Subst}\left(\int e^{3x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{24a^3} -$$

$$= \frac{1}{3}x^3\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^3} - \frac{\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{48a^3} - \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\cosh^{-1}(a}\right.}{16a^3}$$

Mathematica [A] time = 0.0830565, size = 100, normalized size = 0.83

$$\frac{\sqrt{3}\sqrt{\cosh^{-1}(ax)}\,\text{Gamma}\left(\frac{3}{2}, -3\cosh^{-1}(ax)\right) + 9\sqrt{\cosh^{-1}(ax)}\,\text{Gamma}\left(\frac{3}{2}, -\cosh^{-1}(ax)\right) + \sqrt{-\cosh^{-1}(ax)}\left(9\text{Gamma}\right.}{72a^3\sqrt{-\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (Sqrt[3]*Sqrt[ArcCosh[a*x]]*Gamma[3/2, -3*ArcCosh[a*x]] + 9*Sqrt[ArcCosh[a*
x]]*Gamma[3/2, -ArcCosh[a*x]] + Sqrt[-ArcCosh[a*x]]*(9*Gamma[3/2, ArcCosh[a
*x]] + Sqrt[3]*Gamma[3/2, 3*ArcCosh[a*x]]))/(72*a^3*Sqrt[-ArcCosh[a*x]])
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int x^2\sqrt{\operatorname{arccosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)^(1/2),x)
```

```
[Out] int(x^2*arccosh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.75 $\int x\sqrt{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=93

$$-\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a^2}-\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a^2}-\frac{\sqrt{\cosh^{-1}(ax)}}{4a^2}+\frac{1}{2}x^2\sqrt{\cosh^{-1}(ax)}$$

```
[Out] -Sqrt[ArcCosh[a*x]]/(4*a^2) + (x^2*Sqrt[ArcCosh[a*x]])/2 - (Sqrt[Pi/2]*Erf[
Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a^2) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCo
sh[a*x]]])/(16*a^2)
```

Rubi [A] time = 0.350346, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a^2}-\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a^2}-\frac{\sqrt{\cosh^{-1}(ax)}}{4a^2}+\frac{1}{2}x^2\sqrt{\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -Sqrt[ArcCosh[a*x]]/(4*a^2) + (x^2*Sqrt[ArcCosh[a*x]])/2 - (Sqrt[Pi/2]*Erf[
Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a^2) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCo
sh[a*x]]])/(16*a^2)
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
```

```
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x\sqrt{\cosh^{-1}(ax)}\,dx = \frac{1}{2}x^2\sqrt{\cosh^{-1}(ax)} - \frac{1}{4}a\int \frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}\,dx$$
$$= \frac{1}{2}x^2\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{4a^2}$$
$$= \frac{1}{2}x^2\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} + \frac{\cosh(2x)}{2\sqrt{x}}\right) dx, x, \cosh^{-1}(ax)\right)}{4a^2}$$
$$= -\frac{\sqrt{\cosh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{8a^2}$$
$$= -\frac{\sqrt{\cosh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{16a^2} - \frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \cosh\right.}{16a^2}$$
$$= -\frac{\sqrt{\cosh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{8a^2} - \frac{\text{Subst}\left(\int e^{2x^2}\,dx, x,\right.}{8a^2}$$
$$= -\frac{\sqrt{\cosh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a^2} - \frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a^2}$$

Mathematica [A] time = 0.0746745, size = 65, normalized size = 0.7

$$\frac{8\sqrt{\cosh^{-1}(ax)}\cosh\left(2\cosh^{-1}(ax)\right) - \sqrt{2\pi}\left(\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right) + \text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)\right)}{32a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (8*Sqrt[ArcCosh[a*x]]*Cosh[2*ArcCosh[a*x]] - Sqrt[2*Pi]*(Erf[Sqrt[2]*Sqrt[A
rcCosh[a*x]]] + Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]]))/(32*a^2)
```

Maple [A] time = 0.089, size = 73, normalized size = 0.8

$$-\frac{\sqrt{2}}{32\sqrt{\pi}a^2}\left(-8\sqrt{2}\sqrt{\text{arccosh}(ax)}\sqrt{\pi}x^2a^2 + 4\sqrt{2}\sqrt{\text{arccosh}(ax)}\sqrt{\pi} + \pi\,\text{Erf}\left(\sqrt{2}\sqrt{\text{arccosh}(ax)}\right) + \pi\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arccosh}(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^(1/2),x)
```

```
[Out] -1/32*2^(1/2)*(-8*2^(1/2)*arccosh(a*x)^(1/2)*Pi^(1/2)*x^2*a^2+4*2^(1/2)*arc
cosh(a*x)^(1/2)*Pi^(1/2)+Pi*erf(2^(1/2)*arccosh(a*x)^(1/2))+Pi*erfi(2^(1/2)
*arccosh(a*x)^(1/2)))/Pi^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\operatorname{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\operatorname{acosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.76 $\int \sqrt{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=53

$$-\frac{\sqrt{\pi}\,\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a}-\frac{\sqrt{\pi}\,\text{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a}+x\sqrt{\cosh^{-1}(ax)}$$

```
[Out] x*Sqrt[ArcCosh[a*x]] - (Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(4*a) - (Sqrt[Pi]
*Erfi[Sqrt[ArcCosh[a*x]]])/(4*a)
```

Rubi [A] time = 0.218892, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5654, 5781, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a}-\frac{\sqrt{\pi}\,\text{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a}+x\sqrt{\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcCosh[a*x]],x]
```

```
[Out] x*Sqrt[ArcCosh[a*x]] - (Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(4*a) - (Sqrt[Pi]
*Erfi[Sqrt[ArcCosh[a*x]]])/(4*a)
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\cosh^{-1}(ax)}\,dx &= x\sqrt{\cosh^{-1}(ax)} - \frac{1}{2}a\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}\,dx\\
&= x\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{2a}\\
&= x\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{4a} - \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{4a}\\
&= x\sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{2a} - \frac{\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{2a}\\
&= x\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a} - \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a}
\end{aligned}
$$

Mathematica [A] time = 0.0355784, size = 45, normalized size = 0.85

$$\frac{\frac{\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\frac{3}{2},-\cosh^{-1}(ax)\right)}{\sqrt{-\cosh^{-1}(ax)}}+\text{Gamma}\left(\frac{3}{2},\cosh^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[ArcCosh[a*x]],x]
```

```
[Out] ((Sqrt[ArcCosh[a*x]]*Gamma[3/2, -ArcCosh[a*x]])/Sqrt[-ArcCosh[a*x]] + Gamma
[3/2, ArcCosh[a*x]])/(2*a)
```

Maple [A] time = 0.074, size = 41, normalized size = 0.8

$$-\frac{1}{4\sqrt{\pi}a}\left(-4\sqrt{\operatorname{arccosh}(ax)}\sqrt{\pi}xa+\pi\operatorname{Erf}\left(\sqrt{\operatorname{arccosh}(ax)}\right)+\pi\operatorname{erfi}\left(\sqrt{\operatorname{arccosh}(ax)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(1/2),x)
```

```
[Out] -1/4*(-4*arccosh(a*x)^(1/2)*Pi^(1/2)*x*a+Pi*erf(arccosh(a*x)^(1/2))+Pi*erfi
(arccosh(a*x)^(1/2)))/Pi^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{acosh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.77 $\int \frac{\sqrt{\cosh^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\sqrt{\cosh^{-1}(ax)}}{x},x\right)$$

```
[Out] Unintegrable[Sqrt[ArcCosh[a*x]]/x, x]
```

Rubi [A] time = 0.0131443, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcCosh[a*x]]/x,x]
```

```
[Out] Defer[Int][Sqrt[ArcCosh[a*x]]/x, x]
```

Rubi steps

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{x} \, dx = \int \frac{\sqrt{\cosh^{-1}(ax)}}{x} \, dx$$

Mathematica [A] time = 0.283121, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcCosh[a*x]]/x,x]
```

```
[Out] Integrate[Sqrt[ArcCosh[a*x]]/x, x]
```

Maple [A] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{\operatorname{arccosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(1/2)/x,x)
```

```
[Out] int(arccosh(a*x)^(1/2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arcosh}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(a*x))/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{acosh}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(acosh(a*x))/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.78 $\int x^4 \cosh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=345

$$-\frac{3\sqrt{\pi}\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{64a^5}-\frac{3\sqrt{3\pi}\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{3200a^5}-\frac{\sqrt{\frac{\pi}{3}}\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{200a^5}-\frac{3\sqrt{\frac{\pi}{5}}\text{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{3200a^5}+$$

```
[Out] (-4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(25*a^5) - (2*x^2*Sqrt
[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(25*a^3) - (3*x^4*Sqrt[-1 + a*
x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(50*a) + (x^5*ArcCosh[a*x]^(3/2))/5 -
(3*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(64*a^5) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqr
t[ArcCosh[a*x]]])/(200*a^5) - (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]]
)/(3200*a^5) - (3*Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(3200*a^5) +
(3*Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(64*a^5) + (Sqrt[Pi/3]*Erfi[Sqrt[3]*S
qrt[ArcCosh[a*x]]])/(200*a^5) + (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x
]]])/(3200*a^5) + (3*Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(3200*a^5
)
```

Rubi [A] time = 1.1148, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 41, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5664, 5759, 5718, 5658, 3308, 2180, 2204, 2205, 5670, 5448}

$$-\frac{3\sqrt{\pi}\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{64a^5}-\frac{3\sqrt{3\pi}\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{3200a^5}-\frac{\sqrt{\frac{\pi}{3}}\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{200a^5}-\frac{3\sqrt{\frac{\pi}{5}}\text{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{3200a^5}+$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCosh[a*x]^(3/2),x]
```


```
[Out] (-4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(25*a^5) - (2*x^2*Sqrt
[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(25*a^3) - (3*x^4*Sqrt[-1 + a*
x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(50*a) + (x^5*ArcCosh[a*x]^(3/2))/5 -
(3*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(64*a^5) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqr
t[ArcCosh[a*x]]])/(200*a^5) - (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]]
)/(3200*a^5) - (3*Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(3200*a^5) +
(3*Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(64*a^5) + (Sqrt[Pi/3]*Erfi[Sqrt[3]*S
qrt[ArcCosh[a*x]]])/(200*a^5) + (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x
]]])/(3200*a^5) + (3*Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(3200*a^5
)
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
```

```
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^4 \cosh^{-1}(ax)^{3/2}\,dx = \frac{1}{5}x^5\cosh^{-1}(ax)^{3/2} - \frac{1}{10}(3a)\int \frac{x^5\sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{3x^4\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{50a} + \frac{1}{5}x^5\cosh^{-1}(ax)^{3/2} + \frac{3}{100}\int \frac{x^4}{\sqrt{\cosh^{-1}(ax)}}\,dx - \frac{6\int \frac{x^3\sqrt{}}{\sqrt{-}}}{}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{50a} + \frac{1}{5}x^5\cosh^{-1}(ax)^{3}$$

$$= -\frac{4\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{25a^5} - \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{-1+ax}\sqrt{1+}}{50a}$$

$$= -\frac{4\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{25a^5} - \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{-1+ax}\sqrt{1+}}{50a}$$

$$= -\frac{4\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{25a^5} - \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{-1+ax}\sqrt{1+}}{50a}$$

$$= -\frac{4\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{25a^5} - \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{-1+ax}\sqrt{1+}}{50a}$$

$$= -\frac{4\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{25a^5} - \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{-1+ax}\sqrt{1+}}{50a}$$

$$= -\frac{4\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{25a^5} - \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{-1+ax}\sqrt{1+}}{50a}$$

Mathematica [A] time = 0.111888, size = 152, normalized size = 0.44

$$\frac{\frac{9\sqrt{5}\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{5}{2},-5\cosh^{-1}(ax)\right)}{\sqrt{\cosh^{-1}(ax)}} + \frac{125\sqrt{3}\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{5}{2},-3\cosh^{-1}(ax)\right)}{\sqrt{\cosh^{-1}(ax)}} + \frac{2250\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{5}{2},-\cosh^{-1}(ax)\right)}{\sqrt{\cosh^{-1}(ax)}} + 2}{36000a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*ArcCosh[a*x]^(3/2),x]
```

```
[Out] ((9*Sqrt[5]*Sqrt[-ArcCosh[a*x]]*Gamma[5/2, -5*ArcCosh[a*x]])/Sqrt[ArcCosh[a
*x]] + (125*Sqrt[3]*Sqrt[-ArcCosh[a*x]]*Gamma[5/2, -3*ArcCosh[a*x]])/Sqrt[A
```

```
rcCosh[a*x]] + (2250*Sqrt[-ArcCosh[a*x]]*Gamma[5/2, -ArcCosh[a*x]])/Sqrt[Ar
cCosh[a*x]] + 2250*Gamma[5/2, ArcCosh[a*x]] + 125*Sqrt[3]*Gamma[5/2, 3*ArcC
osh[a*x]] + 9*Sqrt[5]*Gamma[5/2, 5*ArcCosh[a*x]])/(36000*a^5)
```

Maple [F] time = 0.187, size = 0, normalized size = 0.

$$\int x^4 (\operatorname{arccosh}(ax))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)^(3/2),x)
```

```
[Out] int(x^4*arccosh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arcosh}(ax)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.79 $\int x^3 \cosh^{-1}(ax)^{3/2}\, dx$

Optimal. Leaf size=209

$$-\frac{3\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{2048a^4}-\frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{128a^4}+\frac{3\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{2048a^4}+\frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{128a^4}-$$

```
[Out] (-9*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(64*a^3) - (3*x^3*Sq
rt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(32*a) - (3*ArcCosh[a*x]^(3/
2))/(32*a^4) + (x^4*ArcCosh[a*x]^(3/2))/4 - (3*Sqrt[Pi]*Erf[2*Sqrt[ArcCosh[
a*x]]])/(2048*a^4) - (3*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(128*a^
4) + (3*Sqrt[Pi]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(2048*a^4) + (3*Sqrt[Pi/2]*Erf
i[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(128*a^4)
```

Rubi [A] time = 0.874247, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5664, 5759, 5676, 5670, 5448, 12, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{2048a^4}-\frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{128a^4}+\frac{3\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{2048a^4}+\frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{128a^4}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCosh[a*x]^(3/2),x]
```


```
[Out] (-9*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(64*a^3) - (3*x^3*Sq
rt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(32*a) - (3*ArcCosh[a*x]^(3/
2))/(32*a^4) + (x^4*ArcCosh[a*x]^(3/2))/4 - (3*Sqrt[Pi]*Erf[2*Sqrt[ArcCosh[
a*x]]])/(2048*a^4) - (3*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(128*a^
4) + (3*Sqrt[Pi]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(2048*a^4) + (3*Sqrt[Pi/2]*Erf
i[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(128*a^4)
```


Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^3\cosh^{-1}(ax)^{3/2}\,dx = \frac{1}{4}x^4\cosh^{-1}(ax)^{3/2} - \frac{1}{8}(3a)\int \frac{x^4\sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{32a} + \frac{1}{4}x^4\cosh^{-1}(ax)^{3/2} + \frac{3}{64}\int \frac{x^3}{\sqrt{\cosh^{-1}(ax)}}\,dx - \frac{9\int \frac{x^2\sqrt{}}{\sqrt{-1}}}{}$$

$$= -\frac{9x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{32a} + \frac{1}{4}x^4\cosh^{-1}(ax)^{3/2}$$

$$= -\frac{9x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{32a} - \frac{3\cosh^{-1}(ax)^{3/2}}{32a^4} +$$

$$= -\frac{9x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{32a} - \frac{3\cosh^{-1}(ax)^{3/2}}{32a^4} +$$

$$= -\frac{9x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{32a} - \frac{3\cosh^{-1}(ax)^{3/2}}{32a^4} +$$

$$= -\frac{9x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{32a} - \frac{3\cosh^{-1}(ax)^{3/2}}{32a^4} +$$

$$= -\frac{9x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{32a} - \frac{3\cosh^{-1}(ax)^{3/2}}{32a^4} +$$

$$= -\frac{9x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{32a} - \frac{3\cosh^{-1}(ax)^{3/2}}{32a^4} +$$

Mathematica [A] time = 0.0851511, size = 101, normalized size = 0.48

$$\frac{\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{5}{2},-4\cosh^{-1}(ax)\right) + 8\sqrt{2}\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{5}{2},-2\cosh^{-1}(ax)\right) + \sqrt{\cosh^{-1}(ax)}\left(8\sqrt{2}\text{Ga}\right.}{512a^4\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*ArcCosh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[-ArcCosh[a*x]]*Gamma[5/2, -4*ArcCosh[a*x]] + 8*Sqrt[2]*Sqrt[-ArcCosh[
a*x]]*Gamma[5/2, -2*ArcCosh[a*x]] + Sqrt[ArcCosh[a*x]]*(8*Sqrt[2]*Gamma[5/2
```

```
, 2*ArcCosh[a*x]] + Gamma[5/2, 4*ArcCosh[a*x]]))/(512*a^4*Sqrt[ArcCosh[a*x]
])
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int x^3 (\operatorname{arccosh}(ax))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)^(3/2),x)
```

```
[Out] int(x^3*arccosh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arcosh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.80 $\int x^2 \cosh^{-1}(ax)^{3/2} \, dx$

Optimal. Leaf size=189

$$-\frac{3\sqrt{\pi}\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^3}-\frac{\sqrt{\frac{\pi}{3}}\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{96a^3}+\frac{3\sqrt{\pi}\text{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\text{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{96a^3}-\frac{\sqrt{ax}}{}$$

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(3*a^3) - (x^2*Sqrt[-1 +
 a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(6*a) + (x^3*ArcCosh[a*x]^(3/2))/3
- (3*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(32*a^3) - (Sqrt[Pi/3]*Erf[Sqrt[3]*S
qrt[ArcCosh[a*x]]])/(96*a^3) + (3*Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(32*a^
3) + (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(96*a^3)
```

Rubi [A] time = 0.639637, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5664, 5759, 5718, 5658, 3308, 2180, 2204, 2205, 5670, 5448}

$$-\frac{3\sqrt{\pi}\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^3}-\frac{\sqrt{\frac{\pi}{3}}\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{96a^3}+\frac{3\sqrt{\pi}\text{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\text{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{96a^3}-\frac{\sqrt{ax}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCosh[a*x]^(3/2),x]
```

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(3*a^3) - (x^2*Sqrt[-1 +
 a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(6*a) + (x^3*ArcCosh[a*x]^(3/2))/3
- (3*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(32*a^3) - (Sqrt[Pi/3]*Erf[Sqrt[3]*S
qrt[ArcCosh[a*x]]])/(96*a^3) + (3*Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(32*a^
3) + (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(96*a^3)
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
```

```
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^2 \cosh^{-1}(ax)^{3/2}\,dx = \frac{1}{3}x^3\cosh^{-1}(ax)^{3/2} - \frac{1}{2}a\int \frac{x^3\sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\cosh^{-1}(ax)^{3/2} + \frac{1}{12}\int \frac{x^2}{\sqrt{\cosh^{-1}(ax)}}\,dx - \frac{\int \frac{x\sqrt{\cosh}}{\sqrt{-1+ax}}}{3a}$$

$$= -\frac{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\cosh^{-1}(ax)^{3/2} +$$

$$= -\frac{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\cosh^{-1}(ax)^{3/2} +$$

$$= -\frac{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\cosh^{-1}(ax)^{3/2} +$$

$$= -\frac{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\cosh^{-1}(ax)^{3/2} -$$

$$= -\frac{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\cosh^{-1}(ax)^{3/2} -$$

$$= -\frac{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\cosh^{-1}(ax)^{3/2} -$$

Mathematica [A] time = 0.0873498, size = 100, normalized size = 0.53

$$\frac{\sqrt{3}\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{5}{2},-3\cosh^{-1}(ax)\right) + 27\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{5}{2},-\cosh^{-1}(ax)\right) + \sqrt{\cosh^{-1}(ax)}\left(27\text{Gam}\right.}{216a^3\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcCosh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[3]*Sqrt[-ArcCosh[a*x]]*Gamma[5/2, -3*ArcCosh[a*x]] + 27*Sqrt[-ArcCosh
[a*x]]*Gamma[5/2, -ArcCosh[a*x]] + Sqrt[ArcCosh[a*x]]*(27*Gamma[5/2, ArcCos
h[a*x]] + Sqrt[3]*Gamma[5/2, 3*ArcCosh[a*x]]))/(216*a^3*Sqrt[ArcCosh[a*x]])
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int x^{2} \left(\operatorname{arccosh}\left(a x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)^(3/2),x)
```

```
[Out] int(x^2*arccosh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2} \operatorname{arcosh}\left(a x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2} \operatorname{acosh}^{\frac{3}{2}}{\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)**(3/2),x)
```

```
[Out] Integral(x**2*acosh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.81 $\int x \cosh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{3\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a^2}+\frac{3\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a^2}-\frac{\cosh^{-1}(ax)^{3/2}}{4a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^{3/2}-\frac{3x\sqrt{ax-1}\sqrt{ax+1}}{8a}$$

```
[Out] (-3*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(8*a) - ArcCosh[a*x]
^(3/2)/(4*a^2) + (x^2*ArcCosh[a*x]^(3/2))/2 - (3*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqr
t[ArcCosh[a*x]]])/(64*a^2) + (3*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]]
)/(64*a^2)
```

Rubi [A] time = 0.437723, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5664, 5759, 5676, 5670, 5448, 12, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a^2}+\frac{3\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a^2}-\frac{\cosh^{-1}(ax)^{3/2}}{4a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^{3/2}-\frac{3x\sqrt{ax-1}\sqrt{ax+1}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCosh[a*x]^(3/2),x]
```

```
[Out] (-3*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(8*a) - ArcCosh[a*x]
^(3/2)/(4*a^2) + (x^2*ArcCosh[a*x]^(3/2))/2 - (3*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqr
t[ArcCosh[a*x]]])/(64*a^2) + (3*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]]
)/(64*a^2)
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
```

```
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x\cosh^{-1}(ax)^{3/2}\,dx=\frac{1}{2}x^2\cosh^{-1}(ax)^{3/2}-\frac{1}{4}(3a)\int\frac{x^2\sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$=-\frac{3x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{8a}+\frac{1}{2}x^2\cosh^{-1}(ax)^{3/2}+\frac{3}{16}\int\frac{x}{\sqrt{\cosh^{-1}(ax)}}\,dx-\frac{3\int\frac{\sqrt{\cosh^{-1}}}{\sqrt{-1+ax}\sqrt{}}}{8a}$$

$$=-\frac{3x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{8a}-\frac{\cosh^{-1}(ax)^{3/2}}{4a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^{3/2}+\frac{3\,\mathrm{Subst}\left(\int\frac{\cosh(x)\sin}{\sqrt{x}}\right.}{1}$$

$$=-\frac{3x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{8a}-\frac{\cosh^{-1}(ax)^{3/2}}{4a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^{3/2}+\frac{3\,\mathrm{Subst}\left(\int\frac{\sinh(2x)}{2\sqrt{x}}\,d\right.}{16a^2}$$

$$=-\frac{3x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{8a}-\frac{\cosh^{-1}(ax)^{3/2}}{4a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^{3/2}+\frac{3\,\mathrm{Subst}\left(\int\frac{\sinh(2x)}{\sqrt{x}}\,d\right.}{32a^2}$$

$$=-\frac{3x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{8a}-\frac{\cosh^{-1}(ax)^{3/2}}{4a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^{3/2}-\frac{3\,\mathrm{Subst}\left(\int\frac{e^{-2x}}{\sqrt{x}}\,dx,x\right.}{64a^2}$$

$$=-\frac{3x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{8a}-\frac{\cosh^{-1}(ax)^{3/2}}{4a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^{3/2}-\frac{3\,\mathrm{Subst}\left(\int e^{-2x^2}\,dx,\right.}{32a^2}$$

$$=-\frac{3x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{8a}-\frac{\cosh^{-1}(ax)^{3/2}}{4a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^{3/2}-\frac{3\sqrt{\frac{\pi}{2}}\,\mathbf{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}}\right.}{64a^2}$$

Mathematica [A] time = 0.125301, size = 84, normalized size = 0.66

$$\frac{3\sqrt{2\pi}\left(\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)-\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)\right)+32\cosh\left(2\cosh^{-1}(ax)\right)\cosh^{-1}(ax)^{3/2}-24\sqrt{\cosh^{-1}(ax)}\sinh}{128a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcCosh[a*x]^(3/2),x]
```

```
[Out] (32*ArcCosh[a*x]^(3/2)*Cosh[2*ArcCosh[a*x]] + 3*Sqrt[2*Pi]*(-Erf[Sqrt[2]*Sq
rt[ArcCosh[a*x]]] + Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]]) - 24*Sqrt[ArcCosh[a*x
]]*Sinh[2*ArcCosh[a*x]])/(128*a^2)
```

Maple [A] time = 0.112, size = 105, normalized size = 0.8

$$-\frac{\sqrt{2}}{128\sqrt{\pi}a^2}\left(-32\,(\operatorname{arccosh}(ax))^{3/2}\sqrt{2}\sqrt{\pi}x^2a^2+24\sqrt{2}\sqrt{\operatorname{arccosh}(ax)}\sqrt{\pi}\sqrt{ax+1}\sqrt{ax-1}xa+16\,(\operatorname{arccosh}(ax))^{3/2}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^(3/2),x)
```

```
[Out] -1/128*2^(1/2)*(-32*arccosh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*x^2*a^2+24*2^(1/2)*
arccosh(a*x)^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x*a+16*arccosh(a*x)
^(3/2)*2^(1/2)*Pi^(1/2)+3*Pi*erf(2^(1/2)*arccosh(a*x)^(1/2))-3*Pi*erfi(2^(1
/2)*arccosh(a*x)^(1/2)))/Pi^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcosh}(ax)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{acosh}^{\frac{3}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**(3/2),x)
```

```
[Out] Integral(x*acosh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.82 $\int \cosh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=86

$$-\frac{3\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a}+\frac{3\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a}+x\cosh^{-1}(ax)^{3/2}-\frac{3\sqrt{ax-1}\sqrt{ax+1}\sqrt{\cosh^{-1}(ax)}}{2a}$$

```
[Out] (-3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(2*a) + x*ArcCosh[a*x]
^(3/2) - (3*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(8*a) + (3*Sqrt[Pi]*Erfi[Sqrt
[ArcCosh[a*x]]])/(8*a)
```

Rubi [A] time = 0.220478, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5654, 5718, 5658, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a}+\frac{3\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a}+x\cosh^{-1}(ax)^{3/2}-\frac{3\sqrt{ax-1}\sqrt{ax+1}\sqrt{\cosh^{-1}(ax)}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^(3/2),x]
```

```
[Out] (-3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(2*a) + x*ArcCosh[a*x]
^(3/2) - (3*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(8*a) + (3*Sqrt[Pi]*Erfi[Sqrt
[ArcCosh[a*x]]])/(8*a)
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
```

```
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \cosh^{-1}(ax)^{3/2}\,dx = x\cosh^{-1}(ax)^{3/2} - \frac{1}{2}(3a)\int \frac{x\sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{3\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{2a} + x\cosh^{-1}(ax)^{3/2} + \frac{3}{4}\int \frac{1}{\sqrt{\cosh^{-1}(ax)}}\,dx$$

$$= -\frac{3\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{2a} + x\cosh^{-1}(ax)^{3/2} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{4a}$$

$$= -\frac{3\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{2a} + x\cosh^{-1}(ax)^{3/2} - \frac{3\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{8a} + \frac{3\,\text{Su}}{}$$

$$= -\frac{3\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{2a} + x\cosh^{-1}(ax)^{3/2} - \frac{3\,\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{4a} + \frac{3}{}$$

$$= -\frac{3\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{2a} + x\cosh^{-1}(ax)^{3/2} - \frac{3\sqrt{\pi}\,\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a} + \frac{3\sqrt{\pi}\,\text{erfi}\left(\sqrt{\text{co}}\right.}{8a}$$

Mathematica [A] time = 0.0287667, size = 45, normalized size = 0.52

$$\frac{\frac{\sqrt{-\cosh^{-1}(ax)}\,\text{Gamma}\left(\frac{5}{2}, -\cosh^{-1}(ax)\right)}{\sqrt{\cosh^{-1}(ax)}} + \text{Gamma}\left(\frac{5}{2}, \cosh^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^(3/2),x]
```

```
[Out] ((Sqrt[-ArcCosh[a*x]]*Gamma[5/2, -ArcCosh[a*x]])/Sqrt[ArcCosh[a*x]] + Gamma
[5/2, ArcCosh[a*x]])/(2*a)
```

Maple [A] time = 0.095, size = 68, normalized size = 0.8

$$-\frac{1}{8\sqrt{\pi}a}\left(-8\,(\text{arccosh}\,(ax))^{3/2}\sqrt{\pi}xa + 12\sqrt{\text{arccosh}\,(ax)}\sqrt{\pi}\sqrt{ax+1}\sqrt{ax-1} + 3\,\pi\,\text{Erf}\left(\sqrt{\text{arccosh}\,(ax)}\right) - 3\,\pi\,\text{erfi}\left(\sqrt{\text{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(3/2),x)
```

```
[Out] -1/8*(-8*arccosh(a*x)^(3/2)*Pi^(1/2)*x*a+12*arccosh(a*x)^(1/2)*Pi^(1/2)*(a*
x+1)^(1/2)*(a*x-1)^(1/2)+3*Pi*erf(arccosh(a*x)^(1/2))-3*Pi*erfi(arccosh(a*x
)^(1/2)))/Pi^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcosh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acosh}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(3/2),x)
```

```
[Out] Integral(acosh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.83 $\int \frac{\cosh^{-1}(ax)^{3/2}}{x} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\cosh^{-1}(ax)^{3/2}}{x},x\right)$$

```
[Out] Unintegrable[ArcCosh[a*x]^(3/2)/x, x]
```

Rubi [A] time = 0.0132645, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCosh[a*x]^(3/2)/x,x]
```

```
[Out] Defer[Int][ArcCosh[a*x]^(3/2)/x, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{x} \, dx = \int \frac{\cosh^{-1}(ax)^{3/2}}{x} \, dx$$

Mathematica [A] time = 0.289235, size = 0, normalized size = 0.

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCosh[a*x]^(3/2)/x,x]
```

```
[Out] Integrate[ArcCosh[a*x]^(3/2)/x, x]
```

Maple [A] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\operatorname{arccosh}(ax))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(3/2)/x,x)
```

```
[Out] int(arccosh(a*x)^(3/2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(3/2)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^{\frac{3}{2}}(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(3/2)/x,x)
```

```
[Out] Integral(acosh(a*x)**(3/2)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.84 $\int x^4 \cosh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=394

$$-\frac{15\sqrt{\pi}\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{128a^5}-\frac{\sqrt{3\pi}\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{1280a^5}-\frac{\sqrt{\frac{\pi}{3}}\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{240a^5}-\frac{3\sqrt{\frac{\pi}{5}}\text{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{6400a^5}$$

```
[Out] (2*x*Sqrt[ArcCosh[a*x]])/(5*a^4) + (x^3*Sqrt[ArcCosh[a*x]])/(15*a^2) + (3*x
^5*Sqrt[ArcCosh[a*x]])/100 - (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(
3/2))/(15*a^5) - (2*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(1
5*a^3) - (x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(10*a) + (x^
5*ArcCosh[a*x]^(5/2))/5 - (15*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(128*a^5) -
 (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(240*a^5) - (Sqrt[3*Pi]*Erf[S
qrt[3]*Sqrt[ArcCosh[a*x]]])/(1280*a^5) - (3*Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[Arc
Cosh[a*x]]])/(6400*a^5) - (15*Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(128*a^5)
- (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(240*a^5) - (Sqrt[3*Pi]*Erf
i[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(1280*a^5) - (3*Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt
[ArcCosh[a*x]]])/(6400*a^5)
```

Rubi [A] time = 1.82197, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 44, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5664, 5759, 5718, 5654, 5781, 3307, 2180, 2204, 2205, 3312}

$$-\frac{15\sqrt{\pi}\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{128a^5}-\frac{\sqrt{3\pi}\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{1280a^5}-\frac{\sqrt{\frac{\pi}{3}}\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{240a^5}-\frac{3\sqrt{\frac{\pi}{5}}\text{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{6400a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCosh[a*x]^(5/2),x]
```

```
[Out] (2*x*Sqrt[ArcCosh[a*x]])/(5*a^4) + (x^3*Sqrt[ArcCosh[a*x]])/(15*a^2) + (3*x
^5*Sqrt[ArcCosh[a*x]])/100 - (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(
3/2))/(15*a^5) - (2*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(1
5*a^3) - (x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(10*a) + (x^
5*ArcCosh[a*x]^(5/2))/5 - (15*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(128*a^5) -
 (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(240*a^5) - (Sqrt[3*Pi]*Erf[S
qrt[3]*Sqrt[ArcCosh[a*x]]])/(1280*a^5) - (3*Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[Arc
Cosh[a*x]]])/(6400*a^5) - (15*Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(128*a^5)
- (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(240*a^5) - (Sqrt[3*Pi]*Erf
i[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(1280*a^5) - (3*Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt
```

```
[ArcCosh[a*x]]])/(6400*a^5)
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
```

```
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^4 \cosh^{-1}(ax)^{5/2}\,dx = \frac{1}{5}x^5\cosh^{-1}(ax)^{5/2} - \frac{1}{2}a\int \frac{x^5\cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{10a} + \frac{1}{5}x^5\cosh^{-1}(ax)^{5/2} + \frac{3}{20}\int x^4\sqrt{\cosh^{-1}(ax)}\,dx - \frac{2\int \frac{x^3\text{co}}{\sqrt{-1}}}{}$$

$$= \frac{3}{100}x^5\sqrt{\cosh^{-1}(ax)} - \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{10a}$$

$$= \frac{x^3\sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{-1+ax}\sqrt{1+}}{15}$$

$$= \frac{2x\sqrt{\cosh^{-1}(ax)}}{5a^4} + \frac{x^3\sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{15a^5} -$$

$$= \frac{2x\sqrt{\cosh^{-1}(ax)}}{5a^4} + \frac{x^3\sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{15a^5} -$$

$$= \frac{2x\sqrt{\cosh^{-1}(ax)}}{5a^4} + \frac{x^3\sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{15a^5} -$$

$$= \frac{2x\sqrt{\cosh^{-1}(ax)}}{5a^4} + \frac{x^3\sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{15a^5} -$$

$$= \frac{2x\sqrt{\cosh^{-1}(ax)}}{5a^4} + \frac{x^3\sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{15a^5} -$$

$$= \frac{2x\sqrt{\cosh^{-1}(ax)}}{5a^4} + \frac{x^3\sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{15a^5} -$$

Mathematica [A] time = 0.0973618, size = 162, normalized size = 0.41

$$\frac{27\sqrt{5}\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\frac{7}{2},-5\cosh^{-1}(ax)\right) + 625\sqrt{3}\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\frac{7}{2},-3\cosh^{-1}(ax)\right) + 33750\sqrt{\cosh^{-1}(ax}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*ArcCosh[a*x]^(5/2),x]
```

```
[Out] (27*Sqrt[5]*Sqrt[ArcCosh[a*x]]*Gamma[7/2, -5*ArcCosh[a*x]] + 625*Sqrt[3]*Sq
rt[ArcCosh[a*x]]*Gamma[7/2, -3*ArcCosh[a*x]] + 33750*Sqrt[ArcCosh[a*x]]*Gam
ma[7/2, -ArcCosh[a*x]] + 33750*Sqrt[-ArcCosh[a*x]]*Gamma[7/2, ArcCosh[a*x]]
 + 625*Sqrt[3]*Sqrt[-ArcCosh[a*x]]*Gamma[7/2, 3*ArcCosh[a*x]] + 27*Sqrt[5]*
Sqrt[-ArcCosh[a*x]]*Gamma[7/2, 5*ArcCosh[a*x]])/(540000*a^5*Sqrt[-ArcCosh[a
*x]])
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int x^4 (\operatorname{arccosh}(ax))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)^(5/2),x)
```

```
[Out] int(x^4*arccosh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arcosh}(ax)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.85 $\int x^3 \cosh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=257

$$-\frac{15\sqrt{\pi}\,\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{16384a^4}-\frac{15\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{512a^4}-\frac{15\sqrt{\pi}\,\text{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{16384a^4}-\frac{15\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(a\right.}}{512a^4}$$

```
[Out] (-225*Sqrt[ArcCosh[a*x]])/(2048*a^4) + (45*x^2*Sqrt[ArcCosh[a*x]])/(256*a^2
) + (15*x^4*Sqrt[ArcCosh[a*x]])/256 - (15*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Ar
cCosh[a*x]^(3/2))/(64*a^3) - (5*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*
x]^(3/2))/(32*a) - (3*ArcCosh[a*x]^(5/2))/(32*a^4) + (x^4*ArcCosh[a*x]^(5/2
))/4 - (15*Sqrt[Pi]*Erf[2*Sqrt[ArcCosh[a*x]]])/(16384*a^4) - (15*Sqrt[Pi/2]
*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(512*a^4) - (15*Sqrt[Pi]*Erfi[2*Sqrt[ArcC
osh[a*x]]])/(16384*a^4) - (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/
(512*a^4)
```

Rubi [A] time = 1.37528, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5664, 5759, 5676, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\pi}\,\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{16384a^4}-\frac{15\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{512a^4}-\frac{15\sqrt{\pi}\,\text{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{16384a^4}-\frac{15\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(a\right.}}{512a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCosh[a*x]^(5/2),x]
```


```
[Out] (-225*Sqrt[ArcCosh[a*x]])/(2048*a^4) + (45*x^2*Sqrt[ArcCosh[a*x]])/(256*a^2
) + (15*x^4*Sqrt[ArcCosh[a*x]])/256 - (15*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Ar
cCosh[a*x]^(3/2))/(64*a^3) - (5*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*
x]^(3/2))/(32*a) - (3*ArcCosh[a*x]^(5/2))/(32*a^4) + (x^4*ArcCosh[a*x]^(5/2
))/4 - (15*Sqrt[Pi]*Erf[2*Sqrt[ArcCosh[a*x]]])/(16384*a^4) - (15*Sqrt[Pi/2]
*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(512*a^4) - (15*Sqrt[Pi]*Erfi[2*Sqrt[ArcC
osh[a*x]]])/(16384*a^4) - (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/
(512*a^4)
```


Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
```

```
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^3 \cosh^{-1}(ax)^{5/2}\,dx = \frac{1}{4}x^4\cosh^{-1}(ax)^{5/2} - \frac{1}{8}(5a)\int \frac{x^4\cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{5x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{32a} + \frac{1}{4}x^4\cosh^{-1}(ax)^{5/2} + \frac{15}{64}\int x^3\sqrt{\cosh^{-1}(ax)}\,dx - \frac{15\int \frac{x^2}{\sqrt{}}}{}$$

$$= \frac{15}{256}x^4\sqrt{\cosh^{-1}(ax)} - \frac{15x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/}}{32a}$$

$$= \frac{45x^2\sqrt{\cosh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\cosh^{-1}(ax)} - \frac{15x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{-1+ax}}{}$$

$$= \frac{45x^2\sqrt{\cosh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\cosh^{-1}(ax)} - \frac{15x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{-1+ax}}{}$$

$$= -\frac{45\sqrt{\cosh^{-1}(ax)}}{2048a^4} + \frac{45x^2\sqrt{\cosh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\cosh^{-1}(ax)} - \frac{15x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(a}{64a^3}$$

$$= -\frac{225\sqrt{\cosh^{-1}(ax)}}{2048a^4} + \frac{45x^2\sqrt{\cosh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\cosh^{-1}(ax)} - \frac{15x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(}{64a^3}$$

$$= -\frac{225\sqrt{\cosh^{-1}(ax)}}{2048a^4} + \frac{45x^2\sqrt{\cosh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\cosh^{-1}(ax)} - \frac{15x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(}{64a^3}$$

$$= -\frac{225\sqrt{\cosh^{-1}(ax)}}{2048a^4} + \frac{45x^2\sqrt{\cosh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\cosh^{-1}(ax)} - \frac{15x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(}{64a^3}$$

$$= -\frac{225\sqrt{\cosh^{-1}(ax)}}{2048a^4} + \frac{45x^2\sqrt{\cosh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\cosh^{-1}(ax)} - \frac{15x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(}{64a^3}$$

Mathematica [A] time = 0.0827625, size = 101, normalized size = 0.39

$$\frac{\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\frac{7}{2},-4\cosh^{-1}(ax)\right) + 16\sqrt{2}\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\frac{7}{2},-2\cosh^{-1}(ax)\right) + \sqrt{-\cosh^{-1}(ax)}\left(16\sqrt{2}\text{Ga}\right.}{2048a^4\sqrt{-\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*ArcCosh[a*x]^(5/2),x]
```

```
[Out] (Sqrt[ArcCosh[a*x]]*Gamma[7/2, -4*ArcCosh[a*x]] + 16*Sqrt[2]*Sqrt[ArcCosh[a
*x]]*Gamma[7/2, -2*ArcCosh[a*x]] + Sqrt[-ArcCosh[a*x]]*(16*Sqrt[2]*Gamma[7/
2, 2*ArcCosh[a*x]] + Gamma[7/2, 4*ArcCosh[a*x]]))/(2048*a^4*Sqrt[-ArcCosh[a
*x]])
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int x^{3} (\operatorname{arccosh}(ax))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)^(5/2),x)
```

```
[Out] int(x^3*arccosh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{3} \operatorname{arcosh}(ax)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.86 $\int x^2 \cosh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=220

$$-\frac{15\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{64a^3}-\frac{5\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{576a^3}-\frac{15\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{64a^3}-\frac{5\sqrt{\frac{\pi}{3}}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{576a^3}$$

```
[Out] (5*x*Sqrt[ArcCosh[a*x]])/(6*a^2) + (5*x^3*Sqrt[ArcCosh[a*x]])/36 - (5*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(9*a^3) - (5*x^2*Sqrt[-1 + a*x]
*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(18*a) + (x^3*ArcCosh[a*x]^(5/2))/3 - (1
5*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(64*a^3) - (5*Sqrt[Pi/3]*Erf[Sqrt[3]*Sq
rt[ArcCosh[a*x]]])/(576*a^3) - (15*Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(64*a
^3) - (5*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(576*a^3)
```

Rubi [A] time = 1.05463, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5664, 5759, 5718, 5654, 5781, 3307, 2180, 2204, 2205, 3312}

$$-\frac{15\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{64a^3}-\frac{5\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{576a^3}-\frac{15\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{64a^3}-\frac{5\sqrt{\frac{\pi}{3}}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{576a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCosh[a*x]^(5/2),x]
```


```
[Out] (5*x*Sqrt[ArcCosh[a*x]])/(6*a^2) + (5*x^3*Sqrt[ArcCosh[a*x]])/36 - (5*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(9*a^3) - (5*x^2*Sqrt[-1 + a*x]
*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(18*a) + (x^3*ArcCosh[a*x]^(5/2))/3 - (1
5*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(64*a^3) - (5*Sqrt[Pi/3]*Erf[Sqrt[3]*Sq
rt[ArcCosh[a*x]]])/(576*a^3) - (15*Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(64*a
^3) - (5*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(576*a^3)
```


Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^2\cosh^{-1}(ax)^{5/2}\,dx = \frac{1}{3}x^3\cosh^{-1}(ax)^{5/2} - \frac{1}{6}(5a)\int \frac{x^3\cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{5x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{18a} + \frac{1}{3}x^3\cosh^{-1}(ax)^{5/2} + \frac{5}{12}\int x^2\sqrt{\cosh^{-1}(ax)}\,dx - \frac{5\int \frac{x\,\mathrm{co}}{\sqrt{-}}}{}$$

$$= \frac{5}{36}x^3\sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{18a} +$$

$$= \frac{5x\sqrt{\cosh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{-1+ax}\sqrt{1+}}{18}$$

$$= \frac{5x\sqrt{\cosh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{-1+ax}\sqrt{1+}}{18}$$

$$= \frac{5x\sqrt{\cosh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{-1+ax}\sqrt{1+}}{18}$$

$$= \frac{5x\sqrt{\cosh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{-1+ax}\sqrt{1+}}{18}$$

$$= \frac{5x\sqrt{\cosh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{-1+ax}\sqrt{1+}}{18}$$

$$= \frac{5x\sqrt{\cosh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{-1+ax}\sqrt{1+}}{18}$$

Mathematica [A] time = 0.0818516, size = 100, normalized size = 0.45

$$\frac{\sqrt{3}\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\frac{7}{2},-3\cosh^{-1}(ax)\right) + 81\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\frac{7}{2},-\cosh^{-1}(ax)\right) + \sqrt{-\cosh^{-1}(ax)}\left(81\text{Gamma}\right.}{648a^3\sqrt{-\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcCosh[a*x]^(5/2),x]
```

```
[Out] (Sqrt[3]*Sqrt[ArcCosh[a*x]]*Gamma[7/2, -3*ArcCosh[a*x]] + 81*Sqrt[ArcCosh[a
*x]]*Gamma[7/2, -ArcCosh[a*x]] + Sqrt[-ArcCosh[a*x]]*(81*Gamma[7/2, ArcCosh
[a*x]] + Sqrt[3]*Gamma[7/2, 3*ArcCosh[a*x]]))/(648*a^3*Sqrt[-ArcCosh[a*x]])
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int x^2 (\operatorname{arccosh}(ax))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)^(5/2),x)
```

```
[Out] int(x^2*arccosh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcosh}(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.87 $\int x \cosh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=157

$$-\frac{15\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a^2}-\frac{15\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a^2}-\frac{\cosh^{-1}(ax)^{5/2}}{4a^2}-\frac{15\sqrt{\cosh^{-1}(ax)}}{64a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^{5/2}$$

```
[Out] (-15*Sqrt[ArcCosh[a*x]])/(64*a^2) + (15*x^2*Sqrt[ArcCosh[a*x]])/32 - (5*x*S
qrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(8*a) - ArcCosh[a*x]^(5/2)/
(4*a^2) + (x^2*ArcCosh[a*x]^(5/2))/2 - (15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcC
osh[a*x]]])/(256*a^2) - (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(2
56*a^2)
```

Rubi [A] time = 0.708933, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {5664, 5759, 5676, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a^2}-\frac{15\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a^2}-\frac{\cosh^{-1}(ax)^{5/2}}{4a^2}-\frac{15\sqrt{\cosh^{-1}(ax)}}{64a^2}+\frac{1}{2}x^2\cosh^{-1}(ax)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCosh[a*x]^(5/2),x]
```

```
[Out] (-15*Sqrt[ArcCosh[a*x]])/(64*a^2) + (15*x^2*Sqrt[ArcCosh[a*x]])/32 - (5*x*S
qrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(8*a) - ArcCosh[a*x]^(5/2)/
(4*a^2) + (x^2*ArcCosh[a*x]^(5/2))/2 - (15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcC
osh[a*x]]])/(256*a^2) - (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(2
56*a^2)
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
```

```
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x\cosh^{-1}(ax)^{5/2}\,dx = \frac{1}{2}x^2\cosh^{-1}(ax)^{5/2} - \frac{1}{4}(5a)\int \frac{x^2\cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{5x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{8a} + \frac{1}{2}x^2\cosh^{-1}(ax)^{5/2} + \frac{15}{16}\int x\sqrt{\cosh^{-1}(ax)}\,dx - \frac{5\int \frac{\cosh}{\sqrt{-1+}}}{}$$

$$= \frac{15}{32}x^2\sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{8a} - \frac{\cosh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\cosh^{-1}(ax)^{5/2} -$$

$$= \frac{15}{32}x^2\sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{8a} - \frac{\cosh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\cosh^{-1}(ax)^{5/2} -$$

$$= \frac{15}{32}x^2\sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{8a} - \frac{\cosh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\cosh^{-1}(ax)^{5/2} -$$

$$= -\frac{15\sqrt{\cosh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2\sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{8a} - \frac{\cosh^{-1}(ax)^{5/2}}{4a^2} +$$

$$= -\frac{15\sqrt{\cosh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2\sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{8a} - \frac{\cosh^{-1}(ax)^{5/2}}{4a^2} +$$

$$= -\frac{15\sqrt{\cosh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2\sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{8a} - \frac{\cosh^{-1}(ax)^{5/2}}{4a^2} +$$

$$= -\frac{15\sqrt{\cosh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2\sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{8a} - \frac{\cosh^{-1}(ax)^{5/2}}{4a^2} +$$

Mathematica [A] time = 0.164887, size = 92, normalized size = 0.59

$$\frac{-15\sqrt{2\pi}\left(\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)\right)+8\left(16\cosh^{-1}(ax)^2+15\right)\cosh\left(2\cosh^{-1}(ax)\right)\sqrt{\cosh^{-1}(ax)}}{512a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcCosh[a*x]^(5/2),x]
```

```
[Out] (8*Sqrt[ArcCosh[a*x]]*(15 + 16*ArcCosh[a*x]^2)*Cosh[2*ArcCosh[a*x]] - 15*Sq
rt[2*Pi]*(Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]
]) - 160*ArcCosh[a*x]^(3/2)*Sinh[2*ArcCosh[a*x]])/(512*a^2)
```

Maple [A] time = 0.115, size = 139, normalized size = 0.9

$$-\frac{\sqrt{2}}{512\sqrt{\pi}a^2}\left(-128\,(\mathrm{arccosh}\,(ax))^{5/2}\sqrt{2}\sqrt{\pi}x^2a^2+160\,(\mathrm{arccosh}\,(ax))^{3/2}\sqrt{2}\sqrt{\pi}\sqrt{ax+1}\sqrt{ax-1}xa-120\sqrt{2}\sqrt{\mathrm{arccosh}\,(ax)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^(5/2),x)
```

```
[Out] -1/512*2^(1/2)*(-128*arccosh(a*x)^(5/2)*2^(1/2)*Pi^(1/2)*x^2*a^2+160*arccos
h(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x*a-120*2^(1/2)*a
rccosh(a*x)^(1/2)*Pi^(1/2)*x^2*a^2+64*arccosh(a*x)^(5/2)*2^(1/2)*Pi^(1/2)+6
0*2^(1/2)*arccosh(a*x)^(1/2)*Pi^(1/2)+15*Pi*erf(2^(1/2)*arccosh(a*x)^(1/2))
+15*Pi*erfi(2^(1/2)*arccosh(a*x)^(1/2)))/Pi^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{arcosh}\,(ax)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.88 $\int \cosh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=99

$$-\frac{15\sqrt{\pi}\mathrm{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a}-\frac{15\sqrt{\pi}\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a}+x\cosh^{-1}(ax)^{5/2}-\frac{5\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{3/2}}{2a}+\frac{15}{4}x\sqrt{\mathrm{co}}$$

```
[Out] (15*x*Sqrt[ArcCosh[a*x]])/4 - (5*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^
(3/2))/(2*a) + x*ArcCosh[a*x]^(5/2) - (15*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])
/(16*a) - (15*Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(16*a)
```

Rubi [A] time = 0.395465, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5654, 5718, 5781, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\pi}\mathrm{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a}-\frac{15\sqrt{\pi}\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a}+x\cosh^{-1}(ax)^{5/2}-\frac{5\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{3/2}}{2a}+\frac{15}{4}x\sqrt{\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^(5/2),x]
```

```
[Out] (15*x*Sqrt[ArcCosh[a*x]])/4 - (5*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^
(3/2))/(2*a) + x*ArcCosh[a*x]^(5/2) - (15*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])
/(16*a) - (15*Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(16*a)
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
```

```
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \cosh^{-1}(ax)^{5/2}\,dx = x\cosh^{-1}(ax)^{5/2} - \frac{1}{2}(5a)\int \frac{x\cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{2a} + x\cosh^{-1}(ax)^{5/2} + \frac{15}{4}\int \sqrt{\cosh^{-1}(ax)}\,dx$$

$$= \frac{15}{4}x\sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{2a} + x\cosh^{-1}(ax)^{5/2} - \frac{1}{8}(15a)\int \frac{}{\sqrt{-1+ax}\sqrt{1}}$$

$$= \frac{15}{4}x\sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{2a} + x\cosh^{-1}(ax)^{5/2} - \frac{15\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx,\right.}{8a}$$

$$= \frac{15}{4}x\sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{2a} + x\cosh^{-1}(ax)^{5/2} - \frac{15\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \text{c}\right.}{16a}$$

$$= \frac{15}{4}x\sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{2a} + x\cosh^{-1}(ax)^{5/2} - \frac{15\,\text{Subst}\left(\int e^{-x^2}\,dx, x,\right.}{8a}$$

$$= \frac{15}{4}x\sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{2a} + x\cosh^{-1}(ax)^{5/2} - \frac{15\sqrt{\pi}\,\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right.}{16a}$$

Mathematica [A] time = 0.035386, size = 45, normalized size = 0.45

$$\frac{\frac{\sqrt{\cosh^{-1}(ax)}\,\text{Gamma}\left(\frac{7}{2},-\cosh^{-1}(ax)\right)}{\sqrt{-\cosh^{-1}(ax)}} + \text{Gamma}\left(\frac{7}{2},\cosh^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^(5/2),x]
```

```
[Out] ((Sqrt[ArcCosh[a*x]]*Gamma[7/2, -ArcCosh[a*x]])/Sqrt[-ArcCosh[a*x]] + Gamma
[7/2, ArcCosh[a*x]])/(2*a)
```

Maple [A] time = 0.098, size = 81, normalized size = 0.8

$$-\frac{1}{16\sqrt{\pi}a}\left(-16\,(\text{arccosh}\,(ax))^{5/2}\sqrt{\pi}xa + 40\,(\text{arccosh}\,(ax))^{3/2}\sqrt{\pi}\sqrt{ax+1}\sqrt{ax-1} - 60\sqrt{\text{arccosh}\,(ax)}\sqrt{\pi}xa + 15\,\pi\,\text{Erf}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(5/2),x)
```

```
[Out] -1/16*(-16*arccosh(a*x)^(5/2)*Pi^(1/2)*x*a+40*arccosh(a*x)^(3/2)*Pi^(1/2)*(
a*x+1)^(1/2)*(a*x-1)^(1/2)-60*arccosh(a*x)^(1/2)*Pi^(1/2)*x*a+15*Pi*erf(arc
cosh(a*x)^(1/2))+15*Pi*erfi(arccosh(a*x)^(1/2)))/Pi^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcosh}(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.89 $\int \frac{\cosh^{-1}(ax)^{5/2}}{x}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\cosh^{-1}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable[ArcCosh[a*x]^(5/2)/x, x]
```

Rubi [A] time = 0.0136085, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cosh^{-1}(ax)^{5/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCosh[a*x]^(5/2)/x,x]
```

```
[Out] Defer[Int][ArcCosh[a*x]^(5/2)/x, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^{5/2}}{x}\,dx = \int \frac{\cosh^{-1}(ax)^{5/2}}{x}\,dx$$

Mathematica [A] time = 0.293893, size = 0, normalized size = 0.

$$\int \frac{\cosh^{-1}(ax)^{5/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCosh[a*x]^(5/2)/x,x]
```

```
[Out] Integrate[ArcCosh[a*x]^(5/2)/x, x]
```

Maple [A] time = 0.071, size = 0, normalized size = 0.

$$\int \frac{1}{x} \left(\operatorname{arccosh}\left(ax\right)\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(5/2)/x,x)

[Out] int(arccosh(a*x)^(5/2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}\left(ax\right)^{\frac{5}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/x,x, algorithm="maxima")

[Out] integrate(arccosh(a*x)^(5/2)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/x,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.90 $\int \frac{x^4}{\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=163

$$-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^5}-\frac{\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{32a^5}-\frac{\sqrt{\frac{\pi}{5}}\,\mathrm{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{32a^5}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^5}+\frac{\sqrt{3\pi}\,\mathrm{E}}{}$$

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(16*a^5) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt
[ArcCosh[a*x]]])/(32*a^5) - (Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(3
2*a^5) + (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(16*a^5) + (Sqrt[3*Pi]*Erfi[Sq
rt[3]*Sqrt[ArcCosh[a*x]]])/(32*a^5) + (Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcCosh
[a*x]]])/(32*a^5)
```

Rubi [A] time = 0.199441, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^5}-\frac{\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{32a^5}-\frac{\sqrt{\frac{\pi}{5}}\,\mathrm{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{32a^5}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^5}+\frac{\sqrt{3\pi}\,\mathrm{E}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(16*a^5) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt
[ArcCosh[a*x]]])/(32*a^5) - (Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(3
2*a^5) + (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(16*a^5) + (Sqrt[3*Pi]*Erfi[Sq
rt[3]*Sqrt[ArcCosh[a*x]]])/(32*a^5) + (Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcCosh
[a*x]]])/(32*a^5)
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{\cosh^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a^5}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{8\sqrt{x}} + \frac{3\sinh(3x)}{16\sqrt{x}} + \frac{\sinh(5x)}{16\sqrt{x}}\right) dx, x, \cosh^{-1}(ax)\right)}{a^5}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{16a^5} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{8a^5} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{x}}\,dx\right.}{16a^5}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{32a^5} + \frac{\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{32a^5} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \cosh^{-1}\right.}{16a^5}$$

$$= -\frac{\text{Subst}\left(\int e^{-5x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{16a^5} + \frac{\text{Subst}\left(\int e^{5x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{16a^5} - \frac{\text{Subst}\left(\int e^{-x^2}\,dx, x,\right.}{8a^5}$$

$$= -\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^5} - \frac{\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{32a^5} - \frac{\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{32a^5} + \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\right.}}{16}$$

Mathematica [A] time = 0.110387, size = 150, normalized size = 0.92

$$\frac{\frac{\sqrt{5}\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-5\cosh^{-1}(ax)\right)}{\sqrt{\cosh^{-1}(ax)}} + \frac{5\sqrt{3}\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-3\cosh^{-1}(ax)\right)}{\sqrt{\cosh^{-1}(ax)}} + \frac{10\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-\cosh^{-1}(ax)\right)}{\sqrt{\cosh^{-1}(ax)}} + 10\text{Gam}}{160a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] ((Sqrt[5]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -5*ArcCosh[a*x]])/Sqrt[ArcCosh[a*x
]] + (5*Sqrt[3]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -3*ArcCosh[a*x]])/Sqrt[ArcCo
sh[a*x]] + (10*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -ArcCosh[a*x]])/Sqrt[ArcCosh[
a*x]] + 10*Gamma[1/2, ArcCosh[a*x]] + 5*Sqrt[3]*Gamma[1/2, 3*ArcCosh[a*x]]
+ Sqrt[5]*Gamma[1/2, 5*ArcCosh[a*x]])/(160*a^5)
```

Maple [F] time = 0.203, size = 0, normalized size = 0.

$$\int x^4 \frac{1}{\sqrt{\text{arccosh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccosh(a*x)^(1/2),x)
```

```
[Out] int(x^4/arccosh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\operatorname{arcosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\operatorname{acosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.91 $\int \frac{x^3}{\sqrt{\cosh^{-1}(ax)}} dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{\pi}\,\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a^4}-\frac{\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{8a^4}+\frac{\sqrt{\pi}\,\text{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a^4}+\frac{\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{8a^4}$$

```
[Out] -(Sqrt[Pi]*Erf[2*Sqrt[ArcCosh[a*x]]])/(32*a^4) - (Sqrt[Pi/2]*Erf[Sqrt[2]*Sq
rt[ArcCosh[a*x]]])/(8*a^4) + (Sqrt[Pi]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(32*a^4)
 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(8*a^4)
```

Rubi [A] time = 0.146045, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a^4}-\frac{\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{8a^4}+\frac{\sqrt{\pi}\,\text{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a^4}+\frac{\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -(Sqrt[Pi]*Erf[2*Sqrt[ArcCosh[a*x]]])/(32*a^4) - (Sqrt[Pi/2]*Erf[Sqrt[2]*Sq
rt[ArcCosh[a*x]]])/(8*a^4) + (Sqrt[Pi]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(32*a^4)
 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(8*a^4)
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{\cosh^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a^4}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4\sqrt{x}} + \frac{\sinh(4x)}{8\sqrt{x}}\right)dx, x, \cosh^{-1}(ax)\right)}{a^4}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{8a^4} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{4a^4}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{16a^4} + \frac{\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{16a^4} - \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \cosh\right.}{8a^4}$$

$$= -\frac{\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{8a^4} + \frac{\text{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{8a^4} - \frac{\text{Subst}\left(\int e^{-2x^2}\,dx,\right.}{4a}$$

$$= -\frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a^4} - \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{8a^4} + \frac{\sqrt{\pi}\,\text{erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a^4} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{\right.}{}$$

Mathematica [A] time = 0.0858704, size = 101, normalized size = 0.93

$$\frac{\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, -4\cosh^{-1}(ax)\right) + 2\sqrt{2}\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, -2\cosh^{-1}(ax)\right) + \sqrt{\cosh^{-1}(ax)}\left(2\sqrt{2}\text{Ga}\right.}{32a^4\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -4*ArcCosh[a*x]] + 2*Sqrt[2]*Sqrt[-ArcCosh[
a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] + Sqrt[ArcCosh[a*x]]*(2*Sqrt[2]*Gamma[1/2
, 2*ArcCosh[a*x]] + Gamma[1/2, 4*ArcCosh[a*x]]))/(32*a^4*Sqrt[ArcCosh[a*x]]
)
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int x^3 \frac{1}{\sqrt{\text{arccosh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccosh(a*x)^(1/2),x)
```

```
[Out] int(x^3/arccosh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\operatorname{arcosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\operatorname{acosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.92 $\int \frac{x^2}{\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{\pi}\,\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^3}-\frac{\sqrt{\frac{\pi}{3}}\,\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\text{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{8a^3}$$

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(8*a^3) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[
ArcCosh[a*x]]])/(8*a^3) + (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(8*a^3) + (Sq
rt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(8*a^3)
```

Rubi [A] time = 0.139809, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^3}-\frac{\sqrt{\frac{\pi}{3}}\,\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\text{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(8*a^3) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[
ArcCosh[a*x]]])/(8*a^3) + (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(8*a^3) + (Sq
rt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(8*a^3)
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{\cosh^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{4\sqrt{x}} + \frac{\sinh(3x)}{4\sqrt{x}}\right)dx, x, \cosh^{-1}(ax)\right)}{a^3}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{4a^3} + \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{4a^3}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{8a^3} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{8a^3} + \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \cosh^{-1}\right.}{8a^3}$$

$$= -\frac{\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{4a^3} - \frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{4a^3} + \frac{\text{Subst}\left(\int e^{x^2}\,dx, x, \right.}{4a^3}$$

$$= -\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^3} - \frac{\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{8a^3} + \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\sqrt{3}\sqrt{\right.}}{8a^3}$$

Mathematica [A] time = 0.083449, size = 100, normalized size = 0.95

$$\frac{\sqrt{3}\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, -3\cosh^{-1}(ax)\right) + 3\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, -\cosh^{-1}(ax)\right) + \sqrt{\cosh^{-1}(ax)}\left(3\text{Gamma}\right.}{24a^3\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (Sqrt[3]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -3*ArcCosh[a*x]] + 3*Sqrt[-ArcCosh[
a*x]]*Gamma[1/2, -ArcCosh[a*x]] + Sqrt[ArcCosh[a*x]]*(3*Gamma[1/2, ArcCosh[
a*x]] + Sqrt[3]*Gamma[1/2, 3*ArcCosh[a*x]]))/(24*a^3*Sqrt[ArcCosh[a*x]])
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{\text{arccosh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccosh(a*x)^(1/2),x)
```

```
[Out] int(x^2/arccosh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\operatorname{arcosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\operatorname{acosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.93 $\int \frac{x}{\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{\frac{\pi}{2}}\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a^2}-\frac{\sqrt{\frac{\pi}{2}}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a^2}$$

```
[Out] -(Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(4*a^2) + (Sqrt[Pi/2]*Erfi[Sq
rt[2]*Sqrt[ArcCosh[a*x]]])/(4*a^2)
```

Rubi [A] time = 0.0772482, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5670, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a^2}-\frac{\sqrt{\frac{\pi}{2}}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -(Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(4*a^2) + (Sqrt[Pi/2]*Erfi[Sq
rt[2]*Sqrt[ArcCosh[a*x]]])/(4*a^2)
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x}{\sqrt{\cosh^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{2a^2}$$
$$= -\frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{4a^2} + \frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{4a^2}$$
$$= -\frac{\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{2a^2} + \frac{\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{2a^2}$$
$$= -\frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a^2} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a^2}$$

Mathematica [A] time = 0.0357936, size = 49, normalized size = 0.78

$$\frac{\sqrt{\frac{\pi}{2}}\left(\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right) - \text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (Sqrt[Pi/2]*(-Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + Erfi[Sqrt[2]*Sqrt[ArcCosh[a
*x]]]))/(4*a^2)
```

Maple [A] time = 0.053, size = 37, normalized size = 0.6

$$-\frac{\sqrt{\pi}\sqrt{2}}{8\,a^2}\left(\text{Erf}\left(\sqrt{2}\sqrt{\text{arccosh}\,(ax)}\right) - \text{erfi}\left(\sqrt{2}\sqrt{\text{arccosh}\,(ax)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccosh(a*x)^(1/2),x)
```

```
[Out] -1/8*Pi^(1/2)*2^(1/2)*(erf(2^(1/2)*arccosh(a*x)^(1/2))-erfi(2^(1/2)*arccosh
(a*x)^(1/2)))/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\operatorname{arcosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\operatorname{acosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.94 $\int \frac{1}{\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{2a}-\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{2a}$$

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(2*a) + (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x
]]])/(2*a)
```

Rubi [A] time = 0.0466218, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5658, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{2a}-\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(2*a) + (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x
]]])/(2*a)
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\cosh^{-1}(ax)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a}\\
&= -\frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{2a} + \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{2a}\\
&= -\frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a} + \frac{\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a}\\
&= -\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{2a} + \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0282835, size = 45, normalized size = 1.05

$$\frac{\frac{\sqrt{-\cosh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -\cosh^{-1}(ax)\right)}{\sqrt{\cosh^{-1}(ax)}} + \text{Gamma}\left(\frac{1}{2}, \cosh^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] ((Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -ArcCosh[a*x]])/Sqrt[ArcCosh[a*x]] + Gamma
[1/2, ArcCosh[a*x]])/(2*a)
```

Maple [A] time = 0.043, size = 26, normalized size = 0.6

$$-\frac{\sqrt{\pi}}{2\,a}\left(\mathrm{Erf}\left(\sqrt{\mathrm{arccosh}\,(ax)}\right)-\mathrm{erfi}\left(\sqrt{\mathrm{arccosh}\,(ax)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x)^(1/2),x)
```

```
[Out] -1/2*Pi^(1/2)*(erf(arccosh(a*x)^(1/2))-erfi(arccosh(a*x)^(1/2)))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\mathrm{arcosh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{acosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.95 $\int \frac{1}{x\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{\cosh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[ArcCosh[a*x]]), x]
```

Rubi [A] time = 0.0142105, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[ArcCosh[a*x]]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[ArcCosh[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\cosh^{-1}(ax)}}\,dx = \int \frac{1}{x\sqrt{\cosh^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.228137, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[ArcCosh[a*x]]),x]
```

```
[Out] Integrate[1/(x*Sqrt[ArcCosh[a*x]]), x]
```

Maple [A] time = 0.071, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{\operatorname{arccosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccosh(a*x)^(1/2),x)
```

```
[Out] int(1/x/arccosh(a*x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\operatorname{arcosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(arccosh(a*x))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\operatorname{acosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(acosh(a*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.96 $\int \frac{1}{x^2\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{\cosh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/(x^2*Sqrt[ArcCosh[a*x]]), x]
```

Rubi [A] time = 0.0136833, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[ArcCosh[a*x]]),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[ArcCosh[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{\cosh^{-1}(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\cosh^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.616681, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[ArcCosh[a*x]]),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[ArcCosh[a*x]]), x]
```

Maple [A] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt{\operatorname{arccosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccosh(a*x)^(1/2),x)
```

```
[Out] int(1/x^2/arccosh(a*x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\operatorname{arcosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*sqrt(arccosh(a*x))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

<u>**Sympy**</u> **[A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\operatorname{acosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(acosh(a*x))), x)
```

<u>**Giac**</u> **[A]** time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.97 $\int \frac{x^4}{\cosh^{-1}(ax)^{3/2}} dx$

Optimal. Leaf size=193

$$\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^5}+\frac{3\sqrt{3\pi}\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{16a^5}+\frac{\sqrt{5\pi}\text{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{16a^5}+\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^5}+\frac{3\sqrt{3\pi}}{}$$

```
[Out] (-2*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi]*Er
f[Sqrt[ArcCosh[a*x]]])/(8*a^5) + (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x
]]])/(16*a^5) + (Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(16*a^5) + (Sq
rt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(8*a^5) + (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[
ArcCosh[a*x]]])/(16*a^5) + (Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(1
6*a^5)
```

Rubi [A] time = 0.184238, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^5}+\frac{3\sqrt{3\pi}\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{16a^5}+\frac{\sqrt{5\pi}\text{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{16a^5}+\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^5}+\frac{3\sqrt{3\pi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (-2*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi]*Er
f[Sqrt[ArcCosh[a*x]]])/(8*a^5) + (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x
]]])/(16*a^5) + (Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(16*a^5) + (Sq
rt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(8*a^5) + (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[
ArcCosh[a*x]]])/(16*a^5) + (Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(1
6*a^5)
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
```

```
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^4}{\cosh^{-1}(ax)^{3/2}}\,dx = -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} - \frac{2\,\text{Subst}\left(\int\left(-\frac{\cosh(x)}{8\sqrt{x}}-\frac{9\cosh(3x)}{16\sqrt{x}}-\frac{5\cosh(5x)}{16\sqrt{x}}\right)dx, x, \cosh^{-1}(ax)\right)}{a^5}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}dx, x, \cosh^{-1}(ax)\right)}{4a^5} + \frac{5\,\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{x}}dx, x, \cosh^{-1}(\right.}{8a^5}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}dx, x, \cosh^{-1}(ax)\right)}{8a^5} + \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}dx, x, \cosh^{-1}(ax)\right)}{8a^5} + \frac{5}{}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int e^{-x^2}dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{4a^5} + \frac{\text{Subst}\left(\int e^{x^2}dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{4a^5}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^5} + \frac{3\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{16a^5} + \frac{\sqrt{5\pi}\,\text{erf}\left(\sqrt{5}\right.}{16}$$

Mathematica [A] time = 0.29186, size = 201, normalized size = 1.04

$$-\frac{-\sqrt{5}\sqrt{-\cosh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-5\cosh^{-1}(ax)\right)-3\sqrt{3}\sqrt{-\cosh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3\cosh^{-1}(ax)\right)-2\sqrt{-\cosh^{-1}(ax)}\,\text{G}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/ArcCosh[a*x]^(3/2),x]
```

```
[Out] -(4*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) - Sqrt[5]*Sqrt[-ArcCosh[a*x]]*Gamm
a[1/2, -5*ArcCosh[a*x]] - 3*Sqrt[3]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -3*ArcCo
sh[a*x]] - 2*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -ArcCosh[a*x]] + 2*Sqrt[ArcCosh
[a*x]]*Gamma[1/2, ArcCosh[a*x]] + 3*Sqrt[3]*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 3
*ArcCosh[a*x]] + Sqrt[5]*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 5*ArcCosh[a*x]] + 6*
Sinh[3*ArcCosh[a*x]] + 2*Sinh[5*ArcCosh[a*x]])/(16*a^5*Sqrt[ArcCosh[a*x]])
```

Maple [F] time = 0.183, size = 0, normalized size = 0.

$$\int x^4 \left(\operatorname{arccosh}(ax)\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccosh(a*x)^(3/2),x)
```

```
[Out] int(x^4/arccosh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arcosh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{acosh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acosh(a*x)**(3/2),x)
```

```
[Out] Integral(x**4/acosh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.98 $\int \frac{x^3}{\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{\sqrt{\pi}\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a^4}+\frac{\sqrt{\frac{\pi}{2}}\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{2a^4}+\frac{\sqrt{\pi}\mathrm{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a^4}+\frac{\sqrt{\frac{\pi}{2}}\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{2a^4}-\frac{2x^3\cdots}{a\cdots}$$

```
[Out] (-2*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi]*Er
f[2*Sqrt[ArcCosh[a*x]]])/(4*a^4) + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x
]]])/(2*a^4) + (Sqrt[Pi]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(4*a^4) + (Sqrt[Pi/2]*
Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(2*a^4)
```

Rubi [A] time = 0.128221, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a^4}+\frac{\sqrt{\frac{\pi}{2}}\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{2a^4}+\frac{\sqrt{\pi}\mathrm{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a^4}+\frac{\sqrt{\frac{\pi}{2}}\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{2a^4}-\frac{2x^3\cdots}{a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (-2*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi]*Er
f[2*Sqrt[ArcCosh[a*x]]])/(4*a^4) + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x
]]])/(2*a^4) + (Sqrt[Pi]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(4*a^4) + (Sqrt[Pi/2]*
Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(2*a^4)
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^3}{\cosh^{-1}(ax)^{3/2}}\,dx = -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} - \frac{2\,\text{Subst}\left(\int \left(-\frac{\cosh(2x)}{2\sqrt{x}} - \frac{\cosh(4x)}{2\sqrt{x}}\right)dx, x, \cosh^{-1}(ax)\right)}{a^4}$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a^4} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \cosh^{-1}\right.}{a^4}$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{2a^4} + \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{2a^4} +$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a^4} + \frac{\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\cosh^{-1}(\right.}{a^4}$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a^4} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{2a^4} + \frac{\sqrt{\pi}\,\text{erfi}\left(2\sqrt{c\right.}{4a}$$

Mathematica [A] time = 0.159266, size = 124, normalized size = 0.87

$$-\frac{-\sqrt{-\cosh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -4\cosh^{-1}(ax)\right) - \sqrt{2}\sqrt{-\cosh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -2\cosh^{-1}(ax)\right) + \sqrt{2}\sqrt{\cosh^{-1}(ax)}\,\text{Ga}}{4a^4\sqrt{\cosh}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/ArcCosh[a*x]^(3/2),x]
```

```
[Out] -(-(Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -4*ArcCosh[a*x]]) - Sqrt[2]*Sqrt[-ArcCos
h[a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] + Sqrt[2]*Sqrt[ArcCosh[a*x]]*Gamma[1/2,
 2*ArcCosh[a*x]] + Sqrt[ArcCosh[a*x]]*Gamma[1/2, 4*ArcCosh[a*x]] + 2*Sinh[2
*ArcCosh[a*x]] + Sinh[4*ArcCosh[a*x]])/(4*a^4*Sqrt[ArcCosh[a*x]])
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int x^3 \left(\text{arccosh}\,(ax)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccosh(a*x)^(3/2),x)
```

```
[Out] int(x^3/arccosh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arcosh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acosh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acosh(a*x)**(3/2),x)
```

```
[Out] Integral(x**3/acosh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.99 $\int \frac{x^2}{\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a^3}+\frac{\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{4a^3}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a^3}+\frac{\sqrt{3\pi}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{4a^3}-\frac{2x^2\sqrt{}}{a\sqrt{}}$$

```
[Out] (-2*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi]*Er
f[Sqrt[ArcCosh[a*x]]])/(4*a^3) + (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]
])/(4*a^3) + (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(4*a^3) + (Sqrt[3*Pi]*Erfi
[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(4*a^3)
```

Rubi [A] time = 0.124145, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a^3}+\frac{\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{4a^3}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a^3}+\frac{\sqrt{3\pi}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{4a^3}-\frac{2x^2\sqrt{}}{a\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (-2*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi]*Er
f[Sqrt[ArcCosh[a*x]]])/(4*a^3) + (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]
])/(4*a^3) + (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(4*a^3) + (Sqrt[3*Pi]*Erfi
[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(4*a^3)
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^2}{\cosh^{-1}(ax)^{3/2}}\,dx = -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} - \frac{2\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{x}} - \frac{3\cosh(3x)}{4\sqrt{x}}\right)dx, x, \cosh^{-1}(ax)\right)}{a^3}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}dx, x, \cosh^{-1}(ax)\right)}{2a^3} + \frac{3\,\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}}dx, x, \cosh^{-1}\right.}{2a^3}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}dx, x, \cosh^{-1}(ax)\right)}{4a^3} + \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}dx, x, \cosh^{-1}(ax)\right)}{4a^3} + 3$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int e^{-x^2}dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{2a^3} + \frac{\text{Subst}\left(\int e^{x^2}dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{2a^3}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a^3} + \frac{\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{4a^3} + \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\cosh\right.}}{4a^3}$$

Mathematica [A] time = 0.166039, size = 139, normalized size = 1.03

$$-\frac{-\sqrt{3}\sqrt{-\cosh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -3\cosh^{-1}(ax)\right) - \sqrt{-\cosh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -\cosh^{-1}(ax)\right) + \sqrt{\cosh^{-1}(ax)}\,\text{Gamma}}{4a^3\sqrt{\cosh^{-1}(ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/ArcCosh[a*x]^(3/2),x]
```

```
[Out] -(2*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) - Sqrt[3]*Sqrt[-ArcCosh[a*x]]*Gamm
a[1/2, -3*ArcCosh[a*x]] - Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -ArcCosh[a*x]] + S
qrt[ArcCosh[a*x]]*Gamma[1/2, ArcCosh[a*x]] + Sqrt[3]*Sqrt[ArcCosh[a*x]]*Gam
ma[1/2, 3*ArcCosh[a*x]] + 2*Sinh[3*ArcCosh[a*x]])/(4*a^3*Sqrt[ArcCosh[a*x]]
)
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int x^2 \left(\text{arccosh}\,(ax)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccosh(a*x)^(3/2),x)
```

```
[Out] int(x^2/arccosh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arcosh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{acosh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acosh(a*x)**(3/2),x)
```

```
[Out] Integral(x**2/acosh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.100 $\int \frac{x}{\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a^2}+\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a^2}-\frac{2x\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{\cosh^{-1}(ax)}}$$

```
[Out] (-2*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi/2]*Er
f[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/a^2 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[
a*x]]])/a^2
```

Rubi [A] time = 0.0658254, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a^2}+\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a^2}-\frac{2x\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (-2*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi/2]*Er
f[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/a^2 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[
a*x]]])/a^2
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\cosh^{-1}(ax)^{3/2}}\,dx &= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}}+\frac{2\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx,x,\cosh^{-1}(ax)\right)}{a^2}\\
&= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}}+\frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx,x,\cosh^{-1}(ax)\right)}{a^2}+\frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx,x,\cosh^{-1}(ax)\right)}{a^2}\\
&= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}}+\frac{2\,\text{Subst}\left(\int e^{-2x^2}\,dx,x,\sqrt{\cosh^{-1}(ax)}\right)}{a^2}+\frac{2\,\text{Subst}\left(\int e^{2x^2}\,dx,x,\sqrt{\cosh^{-1}(ax)}\right)}{a^2}\\
&= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}}+\frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a^2}+\frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.10018, size = 63, normalized size = 0.71

$$\frac{\sqrt{\frac{\pi}{2}}\left(\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)\right)-\frac{\sinh\left(2\cosh^{-1}(ax)\right)}{\sqrt{\cosh^{-1}(ax)}}}{a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[Pi/2]*(Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + Erfi[Sqrt[2]*Sqrt[ArcCosh[a*
x]]]) - Sinh[2*ArcCosh[a*x]]/Sqrt[ArcCosh[a*x]])/a^2
```

Maple [A] time = 0.111, size = 83, normalized size = 0.9

$$\frac{\sqrt{2}}{2\sqrt{\pi}a^2\text{arccosh}(ax)}\left(-2\sqrt{2}\sqrt{\text{arccosh}(ax)}\sqrt{\pi}\sqrt{ax+1}\sqrt{ax-1}xa+\text{arccosh}(ax)\pi\,\text{Erf}\left(\sqrt{2}\sqrt{\text{arccosh}(ax)}\right)+\text{arccosh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccosh(a*x)^(3/2),x)
```

```
[Out] 1/2*2^(1/2)*(-2*2^(1/2)*arccosh(a*x)^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(
1/2)*x*a+arccosh(a*x)*Pi*erf(2^(1/2)*arccosh(a*x)^(1/2))+arccosh(a*x)*Pi*er
fi(2^(1/2)*arccosh(a*x)^(1/2)))/Pi^(1/2)/a^2/arccosh(a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\text{arcosh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{acosh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acosh(a*x)**(3/2),x)
```

```
[Out] Integral(x/acosh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.101 $\int \frac{1}{\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{a}+\frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{a}-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{\cosh^{-1}(ax)}}$$

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi]*Erf[Sq
rt[ArcCosh[a*x]]])/a + (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/a
```

Rubi [A] time = 0.217745, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5656, 5781, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{a}+\frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{a}-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^(-3/2),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi]*Erf[Sq
rt[ArcCosh[a*x]]])/a + (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/a
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
```

```
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\cosh^{-1}(ax)^{3/2}}\,dx = -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + (2a)\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}\,dx$$

$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{2\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a}$$

$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a} + \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a}$$

$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{2\,\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a} + \frac{2\,\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\cosh^{-1}(a\right.}}{a}$$

$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\sqrt{\pi}\,\textbf{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{a} + \frac{\sqrt{\pi}\,\textbf{erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{a}$$

Mathematica [A] time = 0.0604272, size = 76, normalized size = 1.12

$$\frac{\sqrt{-\cosh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -\cosh^{-1}(ax)\right) - \sqrt{\cosh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, \cosh^{-1}(ax)\right) - 2\sqrt{\frac{ax-1}{ax+1}}(ax+1)}{a\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^(-3/2),x]
```

```
[Out] (-2*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) + Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -
ArcCosh[a*x]] - Sqrt[ArcCosh[a*x]]*Gamma[1/2, ArcCosh[a*x]])/(a*Sqrt[ArcCos
h[a*x]])
```

Maple [A] time = 0.095, size = 66, normalized size = 1.

$$\frac{1}{\sqrt{\pi}a\text{arccosh}\,(ax)}\left(-2\sqrt{\text{arccosh}\,(ax)}\sqrt{\pi}\sqrt{ax+1}\sqrt{ax-1} + \text{arccosh}\,(ax)\,\pi\,\text{Erf}\left(\sqrt{\text{arccosh}\,(ax)}\right) + \text{arccosh}\,(ax)\,\pi\,\text{erfi}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x)^(3/2),x)
```

```
[Out] (-2*arccosh(a*x)^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)+arccosh(a*x)*Pi
*erf(arccosh(a*x)^(1/2))+arccosh(a*x)*Pi*erfi(arccosh(a*x)^(1/2)))/Pi^(1/2)
/a/arccosh(a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arcosh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acosh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)**(3/2),x)
```

```
[Out] Integral(acosh(a*x)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.102 $\int \frac{1}{x \cosh^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x \cosh^{-1}(ax)^{3/2}}, x\right)$$

[Out] Unintegrable[1/(x*ArcCosh[a*x]^(3/2)), x]

Rubi [A] time = 0.0136584, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \cosh^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*ArcCosh[a*x]^(3/2)),x]

[Out] Defer[Int][1/(x*ArcCosh[a*x]^(3/2)), x]

Rubi steps

$$\int \frac{1}{x \cosh^{-1}(ax)^{3/2}} \, dx = \int \frac{1}{x \cosh^{-1}(ax)^{3/2}} \, dx$$

Mathematica [A] time = 0.340927, size = 0, normalized size = 0.

$$\int \frac{1}{x \cosh^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*ArcCosh[a*x]^(3/2)),x]

[Out] Integrate[1/(x*ArcCosh[a*x]^(3/2)), x]

Maple [A] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\operatorname{arccosh}(ax))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccosh(a*x)^(3/2),x)
```

```
[Out] int(1/x/arccosh(a*x)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arcosh}(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arccosh(a*x)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{acosh}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acosh(a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*acosh(a*x)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.103 $\int \frac{x^4}{\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=228

$$-\frac{\sqrt{\pi}\,\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{12a^5}-\frac{3\sqrt{3\pi}\,\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{8a^5}-\frac{5\sqrt{5\pi}\,\text{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{24a^5}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{12a^5}+\cdots$$

```
[Out] (-2*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^(3/2)) + (16*x^3)/(
3*a^2*Sqrt[ArcCosh[a*x]]) - (20*x^5)/(3*Sqrt[ArcCosh[a*x]]) - (Sqrt[Pi]*Erf
[Sqrt[ArcCosh[a*x]]])/(12*a^5) - (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x
]]])/(8*a^5) - (5*Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(24*a^5) + (S
qrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(12*a^5) + (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqr
t[ArcCosh[a*x]]])/(8*a^5) + (5*Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ArcCosh[a*x]]])
/(24*a^5)
```

Rubi [A] time = 0.864105, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5668, 5775, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,\text{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{12a^5}-\frac{3\sqrt{3\pi}\,\text{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{8a^5}-\frac{5\sqrt{5\pi}\,\text{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{24a^5}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{12a^5}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCosh[a*x]^(5/2),x]
```

```
[Out] (-2*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^(3/2)) + (16*x^3)/(
3*a^2*Sqrt[ArcCosh[a*x]]) - (20*x^5)/(3*Sqrt[ArcCosh[a*x]]) - (Sqrt[Pi]*Erf
[Sqrt[ArcCosh[a*x]]])/(12*a^5) - (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x
]]])/(8*a^5) - (5*Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(24*a^5) + (S
qrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(12*a^5) + (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqr
t[ArcCosh[a*x]]])/(8*a^5) + (5*Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ArcCosh[a*x]]])
/(24*a^5)
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
```

```
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^4}{\cosh^{-1}(ax)^{5/2}}\,dx = -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{8\int \frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}\,dx}{3a} + \frac{1}{3}(10a)\int \frac{x^5}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} + \frac{100}{3}\int \frac{x^4}{\sqrt{\cosh^{-1}(ax)}}\,dx - \frac{16\int}{}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} - \frac{16\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{x}}\,dx, x,\right.}{a^5}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} - \frac{16\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4\sqrt{x}} + \frac{\sinh(3x)}{4\sqrt{x}}\right)\right.}{a^5}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} + \frac{25\,\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{x}}\,dx, x, \cosh^{}\right.}{12a^5}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} - \frac{25\,\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(a\right.}{24a^5}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} - \frac{25\,\text{Subst}\left(\int e^{-5x^2}\,dx, x, \sqrt{\cosh^{}}\right.}{12a^5}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} - \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{12a^5} - \frac{3\sqrt{3\pi}}{}$$

Mathematica [A] time = 1.5425, size = 278, normalized size = 1.22

$$-\frac{2\left(-\cosh^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2}, -\cosh^{-1}(ax)\right) - 2\cosh^{-1}(ax)^{3/2}\text{Gamma}\left(\frac{1}{2}, \cosh^{-1}(ax)\right) + 5\cosh^{-1}(ax)\left(-\sqrt{5}\sqrt{-\cosh}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/ArcCosh[a*x]^(5/2),x]
```

```
[Out] -(2*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) + (2*ArcCosh[a*x])/E^ArcCosh[a*x]
+ 2*E^ArcCosh[a*x]*ArcCosh[a*x] + 2*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -ArcCo
sh[a*x]] - 2*ArcCosh[a*x]^(3/2)*Gamma[1/2, ArcCosh[a*x]] + 5*ArcCosh[a*x]*(
E^(-5*ArcCosh[a*x]) + E^(5*ArcCosh[a*x]) - Sqrt[5]*Sqrt[-ArcCosh[a*x]]*Gamm
a[1/2, -5*ArcCosh[a*x]] - Sqrt[5]*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 5*ArcCosh[a
*x]]) + 3*((3*ArcCosh[a*x])/E^(3*ArcCosh[a*x]) + 3*E^(3*ArcCosh[a*x])*ArcCo
sh[a*x] + 3*Sqrt[3]*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -3*ArcCosh[a*x]] - 3*S
qrt[3]*ArcCosh[a*x]^(3/2)*Gamma[1/2, 3*ArcCosh[a*x]] + Sinh[3*ArcCosh[a*x]]
) + Sinh[5*ArcCosh[a*x]])/(24*a^5*ArcCosh[a*x]^(3/2))
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int x^{4} \left(\operatorname{arccosh}\left(a x\right)\right)^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccosh(a*x)^(5/2),x)
```

```
[Out] int(x^4/arccosh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\operatorname{arcosh}\left(a x\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{acosh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acosh(a*x)**(5/2),x)
```

```
[Out] Integral(x**4/acosh(a*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.104 $\int \frac{x^3}{\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{2\sqrt{\pi}\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{3a^4}-\frac{\sqrt{2\pi}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^4}+\frac{2\sqrt{\pi}\text{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{3a^4}+\frac{\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^4}+$$

```
[Out] (-2*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^(3/2)) + (4*x^2)/(a
^2*Sqrt[ArcCosh[a*x]]) - (16*x^4)/(3*Sqrt[ArcCosh[a*x]]) - (2*Sqrt[Pi]*Erf[
2*Sqrt[ArcCosh[a*x]]])/(3*a^4) - (Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]
])/(3*a^4) + (2*Sqrt[Pi]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(3*a^4) + (Sqrt[2*Pi]*
Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(3*a^4)
```

Rubi [A] time = 0.755236, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5668, 5775, 5670, 5448, 3308, 2180, 2204, 2205, 12}

$$-\frac{2\sqrt{\pi}\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{3a^4}-\frac{\sqrt{2\pi}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^4}+\frac{2\sqrt{\pi}\text{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{3a^4}+\frac{\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^4}+$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCosh[a*x]^(5/2),x]
```


```
[Out] (-2*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^(3/2)) + (4*x^2)/(a
^2*Sqrt[ArcCosh[a*x]]) - (16*x^4)/(3*Sqrt[ArcCosh[a*x]]) - (2*Sqrt[Pi]*Erf[
2*Sqrt[ArcCosh[a*x]]])/(3*a^4) - (Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]
])/(3*a^4) + (2*Sqrt[Pi]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(3*a^4) + (Sqrt[2*Pi]*
Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(3*a^4)
```


Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
```

```
x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{x^3}{\cosh^{-1}(ax)^{5/2}}\,dx = -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{2\int \frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}\,dx}{a} + \frac{1}{3}(8a)\int \frac{x^4}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4x^2}{a^2\sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\cosh^{-1}(ax)}} + \frac{64}{3}\int \frac{x^3}{\sqrt{\cosh^{-1}(ax)}}\,dx - \frac{8\int \frac{}{\sqrt{\cosh}}}{a}$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4x^2}{a^2\sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\cosh^{-1}(ax)}} - \frac{8\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \cosh\right.}{a^4}$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4x^2}{a^2\sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\cosh^{-1}(ax)}} - \frac{8\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \cosh^{-1}(a\right.}{a^4}$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4x^2}{a^2\sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\cosh^{-1}(ax)}} + \frac{8\,\mathrm{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(a\right.}{3a^4}$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4x^2}{a^2\sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\cosh^{-1}(ax)}} - \frac{4\,\mathrm{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{3a^4}$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4x^2}{a^2\sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\cosh^{-1}(ax)}} - \frac{8\,\mathrm{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\cosh^{-1}(a}\right.}{3a^4}$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4x^2}{a^2\sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2\sqrt{\pi}\,\mathrm{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{3a^4} - \frac{\sqrt{2\pi}\,\mathrm{er}}{}$$

Mathematica [A] time = 0.758529, size = 175, normalized size = 1.02

$$\frac{-4\cosh^{-1}(ax)\left(-2\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-4\cosh^{-1}(ax)\right)-2\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},4\cosh^{-1}(ax)\right)+e^{-4\cosh^{-1}(a}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/ArcCosh[a*x]^(5/2),x]
```

```
[Out] (-4*ArcCosh[a*x]*(E^(-4*ArcCosh[a*x]) + E^(4*ArcCosh[a*x]) - 2*Sqrt[-ArcCos
h[a*x]]*Gamma[1/2, -4*ArcCosh[a*x]] - 2*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 4*Arc
Cosh[a*x]]) - 2*(2*ArcCosh[a*x]*(E^(-2*ArcCosh[a*x]) + E^(2*ArcCosh[a*x]) -
 Sqrt[2]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] - Sqrt[2]*Sqrt[Arc
Cosh[a*x]]*Gamma[1/2, 2*ArcCosh[a*x]]) + Sinh[2*ArcCosh[a*x]]) - Sinh[4*Arc
Cosh[a*x]])/(12*a^4*ArcCosh[a*x]^(3/2))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int x^3 (\operatorname{arccosh}(ax))^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccosh(a*x)^(5/2),x)
```

```
[Out] int(x^3/arccosh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arcosh}(ax)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acosh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acosh(a*x)**(5/2),x)
```

```
[Out] Integral(x**3/acosh(a*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.105 $\int \frac{x^2}{\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{6a^3}-\frac{\sqrt{3\pi}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{2a^3}+\frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{6a^3}+\frac{\sqrt{3\pi}\operatorname{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{2a^3}+\frac{}{3a}$$

```
[Out] (-2*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^(3/2)) + (8*x)/(3*a
^2*Sqrt[ArcCosh[a*x]]) - (4*x^3)/Sqrt[ArcCosh[a*x]] - (Sqrt[Pi]*Erf[Sqrt[Ar
cCosh[a*x]]])/(6*a^3) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(2*a^3
) + (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(6*a^3) + (Sqrt[3*Pi]*Erfi[Sqrt[3]*
Sqrt[ArcCosh[a*x]]])/(2*a^3)
```

Rubi [A] time = 0.626936, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5668, 5775, 5670, 5448, 3308, 2180, 2204, 2205, 5658}

$$-\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{6a^3}-\frac{\sqrt{3\pi}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{2a^3}+\frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{6a^3}+\frac{\sqrt{3\pi}\operatorname{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{2a^3}+\frac{}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCosh[a*x]^(5/2),x]
```


```
[Out] (-2*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^(3/2)) + (8*x)/(3*a
^2*Sqrt[ArcCosh[a*x]]) - (4*x^3)/Sqrt[ArcCosh[a*x]] - (Sqrt[Pi]*Erf[Sqrt[Ar
cCosh[a*x]]])/(6*a^3) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(2*a^3
) + (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(6*a^3) + (Sqrt[3*Pi]*Erfi[Sqrt[3]*
Sqrt[ArcCosh[a*x]]])/(2*a^3)
```


Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
```

```
x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rubi steps

$$\int \frac{x^2}{\cosh^{-1}(ax)^{5/2}}\,dx = -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{4\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}\,dx}{3a} + (2a)\int \frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} + 12\int \frac{x^2}{\sqrt{\cosh^{-1}(ax)}}\,dx - \frac{8\int \frac{}{\sqrt{\cos}}}{3}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} - \frac{8\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(a\right.}{3a^3}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} + \frac{4\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{3a^3}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} + \frac{8\,\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\cosh^{-1}(a}\right.}{3a^3}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} + \frac{4\sqrt{\pi}\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a^3} - \frac{4\sqrt{\pi}\text{e}}{}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} + \frac{4\sqrt{\pi}\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a^3} - \frac{4\sqrt{\pi}\text{e}}{}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} - \frac{\sqrt{\pi}\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{6a^3} - \frac{\sqrt{3\pi}\text{erf}}{}$$

Mathematica [A] time = 0.659596, size = 194, normalized size = 1.17

$$-3\sqrt{3}\left(-\cosh^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-3\cosh^{-1}(ax)\right)-\left(-\cosh^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\cosh^{-1}(ax)\right)+\cosh^{-1}(ax)^{3/2}\text{Ga}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/ArcCosh[a*x]^(5/2),x]
```

```
[Out] (-(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)) - (3*ArcCosh[a*x])/E^(3*ArcCosh[a*
x]) - ArcCosh[a*x]/E^ArcCosh[a*x] - E^ArcCosh[a*x]*ArcCosh[a*x] - 3*E^(3*Ar
cCosh[a*x])*ArcCosh[a*x] - 3*Sqrt[3]*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -3*Ar
cCosh[a*x]] - (-ArcCosh[a*x])^(3/2)*Gamma[1/2, -ArcCosh[a*x]] + ArcCosh[a*x
]^(3/2)*Gamma[1/2, ArcCosh[a*x]] + 3*Sqrt[3]*ArcCosh[a*x]^(3/2)*Gamma[1/2,
3*ArcCosh[a*x]] - Sinh[3*ArcCosh[a*x]])/(6*a^3*ArcCosh[a*x]^(3/2))
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int x^2 (\operatorname{arccosh}(ax))^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccosh(a*x)^(5/2),x)
```

```
[Out] int(x^2/arccosh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arcosh}(ax)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{acosh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acosh(a*x)**(5/2),x)
```

```
[Out] Integral(x**2/acosh(a*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.106 $\int \frac{x}{\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{2\sqrt{2\pi}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^2}+\frac{2\sqrt{2\pi}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^2}+\frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}}-\frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}}-\frac{2x\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)^{3/2}}$$

```
[Out] (-2*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^(3/2)) + 4/(3*a^2*Sqr
t[ArcCosh[a*x]]) - (8*x^2)/(3*Sqrt[ArcCosh[a*x]]) - (2*Sqrt[2*Pi]*Erf[Sqrt[
2]*Sqrt[ArcCosh[a*x]]])/(3*a^2) + (2*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a
*x]]])/(3*a^2)
```

Rubi [A] time = 0.482776, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5668, 5775, 5670, 5448, 12, 3308, 2180, 2204, 2205, 5676}

$$-\frac{2\sqrt{2\pi}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^2}+\frac{2\sqrt{2\pi}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^2}+\frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}}-\frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}}-\frac{2x\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCosh[a*x]^(5/2),x]
```

```
[Out] (-2*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^(3/2)) + 4/(3*a^2*Sqr
t[ArcCosh[a*x]]) - (8*x^2)/(3*Sqrt[ArcCosh[a*x]]) - (2*Sqrt[2*Pi]*Erf[Sqrt[
2]*Sqrt[ArcCosh[a*x]]])/(3*a^2) + (2*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a
*x]]])/(3*a^2)
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{x}{\cosh^{-1}(ax)^{5/2}}\,dx = -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{2\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}\,dx}{3a} + \frac{1}{3}(4a)\int \frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3}$$

$$= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} + \frac{16}{3}\int \frac{x}{\sqrt{\cosh^{-1}(ax)}}\,dx$$

$$= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} + \frac{16\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \cos}{3a^2}$$

$$= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} + \frac{16\,\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \cosh^{-1}(}{3a^2}$$

$$= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} + \frac{8\,\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(a}{3a^2}$$

$$= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} - \frac{4\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{3a^2}$$

$$= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} - \frac{8\,\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\cosh^{-1}(a}{3a^2}$$

$$= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2\sqrt{2\pi}\,\text{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^2} + \frac{2\sqrt{}}{}$$

Mathematica [A] time = 0.255095, size = 83, normalized size = 0.67

$$-\frac{2\sqrt{2\pi}\left(\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)-\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)\right)+\frac{4\cosh\left(2\cosh^{-1}(ax)\right)}{\sqrt{\cosh^{-1}(ax)}}+\frac{\sinh\left(2\cosh^{-1}(ax)\right)}{\cosh^{-1}(ax)^{3/2}}}{3a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/ArcCosh[a*x]^(5/2),x]
```

```
[Out] -((4*Cosh[2*ArcCosh[a*x]])/Sqrt[ArcCosh[a*x]] + 2*Sqrt[2*Pi]*(Erf[Sqrt[2]*S
qrt[ArcCosh[a*x]]] - Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]]) + Sinh[2*ArcCosh[a*x
]]/ArcCosh[a*x]^(3/2))/(3*a^2)
```

Maple [A] time = 0.113, size = 122, normalized size = 1.

$$-\frac{\sqrt{2}}{3\sqrt{\pi}a^2\left(\text{arccosh}\left(ax\right)\right)^2}\left(4\left(\text{arccosh}\left(ax\right)\right)^{3/2}\sqrt{2}\sqrt{\pi}x^2a^2+\sqrt{2}\sqrt{\text{arccosh}\left(ax\right)}\sqrt{\pi}\sqrt{ax+1}\sqrt{ax-1}xa+2\left(\text{arccosh}\left(ax\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccosh(a*x)^(5/2),x)
```

```
[Out] -1/3*2^(1/2)*(4*arccosh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*x^2*a^2+2^(1/2)*arccosh
(a*x)^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x*a+2*arccosh(a*x)^2*Pi*er
f(2^(1/2)*arccosh(a*x)^(1/2))-2*arccosh(a*x)^2*Pi*erfi(2^(1/2)*arccosh(a*x)
^(1/2))-2*arccosh(a*x)^(3/2)*2^(1/2)*Pi^(1/2))/Pi^(1/2)/a^2/arccosh(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\text{arcosh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{acosh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acosh(a*x)**(5/2),x)
```

```
[Out] Integral(x/acosh(a*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.107 $\int \frac{1}{\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a}+\frac{2\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a}-\frac{4x}{3\sqrt{\cosh^{-1}(ax)}}-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)^{3/2}}$$

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^(3/2)) - (4*x)/(3*Sqrt[
ArcCosh[a*x]]) - (2*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(3*a) + (2*Sqrt[Pi]*E
rfi[Sqrt[ArcCosh[a*x]]])/(3*a)
```

Rubi [A] time = 0.234149, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5656, 5775, 5658, 3308, 2180, 2204, 2205}

$$-\frac{2\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a}+\frac{2\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a}-\frac{4x}{3\sqrt{\cosh^{-1}(ax)}}-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^(-5/2),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]^(3/2)) - (4*x)/(3*Sqrt[
ArcCosh[a*x]]) - (2*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(3*a) + (2*Sqrt[Pi]*E
rfi[Sqrt[ArcCosh[a*x]]])/(3*a)
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
```

```
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\cosh^{-1}(ax)^{5/2}}\,dx = -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{1}{3}(2a)\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}\,dx$$

$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\cosh^{-1}(ax)}} + \frac{4}{3}\int \frac{1}{\sqrt{\cosh^{-1}(ax)}}\,dx$$

$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\cosh^{-1}(ax)}} + \frac{4\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{3a}$$

$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{3a} + \frac{2\,\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx,\right.}{3a}$$

$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\cosh^{-1}(ax)}} - \frac{4\,\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{3a} + \frac{4\,\text{Subst}\left(\int e^{x^2}\right.}{}$$

$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2\sqrt{\pi}\,\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a} + \frac{2\sqrt{\pi}\,\text{erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a}$$

Mathematica [A] time = 0.150791, size = 121, normalized size = 1.36

$$-\frac{2e^{-\cosh^{-1}(ax)}\left(e^{\cosh^{-1}(ax)}\left(-\cosh^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2}, -\cosh^{-1}(ax)\right) - e^{\cosh^{-1}(ax)}\cosh^{-1}(ax)^{3/2}\text{Gamma}\left(\frac{1}{2}, \cosh^{-1}(a\right.\right.}{3a\cosh^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^(-5/2),x]
```

```
[Out] (-2*(E^ArcCosh[a*x]*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) + ArcCosh[a*x] + E
^(2*ArcCosh[a*x])*ArcCosh[a*x] + E^ArcCosh[a*x]*(-ArcCosh[a*x])^(3/2)*Gamma
[1/2, -ArcCosh[a*x]] - E^ArcCosh[a*x]*ArcCosh[a*x]^(3/2)*Gamma[1/2, ArcCosh
[a*x]]))/(3*a*E^ArcCosh[a*x]*ArcCosh[a*x]^(3/2))
```

Maple [A] time = 0.106, size = 84, normalized size = 0.9

$$-\frac{2}{3\sqrt{\pi}a\left(\text{arccosh}\,(ax)\right)^2}\left(2\,\left(\text{arccosh}\,(ax)\right)^{3/2}\sqrt{\pi}xa + \left(\text{arccosh}\,(ax)\right)^2\pi\,\text{Erf}\left(\sqrt{\text{arccosh}\,(ax)}\right) - \left(\text{arccosh}\,(ax)\right)^2\pi\,\text{erfi}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x)^(5/2),x)
```

```
[Out] -2/3*(2*arccosh(a*x)^(3/2)*Pi^(1/2)*x*a+arccosh(a*x)^2*Pi*erf(arccosh(a*x)^
(1/2))-arccosh(a*x)^2*Pi*erfi(arccosh(a*x)^(1/2))+arccosh(a*x)^(1/2)*Pi^(1/
2)*(a*x+1)^(1/2)*(a*x-1)^(1/2))/Pi^(1/2)/a/arccosh(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arcosh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acosh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)**(5/2),x)
```

```
[Out] Integral(acosh(a*x)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.108 $\int \frac{1}{x \cosh^{-1}(ax)^{5/2}} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x \cosh^{-1}(ax)^{5/2}}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcCosh[a*x]^(5/2)), x]
```

Rubi [A] time = 0.0148498, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \cosh^{-1}(ax)^{5/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][1/(x*ArcCosh[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{x \cosh^{-1}(ax)^{5/2}} \, dx = \int \frac{1}{x \cosh^{-1}(ax)^{5/2}} \, dx$$

Mathematica [A] time = 0.342992, size = 0, normalized size = 0.

$$\int \frac{1}{x \cosh^{-1}(ax)^{5/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x*ArcCosh[a*x]^(5/2)), x]
```

Maple [A] time = 0.067, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\operatorname{arccosh}(ax))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccosh(a*x)^(5/2),x)
```

```
[Out] int(1/x/arccosh(a*x)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arcosh}(ax)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arccosh(a*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{acosh}^{\frac{5}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acosh(a*x)**(5/2),x)
```

```
[Out] Integral(1/(x*acosh(a*x)**(5/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.109 $\int \frac{x^4}{\cosh^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=300

$$\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{30a^5}+\frac{9\sqrt{3\pi}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{20a^5}+\frac{5\sqrt{5\pi}\operatorname{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{12a^5}+\frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{30a^5}+\frac{9\cdots}{}$$

```
[Out] (-2*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(5*a*ArcCosh[a*x]^(5/2)) + (16*x^3)/(
15*a^2*ArcCosh[a*x]^(3/2)) - (4*x^5)/(3*ArcCosh[a*x]^(3/2)) + (32*x^2*Sqrt[
-1 + a*x]*Sqrt[1 + a*x])/(5*a^3*Sqrt[ArcCosh[a*x]]) - (40*x^4*Sqrt[-1 + a*x
]*Sqrt[1 + a*x])/(3*a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]
]])/(30*a^5) + (9*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(20*a^5) + (5
*Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(12*a^5) + (Sqrt[Pi]*Erfi[Sqrt
[ArcCosh[a*x]]])/(30*a^5) + (9*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])
/(20*a^5) + (5*Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(12*a^5)
```

Rubi [A] time = 0.931954, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5668, 5775, 5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{30a^5}+\frac{9\sqrt{3\pi}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{20a^5}+\frac{5\sqrt{5\pi}\operatorname{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{12a^5}+\frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{30a^5}+\frac{9\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCosh[a*x]^(7/2),x]
```

```
[Out] (-2*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(5*a*ArcCosh[a*x]^(5/2)) + (16*x^3)/(
15*a^2*ArcCosh[a*x]^(3/2)) - (4*x^5)/(3*ArcCosh[a*x]^(3/2)) + (32*x^2*Sqrt[
-1 + a*x]*Sqrt[1 + a*x])/(5*a^3*Sqrt[ArcCosh[a*x]]) - (40*x^4*Sqrt[-1 + a*x
]*Sqrt[1 + a*x])/(3*a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]
]])/(30*a^5) + (9*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(20*a^5) + (5
*Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(12*a^5) + (Sqrt[Pi]*Erfi[Sqrt
[ArcCosh[a*x]]])/(30*a^5) + (9*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])
/(20*a^5) + (5*Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(12*a^5)
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^4}{\cosh^{-1}(ax)^{7/2}}\,dx = -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} - \frac{8\int \frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}}\,dx}{5a} + (2a)\int \frac{x^5}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{16x^3}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\cosh^{-1}(ax)^{3/2}} + \frac{20}{3}\int \frac{x^4}{\cosh^{-1}(ax)^{3/2}}\,dx - \frac{16}{}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{16x^3}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\cosh^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} - \frac{40x}{}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{16x^3}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\cosh^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} - \frac{40x}{}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{16x^3}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\cosh^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} - \frac{40x}{}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{16x^3}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\cosh^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} - \frac{40x}{}$$

$$= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{16x^3}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\cosh^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} - \frac{40x}{}$$

Mathematica [A] time = 1.98824, size = 374, normalized size = 1.25

$$\frac{4\left(-\cosh^{-1}(ax)\right)^{5/2}\text{Gamma}\left(\frac{1}{2},-\cosh^{-1}(ax)\right) - 4\cosh^{-1}(ax)^{5/2}\text{Gamma}\left(\frac{1}{2},\cosh^{-1}(ax)\right) - 5\cosh^{-1}(ax)\left(10\sqrt{5}\left(-\cos\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/ArcCosh[a*x]^(7/2),x]
```

```
[Out] (-6*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) - (2*ArcCosh[a*x])/E^ArcCosh[a*x]
- 2*E^ArcCosh[a*x]*ArcCosh[a*x] + (4*ArcCosh[a*x]^2)/E^ArcCosh[a*x] - 4*E^A
rcCosh[a*x]*ArcCosh[a*x]^2 + 4*(-ArcCosh[a*x])^(5/2)*Gamma[1/2, -ArcCosh[a*
x]] - 4*ArcCosh[a*x]^(5/2)*Gamma[1/2, ArcCosh[a*x]] - 5*ArcCosh[a*x]*((1 -
10*ArcCosh[a*x])/E^(5*ArcCosh[a*x]) + E^(5*ArcCosh[a*x])*(1 + 10*ArcCosh[a*
x]) + 10*Sqrt[5]*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -5*ArcCosh[a*x]] + 10*Sqr
t[5]*ArcCosh[a*x]^(3/2)*Gamma[1/2, 5*ArcCosh[a*x]]) - (9*(ArcCosh[a*x] + E^
(6*ArcCosh[a*x])*ArcCosh[a*x] - 6*ArcCosh[a*x]^2 + 6*E^(6*ArcCosh[a*x])*Arc
Cosh[a*x]^2 - 6*Sqrt[3]*E^(3*ArcCosh[a*x])*(-ArcCosh[a*x])^(5/2)*Gamma[1/2,
 -3*ArcCosh[a*x]] + 6*Sqrt[3]*E^(3*ArcCosh[a*x])*ArcCosh[a*x]^(5/2)*Gamma[1
/2, 3*ArcCosh[a*x]] + E^(3*ArcCosh[a*x])*Sinh[3*ArcCosh[a*x]]))/E^(3*ArcCos
h[a*x]) - 3*Sinh[5*ArcCosh[a*x]])/(120*a^5*ArcCosh[a*x]^(5/2))
```

Maple [F] time = 0.172, size = 0, normalized size = 0.

$$\int x^{4} \left(\operatorname{arccosh}\left(ax\right)\right)^{-\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccosh(a*x)^(7/2),x)
```

```
[Out] int(x^4/arccosh(a*x)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\operatorname{arcosh}\left(ax\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/arccosh(a*x)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acosh(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.110 $\int \frac{x^3}{\cosh^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=244

$$\frac{16\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{15a^4}+\frac{4\sqrt{2\pi}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{15a^4}+\frac{16\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{15a^4}+\frac{4\sqrt{2\pi}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{15a^4}$$

```
[Out] (-2*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(5*a*ArcCosh[a*x]^(5/2)) + (4*x^2)/(5
*a^2*ArcCosh[a*x]^(3/2)) - (16*x^4)/(15*ArcCosh[a*x]^(3/2)) + (16*x*Sqrt[-1
 + a*x]*Sqrt[1 + a*x])/(5*a^3*Sqrt[ArcCosh[a*x]]) - (128*x^3*Sqrt[-1 + a*x]
*Sqrt[1 + a*x])/(15*a*Sqrt[ArcCosh[a*x]]) + (16*Sqrt[Pi]*Erf[2*Sqrt[ArcCosh
[a*x]]])/(15*a^4) + (4*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(15*a^4)
 + (16*Sqrt[Pi]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(15*a^4) + (4*Sqrt[2*Pi]*Erfi[S
qrt[2]*Sqrt[ArcCosh[a*x]]])/(15*a^4)
```

Rubi [A] time = 0.756945, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5668, 5775, 5666, 3307, 2180, 2204, 2205}

$$\frac{16\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{15a^4}+\frac{4\sqrt{2\pi}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{15a^4}+\frac{16\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{15a^4}+\frac{4\sqrt{2\pi}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{15a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCosh[a*x]^(7/2),x]
```


```
[Out] (-2*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(5*a*ArcCosh[a*x]^(5/2)) + (4*x^2)/(5
*a^2*ArcCosh[a*x]^(3/2)) - (16*x^4)/(15*ArcCosh[a*x]^(3/2)) + (16*x*Sqrt[-1
 + a*x]*Sqrt[1 + a*x])/(5*a^3*Sqrt[ArcCosh[a*x]]) - (128*x^3*Sqrt[-1 + a*x]
*Sqrt[1 + a*x])/(15*a*Sqrt[ArcCosh[a*x]]) + (16*Sqrt[Pi]*Erf[2*Sqrt[ArcCosh
[a*x]]])/(15*a^4) + (4*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(15*a^4)
 + (16*Sqrt[Pi]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(15*a^4) + (4*Sqrt[2*Pi]*Erfi[S
qrt[2]*Sqrt[ArcCosh[a*x]]])/(15*a^4)
```


Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
```

```
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^3}{\cosh^{-1}(ax)^{7/2}}\,dx = -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} - \frac{6\int \frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}}\,dx}{5a} + \frac{1}{5}(8a)\int \frac{x^4}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4x^2}{5a^2\cosh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\cosh^{-1}(ax)^{3/2}} + \frac{64}{15}\int \frac{x^3}{\cosh^{-1}(ax)^{3/2}}\,dx - \frac{8\int$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4x^2}{5a^2\cosh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\cosh^{-1}(ax)^{3/2}} + \frac{16x\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} - \frac{128x^3}{15a$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4x^2}{5a^2\cosh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\cosh^{-1}(ax)^{3/2}} + \frac{16x\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} - \frac{128x^3}{15a$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4x^2}{5a^2\cosh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\cosh^{-1}(ax)^{3/2}} + \frac{16x\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} - \frac{128x^3}{15a$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4x^2}{5a^2\cosh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\cosh^{-1}(ax)^{3/2}} + \frac{16x\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} - \frac{128x^3}{15a$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4x^2}{5a^2\cosh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\cosh^{-1}(ax)^{3/2}} + \frac{16x\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} - \frac{128x^3}{15a$$

Mathematica [A] time = 0.555799, size = 291, normalized size = 1.19

$$e^{-4\cosh^{-1}(ax)}\left(128e^{4\cosh^{-1}(ax)}\left(-\cosh^{-1}(ax)\right)^{5/2}\text{Gamma}\left(\frac{1}{2},-4\cosh^{-1}(ax)\right) - 8e^{2\cosh^{-1}(ax)}\left(-4\sqrt{2}e^{2\cosh^{-1}(ax)}\left(-\cosh^{-1}(ax\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/ArcCosh[a*x]^(7/2),x]
```

```
[Out] (3 - 3*E^(8*ArcCosh[a*x]) - 8*ArcCosh[a*x] - 8*E^(8*ArcCosh[a*x])*ArcCosh[a
*x] + 64*ArcCosh[a*x]^2 - 64*E^(8*ArcCosh[a*x])*ArcCosh[a*x]^2 + 128*E^(4*A
```

```
rcCosh[a*x])*(-ArcCosh[a*x])^(5/2)*Gamma[1/2, -4*ArcCosh[a*x]] - 8*E^(2*Arc
Cosh[a*x])*(3*a*E^(2*ArcCosh[a*x])*x*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) +
 ArcCosh[a*x] + E^(4*ArcCosh[a*x])*ArcCosh[a*x] - 4*ArcCosh[a*x]^2 + 4*E^(4
*ArcCosh[a*x])*ArcCosh[a*x]^2 - 4*Sqrt[2]*E^(2*ArcCosh[a*x])*(-ArcCosh[a*x]
)^(5/2)*Gamma[1/2, -2*ArcCosh[a*x]] + 4*Sqrt[2]*E^(2*ArcCosh[a*x])*ArcCosh[
a*x]^(5/2)*Gamma[1/2, 2*ArcCosh[a*x]]) - 128*E^(4*ArcCosh[a*x])*ArcCosh[a*x
]^(5/2)*Gamma[1/2, 4*ArcCosh[a*x]])/(120*a^4*E^(4*ArcCosh[a*x])*ArcCosh[a*x
]^(5/2))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int x^{3} \left(\operatorname{arccosh}\left(a x\right)\right)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccosh(a*x)^(7/2),x)

[Out] int(x^3/arccosh(a*x)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\operatorname{arcosh}\left(a x\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(7/2),x, algorithm="maxima")

[Out] integrate(x^3/arccosh(a*x)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acosh(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.111 $\int \frac{x^2}{\cosh^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=237

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15a^3}+\frac{3\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{5a^3}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15a^3}+\frac{3\sqrt{3\pi}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{5a^3}+ -\frac{}{15}$$

```
[Out] (-2*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(5*a*ArcCosh[a*x]^(5/2)) + (8*x)/(15*
a^2*ArcCosh[a*x]^(3/2)) - (4*x^3)/(5*ArcCosh[a*x]^(3/2)) + (16*Sqrt[-1 + a*
x]*Sqrt[1 + a*x])/(15*a^3*Sqrt[ArcCosh[a*x]]) - (24*x^2*Sqrt[-1 + a*x]*Sqrt
[1 + a*x])/(5*a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(1
5*a^3) + (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(5*a^3) + (Sqrt[Pi]
*Erfi[Sqrt[ArcCosh[a*x]]])/(15*a^3) + (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcCo
sh[a*x]]])/(5*a^3)
```

Rubi [A] time = 0.847696, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5668, 5775, 5666, 3307, 2180, 2204, 2205, 5656, 5781}

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15a^3}+\frac{3\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{5a^3}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15a^3}+\frac{3\sqrt{3\pi}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{5a^3}+ -\frac{}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCosh[a*x]^(7/2),x]
```


```
[Out] (-2*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(5*a*ArcCosh[a*x]^(5/2)) + (8*x)/(15*
a^2*ArcCosh[a*x]^(3/2)) - (4*x^3)/(5*ArcCosh[a*x]^(3/2)) + (16*Sqrt[-1 + a*
x]*Sqrt[1 + a*x])/(15*a^3*Sqrt[ArcCosh[a*x]]) - (24*x^2*Sqrt[-1 + a*x]*Sqrt
[1 + a*x])/(5*a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(1
5*a^3) + (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(5*a^3) + (Sqrt[Pi]
*Erfi[Sqrt[ArcCosh[a*x]]])/(15*a^3) + (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcCo
sh[a*x]]])/(5*a^3)
```


Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
```

```
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{x^2}{\cosh^{-1}(ax)^{7/2}}\,dx = -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} - \frac{4\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}}\,dx}{5a} + \frac{1}{5}(6a)\int \frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{8x}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\cosh^{-1}(ax)^{3/2}} + \frac{12}{5}\int \frac{x^2}{\cosh^{-1}(ax)^{3/2}}\,dx - \frac{8\int}{}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{8x}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\cosh^{-1}(ax)^{3/2}} + \frac{16\sqrt{-1+ax}\sqrt{1+ax}}{15a^3\sqrt{\cosh^{-1}(ax)}} - \frac{24x^2\sqrt{}}{5a\sqrt{}}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{8x}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\cosh^{-1}(ax)^{3/2}} + \frac{16\sqrt{-1+ax}\sqrt{1+ax}}{15a^3\sqrt{\cosh^{-1}(ax)}} - \frac{24x^2\sqrt{}}{5a\sqrt{}}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{8x}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\cosh^{-1}(ax)^{3/2}} + \frac{16\sqrt{-1+ax}\sqrt{1+ax}}{15a^3\sqrt{\cosh^{-1}(ax)}} - \frac{24x^2\sqrt{}}{5a\sqrt{}}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{8x}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\cosh^{-1}(ax)^{3/2}} + \frac{16\sqrt{-1+ax}\sqrt{1+ax}}{15a^3\sqrt{\cosh^{-1}(ax)}} - \frac{24x^2\sqrt{}}{5a\sqrt{}}$$

$$= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{8x}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\cosh^{-1}(ax)^{3/2}} + \frac{16\sqrt{-1+ax}\sqrt{1+ax}}{15a^3\sqrt{\cosh^{-1}(ax)}} - \frac{24x^2\sqrt{}}{5a\sqrt{}}$$

<u>Mathematica</u> [A] time = 0.795736, size = 286, normalized size = 1.21

$$e^{-3\cosh^{-1}(ax)}\left(-e^{2\cosh^{-1}(ax)}\left(-2e^{\cosh^{-1}(ax)}\left(-\cosh^{-1}(ax)\right)^{5/2}\text{Gamma}\left(\frac{1}{2},-\cosh^{-1}(ax)\right)+2e^{\cosh^{-1}(ax)}\cosh^{-1}(ax)^{5/2}\text{Gamm}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/ArcCosh[a*x]^(7/2),x]
```

```
[Out] (-(E^(2*ArcCosh[a*x])*(3*E^ArcCosh[a*x]*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x
) + ArcCosh[a*x] + E^(2*ArcCosh[a*x])*ArcCosh[a*x] - 2*ArcCosh[a*x]^2 + 2*E
^(2*ArcCosh[a*x])*ArcCosh[a*x]^2 - 2*E^ArcCosh[a*x]*(-ArcCosh[a*x])^(5/2)*G
amma[1/2, -ArcCosh[a*x]] + 2*E^ArcCosh[a*x]*ArcCosh[a*x]^(5/2)*Gamma[1/2, A
rcCosh[a*x]])) - 3*(ArcCosh[a*x] + E^(6*ArcCosh[a*x])*ArcCosh[a*x] - 6*ArcC
osh[a*x]^2 + 6*E^(6*ArcCosh[a*x])*ArcCosh[a*x]^2 - 6*Sqrt[3]*E^(3*ArcCosh[a
```

```
*x])*(-ArcCosh[a*x])^(5/2)*Gamma[1/2, -3*ArcCosh[a*x]] + 6*Sqrt[3]*E^(3*Arc
Cosh[a*x])*ArcCosh[a*x]^(5/2)*Gamma[1/2, 3*ArcCosh[a*x]] + E^(3*ArcCosh[a*x
])*Sinh[3*ArcCosh[a*x]]))/(30*a^3*E^(3*ArcCosh[a*x])*ArcCosh[a*x]^(5/2))
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int x^{2} \left(\operatorname{arccosh}\left(ax\right)\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccosh(a*x)^(7/2),x)
```

```
[Out] int(x^2/arccosh(a*x)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\operatorname{arcosh}\left(ax\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/arccosh(a*x)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acosh(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.112 $\int \frac{x}{\cosh^{-1}(ax)^{7/2}} \, dx$

Optimal. Leaf size=157

$$\frac{8\sqrt{2\pi}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{15a^2}+\frac{8\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{15a^2}+\frac{4}{15a^2\cosh^{-1}(ax)^{3/2}}-\frac{8x^2}{15\cosh^{-1}(ax)^{3/2}}-\frac{32x\sqrt{ax-1}\sqrt{}}{15a\sqrt{\cosh^{-}}}$$

```
[Out] (-2*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(5*a*ArcCosh[a*x]^(5/2)) + 4/(15*a^2*Ar
cCosh[a*x]^(3/2)) - (8*x^2)/(15*ArcCosh[a*x]^(3/2)) - (32*x*Sqrt[-1 + a*x]*
Sqrt[1 + a*x])/(15*a*Sqrt[ArcCosh[a*x]]) + (8*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[A
rcCosh[a*x]]])/(15*a^2) + (8*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(
15*a^2)
```

Rubi [A] time = 0.495435, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {5668, 5775, 5666, 3307, 2180, 2204, 2205, 5676}

$$\frac{8\sqrt{2\pi}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{15a^2}+\frac{8\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{15a^2}+\frac{4}{15a^2\cosh^{-1}(ax)^{3/2}}-\frac{8x^2}{15\cosh^{-1}(ax)^{3/2}}-\frac{32x\sqrt{ax-1}\sqrt{}}{15a\sqrt{\cosh^{-}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCosh[a*x]^(7/2),x]
```


```
[Out] (-2*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(5*a*ArcCosh[a*x]^(5/2)) + 4/(15*a^2*Ar
cCosh[a*x]^(3/2)) - (8*x^2)/(15*ArcCosh[a*x]^(3/2)) - (32*x*Sqrt[-1 + a*x]*
Sqrt[1 + a*x])/(15*a*Sqrt[ArcCosh[a*x]]) + (8*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[A
rcCosh[a*x]]])/(15*a^2) + (8*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(
15*a^2)
```


Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
```

```
x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{x}{\cosh^{-1}(ax)^{7/2}}\,dx = -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} - \frac{2\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}}\,dx}{5a} + \frac{1}{5}(4a)\int \frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax}$$

$$= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\cosh^{-1}(ax)^{3/2}} + \frac{16}{15}\int \frac{x}{\cosh^{-1}(ax)^{3/2}}\,dx$$

$$= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\cosh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{32\,\mathrm{S}}{}$$

$$= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\cosh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{16\,\mathrm{S}}{}$$

$$= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\cosh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{32\,\mathrm{S}}{}$$

$$= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\cosh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{8\sqrt{2}}{}$$

Mathematica [A] time = 0.285803, size = 91, normalized size = 0.58

$$-\frac{-8\sqrt{2\pi}\left(\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)\right)+\frac{4\cosh\left(2\cosh^{-1}(ax)\right)}{\cosh^{-1}(ax)^{3/2}}+\frac{\left(16\cosh^{-1}(ax)^2+3\right)\sinh\left(2\cosh^{-1}(ax)\right)}{\cosh^{-1}(ax)^{5/2}}}{15a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/ArcCosh[a*x]^(7/2),x]
```

```
[Out] -((4*Cosh[2*ArcCosh[a*x]])/ArcCosh[a*x]^(3/2) - 8*Sqrt[2*Pi]*(Erf[Sqrt[2]*S
qrt[ArcCosh[a*x]]] + Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]]) + ((3 + 16*ArcCosh[a
*x]^2)*Sinh[2*ArcCosh[a*x]])/ArcCosh[a*x]^(5/2))/(15*a^2)
```

Maple [A] time = 0.118, size = 153, normalized size = 1.

$$\frac{\sqrt{2}}{15\sqrt{\pi}a^2 (\operatorname{arccosh}(ax))^3}\left(-16 (\operatorname{arccosh}(ax))^{5/2}\sqrt{2}\sqrt{\pi}\sqrt{ax+1}\sqrt{ax-1}xa - 4 (\operatorname{arccosh}(ax))^{3/2}\sqrt{2}\sqrt{\pi}x^2a^2 - 3\sqrt{2}\sqrt{\operatorname{arcc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccosh(a*x)^(7/2),x)
```

```
[Out] 1/15*2^(1/2)*(-16*arccosh(a*x)^(5/2)*2^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)
^(1/2)*x*a-4*arccosh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*x^2*a^2-3*2^(1/2)*arccosh(
a*x)^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x*a+8*arccosh(a*x)^3*Pi*erf
(2^(1/2)*arccosh(a*x)^(1/2))+8*arccosh(a*x)^3*Pi*erfi(2^(1/2)*arccosh(a*x)^
(1/2))+2*arccosh(a*x)^(3/2)*2^(1/2)*Pi^(1/2))/Pi^(1/2)/a^2/arccosh(a*x)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arcosh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x/arccosh(a*x)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^(7/2),x, algorithm="fricas")
```

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acosh(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.113 $\int \frac{1}{\cosh^{-1}(ax)^{7/2}} dx$

Optimal. Leaf size=122

$$\frac{4\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15a}+\frac{4\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15a}-\frac{4x}{15\cosh^{-1}(ax)^{3/2}}-\frac{8\sqrt{ax-1}\sqrt{ax+1}}{15a\sqrt{\cosh^{-1}(ax)}}-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{5a\cosh^{-1}(ax)^{5/2}}$$

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(5*a*ArcCosh[a*x]^(5/2)) - (4*x)/(15*ArcC
osh[a*x]^(3/2)) - (8*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(15*a*Sqrt[ArcCosh[a*x]]
) + (4*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(15*a) + (4*Sqrt[Pi]*Erfi[Sqrt[Arc
Cosh[a*x]]])/(15*a)
```

Rubi [A] time = 0.443552, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5656, 5775, 5781, 3307, 2180, 2204, 2205}

$$\frac{4\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15a}+\frac{4\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15a}-\frac{4x}{15\cosh^{-1}(ax)^{3/2}}-\frac{8\sqrt{ax-1}\sqrt{ax+1}}{15a\sqrt{\cosh^{-1}(ax)}}-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{5a\cosh^{-1}(ax)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^(-7/2),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(5*a*ArcCosh[a*x]^(5/2)) - (4*x)/(15*ArcC
osh[a*x]^(3/2)) - (8*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(15*a*Sqrt[ArcCosh[a*x]]
) + (4*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(15*a) + (4*Sqrt[Pi]*Erfi[Sqrt[Arc
Cosh[a*x]]])/(15*a)
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\cosh^{-1}(ax)^{7/2}}\,dx = -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{1}{5}(2a)\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}}\,dx$$

$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} - \frac{4x}{15\cosh^{-1}(ax)^{3/2}} + \frac{4}{15}\int \frac{1}{\cosh^{-1}(ax)^{3/2}}\,dx$$

$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} - \frac{4x}{15\cosh^{-1}(ax)^{3/2}} - \frac{8\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{1}{15}(8a)\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c}}$$

$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} - \frac{4x}{15\cosh^{-1}(ax)^{3/2}} - \frac{8\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{8\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}\right.}{15a}$$

$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} - \frac{4x}{15\cosh^{-1}(ax)^{3/2}} - \frac{8\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{4\,\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax\right.}{15a}$$

$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} - \frac{4x}{15\cosh^{-1}(ax)^{3/2}} - \frac{8\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{8\,\mathrm{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\cosh^{-1}(}\right.}{15a}$$

$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} - \frac{4x}{15\cosh^{-1}(ax)^{3/2}} - \frac{8\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{4\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15a} + \frac{4\sqrt{\pi}}{}$$

Mathematica [A] time = 0.181454, size = 147, normalized size = 1.2

$$-\frac{2e^{-\cosh^{-1}(ax)}\left(-2e^{\cosh^{-1}(ax)}\left(-\cosh^{-1}(ax)\right)^{5/2}\mathrm{Gamma}\left(\frac{1}{2}, -\cosh^{-1}(ax)\right) + 2e^{\cosh^{-1}(ax)}\cosh^{-1}(ax)^{5/2}\mathrm{Gamma}\left(\frac{1}{2}, \cosh^{-1}\right.\right.}{15a\cosh^{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^(-7/2),x]
```

```
[Out] (-2*(3*E^ArcCosh[a*x]*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) + ArcCosh[a*x] +
 E^(2*ArcCosh[a*x])*ArcCosh[a*x] - 2*ArcCosh[a*x]^2 + 2*E^(2*ArcCosh[a*x])*
ArcCosh[a*x]^2 - 2*E^ArcCosh[a*x]*(-ArcCosh[a*x])^(5/2)*Gamma[1/2, -ArcCosh
[a*x]] + 2*E^ArcCosh[a*x]*ArcCosh[a*x]^(5/2)*Gamma[1/2, ArcCosh[a*x]]))/(15
*a*E^ArcCosh[a*x]*ArcCosh[a*x]^(5/2))
```

Maple [A] time = 0.101, size = 111, normalized size = 0.9

$$\frac{2}{15\sqrt{\pi}a\left(\operatorname{arccosh}(ax)\right)^{3}}\left(-4\sqrt{ax-1}\left(\operatorname{arccosh}(ax)\right)^{5/2}\sqrt{ax+1}\sqrt{\pi}+2\left(\operatorname{arccosh}(ax)\right)^{3}\pi\operatorname{Erf}\left(\sqrt{\operatorname{arccosh}(ax)}\right)+2\left(\operatorname{arc}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x)^(7/2),x)
```

```
[Out] 2/15*(-4*(a*x-1)^(1/2)*arccosh(a*x)^(5/2)*(a*x+1)^(1/2)*Pi^(1/2)+2*arccosh(
a*x)^3*Pi*erf(arccosh(a*x)^(1/2))+2*arccosh(a*x)^3*Pi*erfi(arccosh(a*x)^(1/
2))-2*arccosh(a*x)^(3/2)*Pi^(1/2)*x*a-3*arccosh(a*x)^(1/2)*Pi^(1/2)*(a*x+1)
^(1/2)*(a*x-1)^(1/2))/Pi^(1/2)/a/arccosh(a*x)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\operatorname{arcosh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(-7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.114 $\int \frac{1}{x \cosh^{-1}(ax)^{7/2}} dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x \cosh^{-1}(ax)^{7/2}}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcCosh[a*x]^(7/2)), x]
```

Rubi [A] time = 0.0166465, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \cosh^{-1}(ax)^{7/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcCosh[a*x]^(7/2)),x]
```

```
[Out] Defer[Int][1/(x*ArcCosh[a*x]^(7/2)), x]
```

Rubi steps

$$\int \frac{1}{x \cosh^{-1}(ax)^{7/2}} dx = \int \frac{1}{x \cosh^{-1}(ax)^{7/2}} dx$$

Mathematica [A] time = 0.345481, size = 0, normalized size = 0.

$$\int \frac{1}{x \cosh^{-1}(ax)^{7/2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcCosh[a*x]^(7/2)),x]
```

```
[Out] Integrate[1/(x*ArcCosh[a*x]^(7/2)), x]
```

Maple [A] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\operatorname{arccosh}(ax))^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccosh(a*x)^(7/2),x)
```

```
[Out] int(1/x/arccosh(a*x)^(7/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arcosh}(ax)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arccosh(a*x)^(7/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acosh(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.115 $\int x^m \cosh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=58

$$\frac{x^{m+1}\cosh^{-1}(ax)^4}{m+1}-\frac{4a\text{Unintegrable}\left(\frac{x^{m+1}\cosh^{-1}(ax)^3}{\sqrt{ax-1}\sqrt{ax+1}},x\right)}{m+1}$$

```
[Out] (x^(1 + m)*ArcCosh[a*x]^4)/(1 + m) - (4*a*Unintegrable[(x^(1 + m)*ArcCosh[a
*x]^3)/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]), x])/(1 + m)
```

Rubi [A] time = 0.277429, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \cosh^{-1}(ax)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*ArcCosh[a*x]^4,x]
```

```
[Out] (x^(1 + m)*ArcCosh[a*x]^4)/(1 + m) - (4*a*Defer[Int][(x^(1 + m)*ArcCosh[a*x
]^3)/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]), x])/(1 + m)
```

Rubi steps

$$\int x^m \cosh^{-1}(ax)^4 \, dx = \frac{x^{1+m}\cosh^{-1}(ax)^4}{1+m}-\frac{(4a)\int \frac{x^{1+m}\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}} \, dx}{1+m}$$

Mathematica [A] time = 1.80911, size = 0, normalized size = 0.

$$\int x^m \cosh^{-1}(ax)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*ArcCosh[a*x]^4,x]
```

```
[Out] Integrate[x^m*ArcCosh[a*x]^4, x]
```

Maple [A] time = 0.646, size = 0, normalized size = 0.

$$\int x^m (\operatorname{arccosh}(ax))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccosh(a*x)^4,x)
```

```
[Out] int(x^m*arccosh(a*x)^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^m \operatorname{arcosh}(ax)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^m*arccosh(a*x)^4, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{acosh}^4(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acosh(a*x)**4,x)
```

```
[Out] Integral(x**m*acosh(a*x)**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arcosh}(ax)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^m*arccosh(a*x)^4, x)
```

3.116 $\int x^m \cosh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=58

$$\frac{x^{m+1}\cosh^{-1}(ax)^3}{m+1} - \frac{3a\text{Unintegrable}\left(\frac{x^{m+1}\cosh^{-1}(ax)^2}{\sqrt{ax-1}\sqrt{ax+1}}, x\right)}{m+1}$$

```
[Out] (x^(1 + m)*ArcCosh[a*x]^3)/(1 + m) - (3*a*Unintegrable[(x^(1 + m)*ArcCosh[a
*x]^2)/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]), x])/(1 + m)
```

Rubi [A] time = 0.298019, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \cosh^{-1}(ax)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*ArcCosh[a*x]^3,x]
```

```
[Out] (x^(1 + m)*ArcCosh[a*x]^3)/(1 + m) - (3*a*Defer[Int][(x^(1 + m)*ArcCosh[a*x
]^2)/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]), x])/(1 + m)
```

Rubi steps

$$\int x^m \cosh^{-1}(ax)^3 \, dx = \frac{x^{1+m}\cosh^{-1}(ax)^3}{1+m} - \frac{(3a)\int \frac{x^{1+m}\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}} \, dx}{1+m}$$

Mathematica [A] time = 1.73522, size = 0, normalized size = 0.

$$\int x^m \cosh^{-1}(ax)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*ArcCosh[a*x]^3,x]
```

```
[Out] Integrate[x^m*ArcCosh[a*x]^3, x]
```

Maple [A] time = 0.569, size = 0, normalized size = 0.

$$\int x^m (\operatorname{arccosh}(ax))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccosh(a*x)^3,x)
```

```
[Out] int(x^m*arccosh(a*x)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^m \operatorname{arcosh}(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*arccosh(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{acosh}^3(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acosh(a*x)**3,x)
```

```
[Out] Integral(x**m*acosh(a*x)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arcosh}(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*arccosh(a*x)^3, x)
```

3.117 $\int x^m \cosh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=154

$$-\frac{2a^2x^{m+3}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2}\right\},\left\{\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2}\right\},a^2x^2\right)}{m^3+6m^2+11m+6}-\frac{2a\sqrt{1-ax}x^{m+2}\cosh^{-1}(ax)\text{Hypergeome}}{\left(m^2+3m+2\right)\sqrt{}}$$

```
[Out] (x^(1 + m)*ArcCosh[a*x]^2)/(1 + m) - (2*a*x^(2 + m)*Sqrt[1 - a*x]*ArcCosh[a
*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + 3*m + m^2)
*Sqrt[-1 + a*x]) - (2*a^2*x^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 +
m/2}, {2 + m/2, 5/2 + m/2}, a^2*x^2])/(6 + 11*m + 6*m^2 + m^3)
```

Rubi [A] time = 0.238126, antiderivative size = 167, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5662, 5763}

$$-\frac{2a^2x^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};a^2x^2\right)}{m^3+6m^2+11m+6}-\frac{2a\sqrt{1-a^2x^2}x^{m+2}\cosh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{\left(m^2+3m+2\right)\sqrt{ax-1}\sqrt{ax+1}}+\frac{x^{m+1}\cosh}{m+}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCosh[a*x]^2,x]
```

```
[Out] (x^(1 + m)*ArcCosh[a*x]^2)/(1 + m) - (2*a*x^(2 + m)*Sqrt[1 - a^2*x^2]*ArcCo
sh[a*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + 3*m +
m^2)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (2*a^2*x^(3 + m)*HypergeometricPFQ[{1,
 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, a^2*x^2])/(6 + 11*m + 6*m^2 +
 m^3)
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int x^m \cosh^{-1}(ax)^2\,dx = \frac{x^{1+m}\cosh^{-1}(ax)^2}{1+m} - \frac{(2a)\int \frac{x^{1+m}\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{1+m}$$
$$= \frac{x^{1+m}\cosh^{-1}(ax)^2}{1+m} - \frac{2ax^{2+m}\sqrt{1-a^2x^2}\cosh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{(2+3m+m^2)\sqrt{-1+ax}\sqrt{1+ax}} - \frac{2a^2x^{3+m}\,{}_3F_2\left(1,\frac{3}{2}\right.}{6+}$$

Mathematica [A] time = 0.354987, size = 143, normalized size = 0.93

$$\frac{x^{m+1}\left(\cosh^{-1}(ax)^2 - \frac{2ax\left(\frac{ax\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2}\right\},\left\{\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2}\right\},a^2x^2\right)}{m+3} + \frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{\sqrt{ax-1}\sqrt{ax+1}}\right)}{m+2}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCosh[a*x]^2,x]
```

```
[Out] (x^(1 + m)*(ArcCosh[a*x]^2 - (2*a*x*((Sqrt[1 - a^2*x^2]*ArcCosh[a*x]*Hyperg
eometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(Sqrt[-1 + a*x]*Sqrt[1 + a
*x]) + (a*x*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/
2}, a^2*x^2])/(3 + m)))/(2 + m)))/(1 + m)
```

Maple [F] time = 0.693, size = 0, normalized size = 0.

$$\int x^m (\mathrm{arccosh}\,(ax))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccosh(a*x)^2,x)
```

```
[Out] int(x^m*arccosh(a*x)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \operatorname{arcosh}(ax)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*arccosh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{acosh}^2(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acosh(a*x)**2,x)
```

```
[Out] Integral(x**m*acosh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arcosh}(ax)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*arccosh(a*x)^2, x)
```

3.118 $\int x^m \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=91

$$\frac{x^{m+1}\cosh^{-1}(ax)}{m+1}-\frac{a\sqrt{1-a^2x^2}x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{(m^2+3m+2)\sqrt{ax-1}\sqrt{ax+1}}$$

```
[Out] (x^(1 + m)*ArcCosh[a*x])/(1 + m) - (a*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeom
etric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + a*
x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.054826, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5662, 126, 365, 364}

$$\frac{x^{m+1}\cosh^{-1}(ax)}{m+1}-\frac{a\sqrt{1-a^2x^2}x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m^2+3m+2)\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCosh[a*x],x]
```

```
[Out] (x^(1 + m)*ArcCosh[a*x])/(1 + m) - (a*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeom
etric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + a*
x]*Sqrt[1 + a*x])
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int x^m \cosh^{-1}(ax)\,dx &= \frac{x^{1+m}\cosh^{-1}(ax)}{1+m} - \frac{a\int \frac{x^{1+m}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{1+m}\\
&= \frac{x^{1+m}\cosh^{-1}(ax)}{1+m} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\int \frac{x^{1+m}}{\sqrt{-1+a^2x^2}}\,dx}{(1+m)\sqrt{-1+ax}\sqrt{1+ax}}\\
&= \frac{x^{1+m}\cosh^{-1}(ax)}{1+m} - \frac{\left(a\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{(1+m)\sqrt{-1+ax}\sqrt{1+ax}}\\
&= \frac{x^{1+m}\cosh^{-1}(ax)}{1+m} - \frac{ax^{2+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{\left(2+3m+m^2\right)\sqrt{-1+ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.0954662, size = 82, normalized size = 0.9

$$\frac{x^{m+1}\left(\cosh^{-1}(ax) - \frac{ax\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{(m+2)\sqrt{ax-1}\sqrt{ax+1}}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCosh[a*x],x]
```

```
[Out] (x^(1 + m)*(ArcCosh[a*x] - (a*x*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1/2, (2
 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[-1 + a*x]*Sqrt[1 + a*x])))/(1 +
 m)
```

Maple [F] time = 0.676, size = 0, normalized size = 0.

$$\int x^m \text{arccosh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccosh(a*x),x)
```

```
[Out] int(x^m*arccosh(a*x),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\,(x^m\,\text{arcosh}\,(ax)\,,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^m*arccosh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{acosh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acosh(a*x),x)
```

```
[Out] Integral(x**m*acosh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arcosh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^m*arccosh(a*x), x)
```

3.119 $\int \frac{x^m}{\cosh^{-1}(ax)} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{x^m}{\cosh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[x^m/ArcCosh[a*x], x]
```

Rubi [A] time = 0.0229181, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\cosh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/ArcCosh[a*x],x]
```

```
[Out] Defer[Int][x^m/ArcCosh[a*x], x]
```

Rubi steps

$$\int \frac{x^m}{\cosh^{-1}(ax)} dx = \int \frac{x^m}{\cosh^{-1}(ax)} dx$$

Mathematica [A] time = 1.30103, size = 0, normalized size = 0.

$$\int \frac{x^m}{\cosh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/ArcCosh[a*x],x]
```

```
[Out] Integrate[x^m/ArcCosh[a*x], x]
```

Maple [A] time = 0.485, size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arccosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arccosh(a*x),x)
```

```
[Out] int(x^m/arccosh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^m/arccosh(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^m}{\operatorname{arcosh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^m/arccosh(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/acosh(a*x),x)
```

```
[Out] Integral(x**m/acosh(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^m/arccosh(a*x), x)
```

3.120 $\int \frac{x^m}{\cosh^{-1}(ax)^2} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{x^m}{\cosh^{-1}(ax)^2}, x\right)$$

```
[Out] Unintegrable[x^m/ArcCosh[a*x]^2, x]
```

Rubi [A] time = 0.0229005, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\cosh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/ArcCosh[a*x]^2,x]
```

```
[Out] Defer[Int][x^m/ArcCosh[a*x]^2, x]
```

Rubi steps

$$\int \frac{x^m}{\cosh^{-1}(ax)^2} dx = \int \frac{x^m}{\cosh^{-1}(ax)^2} dx$$

Mathematica [A] time = 1.31887, size = 0, normalized size = 0.

$$\int \frac{x^m}{\cosh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/ArcCosh[a*x]^2,x]
```

```
[Out] Integrate[x^m/ArcCosh[a*x]^2, x]
```

Maple [A] time = 0.576, size = 0, normalized size = 0.

$$\int \frac{x^m}{(\operatorname{arccosh}(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arccosh(a*x)^2,x)
```

```
[Out] int(x^m/arccosh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{(a^2x^2-1)\sqrt{ax+1}\sqrt{ax-1}x^m+(a^3x^3-ax)x^m}{(a^3x^2+\sqrt{ax+1}\sqrt{ax-1}a^2x-a)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)}+\int\frac{(a^3(m+1)x^3-a(m-1)x)(ax+1)(ax-1)x^m+(2a^4}{(a^5x^5+(ax+1)(ax-1)a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1)*x^m + (a^3*x^3 - a*x)*x^m)/((a^
3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt
(a*x - 1))) + integrate(((a^3*(m + 1)*x^3 - a*(m - 1)*x)*(a*x + 1)*(a*x - 1
)*x^m + (2*a^4*(m + 1)*x^4 - a^2*(3*m + 1)*x^2 + m)*sqrt(a*x + 1)*sqrt(a*x
- 1)*x^m + (a^5*(m + 1)*x^5 - 2*a^3*(m + 1)*x^3 + a*(m + 1)*x)*x^m)/((a^5*x
^5 + (a*x + 1)*(a*x - 1)*a^3*x^3 - 2*a^3*x^3 + 2*(a^4*x^4 - a^2*x^2)*sqrt(a
*x + 1)*sqrt(a*x - 1) + a*x)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^m}{\operatorname{arcosh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/arccosh(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{acosh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/acosh(a*x)**2,x)
```

```
[Out] Integral(x**m/acosh(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arcosh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/arccosh(a*x)^2, x)
```

3.121 $\int \frac{x^m}{\cosh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{x^m}{\cosh^{-1}(ax)^3}, x\right)$$

```
[Out] Unintegrable[x^m/ArcCosh[a*x]^3, x]
```

Rubi [A] time = 0.0196945, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^m}{\cosh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/ArcCosh[a*x]^3,x]
```

```
[Out] Defer[Int][x^m/ArcCosh[a*x]^3, x]
```

Rubi steps

$$\int \frac{x^m}{\cosh^{-1}(ax)^3} \, dx = \int \frac{x^m}{\cosh^{-1}(ax)^3} \, dx$$

Mathematica [A] time = 1.32557, size = 0, normalized size = 0.

$$\int \frac{x^m}{\cosh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/ArcCosh[a*x]^3,x]
```

```
[Out] Integrate[x^m/ArcCosh[a*x]^3, x]
```

Maple [A] time = 0.457, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\operatorname{arccosh}\left(ax\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arccosh(a*x)^3,x)
```

```
[Out] int(x^m/arccosh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((a^5*x^5 - a^3*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2)*x^m + (3*a^6*x^6
- 5*a^4*x^4 + 2*a^2*x^2)*(a*x + 1)*(a*x - 1)*x^m + (3*a^7*x^7 - 7*a^5*x^5 +
 5*a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*x - 1)*x^m + (a^8*x^8 - 3*a^6*x^6 +
3*a^4*x^4 - a^2*x^2)*x^m + ((a^5*(m + 1)*x^5 - 2*a^3*m*x^3 + a*(m - 1)*x)*(
a*x + 1)^(3/2)*(a*x - 1)^(3/2)*x^m + (3*a^6*(m + 1)*x^6 - a^4*(7*m + 3)*x^4
 + 5*a^2*m*x^2 - m)*(a*x + 1)*(a*x - 1)*x^m + (3*a^7*(m + 1)*x^7 - 2*a^5*(4
*m + 3)*x^5 + a^3*(7*m + 4)*x^3 - a*(2*m + 1)*x)*sqrt(a*x + 1)*sqrt(a*x - 1
)*x^m + (a^8*(m + 1)*x^8 - 3*a^6*(m + 1)*x^6 + 3*a^4*(m + 1)*x^4 - a^2*(m +
 1)*x^2)*x^m)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)))/((a^8*x^7 + (a*x + 1)
^(3/2)*(a*x - 1)^(3/2)*a^5*x^4 - 3*a^6*x^5 + 3*a^4*x^3 + 3*(a^6*x^5 - a^4*x
^3)*(a*x + 1)*(a*x - 1) - a^2*x + 3*(a^7*x^6 - 2*a^5*x^4 + a^3*x^2)*sqrt(a*
x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2) + integrate
(1/2*(((m^2 + 2*m + 1)*a^6*x^6 - 2*(m^2 - m)*a^4*x^4 + (m^2 - 4*m + 3)*a^2*
x^2)*(a*x + 1)^2*(a*x - 1)^2*x^m + (4*(m^2 + 2*m + 1)*a^7*x^7 - 2*(5*m^2 +
m + 2)*a^5*x^5 + (8*m^2 - 11*m + 3)*a^3*x^3 - (2*m^2 - 5*m)*a*x)*(a*x + 1)^
(3/2)*(a*x - 1)^(3/2)*x^m + (6*(m^2 + 2*m + 1)*a^8*x^8 - 6*(3*m^2 + 3*m + 2
)*a^6*x^6 + (19*m^2 + 2*m + 3)*a^4*x^4 - (8*m^2 - 5*m - 3)*a^2*x^2 + m^2 -
m)*(a*x + 1)*(a*x - 1)*x^m + (4*(m^2 + 2*m + 1)*a^9*x^9 - 2*(7*m^2 + 11*m +
 6)*a^7*x^7 + 3*(6*m^2 + 7*m + 3)*a^5*x^5 - (10*m^2 + 8*m + 1)*a^3*x^3 + (2
*m^2 + m)*a*x)*sqrt(a*x + 1)*sqrt(a*x - 1)*x^m + ((m^2 + 2*m + 1)*a^10*x^10
 - 4*(m^2 + 2*m + 1)*a^8*x^8 + 6*(m^2 + 2*m + 1)*a^6*x^6 - 4*(m^2 + 2*m + 1
```

```
)*a^4*x^4 + (m^2 + 2*m + 1)*a^2*x^2)*x^m)/((a^10*x^10 + (a*x + 1)^2*(a*x -
1)^2*a^6*x^6 - 4*a^8*x^8 + 6*a^6*x^6 - 4*a^4*x^4 + 4*(a^7*x^7 - a^5*x^5)*(a
*x + 1)^(3/2)*(a*x - 1)^(3/2) + a^2*x^2 + 6*(a^8*x^8 - 2*a^6*x^6 + a^4*x^4)
*(a*x + 1)*(a*x - 1) + 4*(a^9*x^9 - 3*a^7*x^7 + 3*a^5*x^5 - a^3*x^3)*sqrt(a
*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\operatorname{arcosh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/arccosh(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{acosh}^3(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/acosh(a*x)**3,x)
```

```
[Out] Integral(x**m/acosh(a*x)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arcosh}(ax)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/arccosh(a*x)^3, x)
```

3.122 $\int x^m \cosh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(x^m \cosh^{-1}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable[x^m*ArcCosh[a*x]^(3/2), x]
```

Rubi [A] time = 0.0194091, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \cosh^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*ArcCosh[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^m*ArcCosh[a*x]^(3/2), x]
```

Rubi steps

$$\int x^m \cosh^{-1}(ax)^{3/2}\,dx = \int x^m \cosh^{-1}(ax)^{3/2}\,dx$$

Mathematica [A] time = 1.87123, size = 0, normalized size = 0.

$$\int x^m \cosh^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*ArcCosh[a*x]^(3/2),x]
```

```
[Out] Integrate[x^m*ArcCosh[a*x]^(3/2), x]
```

Maple [A] time = 0.07, size = 0, normalized size = 0.

$$\int x^m (\operatorname{arccosh}(ax))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccosh(a*x)^(3/2),x)
```

```
[Out] int(x^m*arccosh(a*x)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arcosh}(ax)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acosh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.123 $\int x^m \sqrt{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(x^m \sqrt{\cosh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^m*Sqrt[ArcCosh[a*x]], x]
```

Rubi [A] time = 0.0156938, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \sqrt{\cosh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] Defer[Int][x^m*Sqrt[ArcCosh[a*x]], x]
```

Rubi steps

$$\int x^m \sqrt{\cosh^{-1}(ax)}\,dx = \int x^m \sqrt{\cosh^{-1}(ax)}\,dx$$

Mathematica [A] time = 2.16921, size = 0, normalized size = 0.

$$\int x^m \sqrt{\cosh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] Integrate[x^m*Sqrt[ArcCosh[a*x]], x]
```

Maple [A] time = 0.067, size = 0, normalized size = 0.

$$\int x^m \sqrt{\operatorname{arccosh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccosh(a*x)^(1/2),x)
```

```
[Out] int(x^m*arccosh(a*x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\operatorname{arcosh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\operatorname{acosh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acosh(a*x)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(acosh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.124 $\int \frac{x^m}{\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{x^m}{\sqrt{\cosh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[x^m/Sqrt[ArcCosh[a*x]], x]
```

Rubi [A] time = 0.0162995, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] Defer[Int][x^m/Sqrt[ArcCosh[a*x]], x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{\cosh^{-1}(ax)}}\,dx = \int \frac{x^m}{\sqrt{\cosh^{-1}(ax)}}\,dx$$

Mathematica [A] time = 2.01138, size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] Integrate[x^m/Sqrt[ArcCosh[a*x]], x]
```

Maple [A] time = 0.071, size = 0, normalized size = 0.

$$\int x^m \frac{1}{\sqrt{\operatorname{arccosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arccosh(a*x)^(1/2),x)
```

```
[Out] int(x^m/arccosh(a*x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\operatorname{arcosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\operatorname{acosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(x**m/sqrt(acosh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.125 $\int \frac{x^m}{\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{x^m}{\cosh^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[x^m/ArcCosh[a*x]^(3/2), x]
```

Rubi [A] time = 0.0164417, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\cosh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/ArcCosh[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^m/ArcCosh[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x^m}{\cosh^{-1}(ax)^{3/2}}\,dx = \int \frac{x^m}{\cosh^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.93164, size = 0, normalized size = 0.

$$\int \frac{x^m}{\cosh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/ArcCosh[a*x]^(3/2),x]
```

```
[Out] Integrate[x^m/ArcCosh[a*x]^(3/2), x]
```

Maple [A] time = 0.069, size = 0, normalized size = 0.

$$\int x^{m} (\operatorname{arccosh}(ax))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arccosh(a*x)^(3/2),x)
```

```
[Out] int(x^m/arccosh(a*x)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\operatorname{arcosh}(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\operatorname{acosh}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/acosh(a*x)**(3/2),x)
```

```
[Out] Integral(x**m/acosh(a*x)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.126 $\int (dx)^m \cosh^{-1}(ax)^n \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left((dx)^m \cosh^{-1}(ax)^n, x\right)$$

```
[Out] Unintegrable[(d*x)^m*ArcCosh[a*x]^n, x]
```

Rubi [A] time = 0.0192157, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (dx)^m \cosh^{-1}(ax)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*ArcCosh[a*x]^n,x]
```

```
[Out] Defer[Int][(d*x)^m*ArcCosh[a*x]^n, x]
```

Rubi steps

$$\int (dx)^m \cosh^{-1}(ax)^n \, dx = \int (dx)^m \cosh^{-1}(ax)^n \, dx$$

Mathematica [A] time = 1.39556, size = 0, normalized size = 0.

$$\int (dx)^m \cosh^{-1}(ax)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*ArcCosh[a*x]^n,x]
```

```
[Out] Integrate[(d*x)^m*ArcCosh[a*x]^n, x]
```

Maple [A] time = 0.082, size = 0, normalized size = 0.

$$\int (dx)^m (\operatorname{arccosh}(ax))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*arccosh(a*x)^n,x)
```

```
[Out] int((d*x)^m*arccosh(a*x)^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{arcosh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*arccosh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m*arccosh(a*x)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx)^m \operatorname{arcosh}(ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*arccosh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*arccosh(a*x)^n, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{acosh}^n(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*acosh(a*x)**n,x)
```

```
[Out] Integral((d*x)**m*acosh(a*x)**n, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*arccosh(a*x)^n,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.127 $\int x^4 \cosh^{-1}(ax)^n \, dx$

Optimal. Leaf size=173

$$\frac{5^{-n-1}\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-5\cosh^{-1}(ax)\right)}{32a^5}+\frac{3^{-n}\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+\right.}{32a^5}$$

```
[Out] (5^(-1 - n)*ArcCosh[a*x]^n*Gamma[1 + n, -5*ArcCosh[a*x]])/(32*a^5*(-ArcCosh
[a*x])^n) + (ArcCosh[a*x]^n*Gamma[1 + n, -3*ArcCosh[a*x]])/(32*3^n*a^5*(-Ar
cCosh[a*x])^n) + (ArcCosh[a*x]^n*Gamma[1 + n, -ArcCosh[a*x]])/(16*a^5*(-Arc
Cosh[a*x])^n) + Gamma[1 + n, ArcCosh[a*x]]/(16*a^5) + Gamma[1 + n, 3*ArcCos
h[a*x]]/(32*3^n*a^5) + (5^(-1 - n)*Gamma[1 + n, 5*ArcCosh[a*x]])/(32*a^5)
```

Rubi [A] time = 0.247297, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5670, 5448, 3308, 2181}

$$\frac{5^{-n-1}\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-5\cosh^{-1}(ax)\right)}{32a^5}+\frac{3^{-n}\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+\right.}{32a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCosh[a*x]^n,x]
```


```
[Out] (5^(-1 - n)*ArcCosh[a*x]^n*Gamma[1 + n, -5*ArcCosh[a*x]])/(32*a^5*(-ArcCosh
[a*x])^n) + (ArcCosh[a*x]^n*Gamma[1 + n, -3*ArcCosh[a*x]])/(32*3^n*a^5*(-Ar
cCosh[a*x])^n) + (ArcCosh[a*x]^n*Gamma[1 + n, -ArcCosh[a*x]])/(16*a^5*(-Arc
Cosh[a*x])^n) + Gamma[1 + n, ArcCosh[a*x]]/(16*a^5) + Gamma[1 + n, 3*ArcCos
h[a*x]]/(32*3^n*a^5) + (5^(-1 - n)*Gamma[1 + n, 5*ArcCosh[a*x]])/(32*a^5)
```


Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^4 \cosh^{-1}(ax)^n\,dx &= \frac{\text{Subst}\left(\int x^n \cosh^4(x)\sinh(x)\,dx, x, \cosh^{-1}(ax)\right)}{a^5}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{8}x^n\sinh(x)+\frac{3}{16}x^n\sinh(3x)+\frac{1}{16}x^n\sinh(5x)\right)dx, x, \cosh^{-1}(ax)\right)}{a^5}\\ &= \frac{\text{Subst}\left(\int x^n\sinh(5x)\,dx, x, \cosh^{-1}(ax)\right)}{16a^5}+\frac{\text{Subst}\left(\int x^n\sinh(x)\,dx, x, \cosh^{-1}(ax)\right)}{8a^5}+\frac{3\,\text{Subst}\left(\right.}{}\\ &= -\frac{\text{Subst}\left(\int e^{-5x}x^n\,dx, x, \cosh^{-1}(ax)\right)}{32a^5}+\frac{\text{Subst}\left(\int e^{5x}x^n\,dx, x, \cosh^{-1}(ax)\right)}{32a^5}-\frac{\text{Subst}\left(\int e^{-x}x^n\,dx,\right.}{16a^5}\\ &= \frac{5^{-1-n}\left(-\cosh^{-1}(ax)\right)^{-n}\cosh^{-1}(ax)^n\Gamma\left(1+n,-5\cosh^{-1}(ax)\right)}{32a^5}+\frac{3^{-n}\left(-\cosh^{-1}(ax)\right)^{-n}\cosh^{-1}(ax}{32a^5}\end{aligned}$$

Mathematica [A] time = 0.193687, size = 144, normalized size = 0.83

$$\frac{5^{-n}\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-5\cosh^{-1}(ax)\right)+5\,3^{-n}\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCosh[a*x]^n,x]
```

```
[Out] ((ArcCosh[a*x]^n*Gamma[1 + n, -5*ArcCosh[a*x]])/(5^n*(-ArcCosh[a*x])^n) + (
5*ArcCosh[a*x]^n*Gamma[1 + n, -3*ArcCosh[a*x]])/(3^n*(-ArcCosh[a*x])^n) + (
```

```
10*ArcCosh[a*x]^n*Gamma[1 + n, -ArcCosh[a*x]])/(-ArcCosh[a*x])^n + 10*Gamma
[1 + n, ArcCosh[a*x]] + (5*Gamma[1 + n, 3*ArcCosh[a*x]])/3^n + Gamma[1 + n,
 5*ArcCosh[a*x]]/5^n)/(160*a^5)
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int x^4 (\operatorname{arccosh}(ax))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)^n,x)
```

```
[Out] int(x^4*arccosh(a*x)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arcosh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(x^4*arccosh(a*x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4 \operatorname{arcosh}(ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^4*arccosh(a*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{acosh}^n(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)**n,x)
```

```
[Out] Integral(x**4*acosh(a*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^n,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.128 $\int x^3 \cosh^{-1}(ax)^n \, dx$

Optimal. Leaf size=117

$$\frac{2^{-2(n+3)} \cosh^{-1}(ax)^n \left(-\cosh^{-1}(ax)\right)^{-n} \text{Gamma}\left(n+1, -4\cosh^{-1}(ax)\right)}{a^4} + \frac{2^{-n-4} \cosh^{-1}(ax)^n \left(-\cosh^{-1}(ax)\right)^{-n} \text{Gamma}\left(\right.}{a^4}$$

```
[Out] (ArcCosh[a*x]^n*Gamma[1 + n, -4*ArcCosh[a*x]])/(2^(2*(3 + n))*a^4*(-ArcCosh
[a*x])^n) + (2^(-4 - n)*ArcCosh[a*x]^n*Gamma[1 + n, -2*ArcCosh[a*x]])/(a^4*
(-ArcCosh[a*x])^n) + (2^(-4 - n)*Gamma[1 + n, 2*ArcCosh[a*x]])/a^4 + Gamma[
1 + n, 4*ArcCosh[a*x]]/(2^(2*(3 + n))*a^4)
```

Rubi [A] time = 0.19092, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5670, 5448, 3308, 2181}

$$\frac{2^{-2(n+3)} \cosh^{-1}(ax)^n \left(-\cosh^{-1}(ax)\right)^{-n} \text{Gamma}\left(n+1, -4\cosh^{-1}(ax)\right)}{a^4} + \frac{2^{-n-4} \cosh^{-1}(ax)^n \left(-\cosh^{-1}(ax)\right)^{-n} \text{Gamma}\left(\right.}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCosh[a*x]^n,x]
```

```
[Out] (ArcCosh[a*x]^n*Gamma[1 + n, -4*ArcCosh[a*x]])/(2^(2*(3 + n))*a^4*(-ArcCosh
[a*x])^n) + (2^(-4 - n)*ArcCosh[a*x]^n*Gamma[1 + n, -2*ArcCosh[a*x]])/(a^4*
(-ArcCosh[a*x])^n) + (2^(-4 - n)*Gamma[1 + n, 2*ArcCosh[a*x]])/a^4 + Gamma[
1 + n, 4*ArcCosh[a*x]]/(2^(2*(3 + n))*a^4)
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^3\cosh^{-1}(ax)^n\,dx=\frac{\text{Subst}\left(\int x^n\cosh^3(x)\sinh(x)\,dx,x,\cosh^{-1}(ax)\right)}{a^4}$$
$$=\frac{\text{Subst}\left(\int\left(\frac{1}{4}x^n\sinh(2x)+\frac{1}{8}x^n\sinh(4x)\right)dx,x,\cosh^{-1}(ax)\right)}{a^4}$$
$$=\frac{\text{Subst}\left(\int x^n\sinh(4x)\,dx,x,\cosh^{-1}(ax)\right)}{8a^4}+\frac{\text{Subst}\left(\int x^n\sinh(2x)\,dx,x,\cosh^{-1}(ax)\right)}{4a^4}$$
$$=-\frac{\text{Subst}\left(\int e^{-4x}x^n\,dx,x,\cosh^{-1}(ax)\right)}{16a^4}+\frac{\text{Subst}\left(\int e^{4x}x^n\,dx,x,\cosh^{-1}(ax)\right)}{16a^4}-\frac{\text{Subst}\left(\int e^{-2x}x^n\,dx\right.}{8a^4}$$
$$=\frac{4^{-3-n}\left(-\cosh^{-1}(ax)\right)^{-n}\cosh^{-1}(ax)^n\Gamma\left(1+n,-4\cosh^{-1}(ax)\right)}{a^4}+\frac{2^{-4-n}\left(-\cosh^{-1}(ax)\right)^{-n}\cosh^{-1}(}{a^4}$$

Mathematica [A] time = 0.0950426, size = 97, normalized size = 0.83

$$\frac{4^{-n-3}\left(-\cosh^{-1}(ax)\right)^{-n}\left(\left(-\cosh^{-1}(ax)\right)^n\left(2^{n+2}\text{Gamma}\left(n+1,2\cosh^{-1}(ax)\right)+\text{Gamma}\left(n+1,4\cosh^{-1}(ax)\right)\right)+\cosh}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCosh[a*x]^n,x]
```

```
[Out] (4^(-3 - n)*(ArcCosh[a*x]^n*Gamma[1 + n, -4*ArcCosh[a*x]] + 2^(2 + n)*ArcCo
sh[a*x]^n*Gamma[1 + n, -2*ArcCosh[a*x]] + (-ArcCosh[a*x])^n*(2^(2 + n)*Gamm
a[1 + n, 2*ArcCosh[a*x]] + Gamma[1 + n, 4*ArcCosh[a*x]])))/(a^4*(-ArcCosh[a
```

```
*x])^n)
```

Maple [F] time = 0.08, size = 0, normalized size = 0.

$$\int x^3 (\operatorname{arccosh}(ax))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)^n,x)
```

```
[Out] int(x^3*arccosh(a*x)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arcosh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(x^3*arccosh(a*x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^3 \operatorname{arcosh}(ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^3*arccosh(a*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{acosh}^n(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**n,x)
```

```
[Out] Integral(x**3*acosh(a*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^n,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.129 $\int x^2 \cosh^{-1}(ax)^n \, dx$

Optimal. Leaf size=113

$$\frac{3^{-n-1}\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-3\cosh^{-1}(ax)\right)}{8a^3}+\frac{\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-\right.}{8a^3}$$

```
[Out] (3^(-1 - n)*ArcCosh[a*x]^n*Gamma[1 + n, -3*ArcCosh[a*x]])/(8*a^3*(-ArcCosh[
a*x])^n) + (ArcCosh[a*x]^n*Gamma[1 + n, -ArcCosh[a*x]])/(8*a^3*(-ArcCosh[a*
x])^n) + Gamma[1 + n, ArcCosh[a*x]]/(8*a^3) + (3^(-1 - n)*Gamma[1 + n, 3*Ar
cCosh[a*x]])/(8*a^3)
```

Rubi [A] time = 0.182161, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5670, 5448, 3308, 2181}

$$\frac{3^{-n-1}\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-3\cosh^{-1}(ax)\right)}{8a^3}+\frac{\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-\right.}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCosh[a*x]^n,x]
```


```
[Out] (3^(-1 - n)*ArcCosh[a*x]^n*Gamma[1 + n, -3*ArcCosh[a*x]])/(8*a^3*(-ArcCosh[
a*x])^n) + (ArcCosh[a*x]^n*Gamma[1 + n, -ArcCosh[a*x]])/(8*a^3*(-ArcCosh[a*
x])^n) + Gamma[1 + n, ArcCosh[a*x]]/(8*a^3) + (3^(-1 - n)*Gamma[1 + n, 3*Ar
cCosh[a*x]])/(8*a^3)
```


Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^2\cosh^{-1}(ax)^n\,dx = \frac{\text{Subst}\left(\int x^n\cosh^2(x)\sinh(x)\,dx, x, \cosh^{-1}(ax)\right)}{a^3}$$

$$= \frac{\text{Subst}\left(\int\left(\frac{1}{4}x^n\sinh(x)+\frac{1}{4}x^n\sinh(3x)\right)dx, x, \cosh^{-1}(ax)\right)}{a^3}$$

$$= \frac{\text{Subst}\left(\int x^n\sinh(x)\,dx, x, \cosh^{-1}(ax)\right)}{4a^3} + \frac{\text{Subst}\left(\int x^n\sinh(3x)\,dx, x, \cosh^{-1}(ax)\right)}{4a^3}$$

$$= -\frac{\text{Subst}\left(\int e^{-3x}x^n\,dx, x, \cosh^{-1}(ax)\right)}{8a^3} - \frac{\text{Subst}\left(\int e^{-x}x^n\,dx, x, \cosh^{-1}(ax)\right)}{8a^3} + \frac{\text{Subst}\left(\int e^{x}x^n\,dx,\right.}{8a^3}$$

$$= \frac{3^{-1-n}\left(-\cosh^{-1}(ax)\right)^{-n}\cosh^{-1}(ax)^n\Gamma\left(1+n,-3\cosh^{-1}(ax)\right)}{8a^3} + \frac{\left(-\cosh^{-1}(ax)\right)^{-n}\cosh^{-1}(ax)^n\Gamma}{8a^3}$$

Mathematica [A] time = 0.110834, size = 95, normalized size = 0.84

$$\frac{3^{-n-1}\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-3\cosh^{-1}(ax)\right)+\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,\right.}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCosh[a*x]^n,x]
```

```
[Out] ((3^(-1 - n)*ArcCosh[a*x]^n*Gamma[1 + n, -3*ArcCosh[a*x]])/(-ArcCosh[a*x])^
n + (ArcCosh[a*x]^n*Gamma[1 + n, -ArcCosh[a*x]])/(-ArcCosh[a*x])^n + Gamma[
1 + n, ArcCosh[a*x]] + 3^(-1 - n)*Gamma[1 + n, 3*ArcCosh[a*x]])/(8*a^3)
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int x^{2} (\operatorname{arccosh}(ax))^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)^n,x)
```

```
[Out] int(x^2*arccosh(a*x)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2} \operatorname{arcosh}(ax)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(x^2*arccosh(a*x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^{2} \operatorname{arcosh}(ax)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^2*arccosh(a*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2} \operatorname{acosh}^{n}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)**n,x)
```

```
[Out] Integral(x**2*acosh(a*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^n,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.130 $\int x \cosh^{-1}(ax)^n \, dx$

Optimal. Leaf size=59

$$\frac{2^{-n-3}\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-2\cosh^{-1}(ax)\right)}{a^2}+\frac{2^{-n-3}\text{Gamma}\left(n+1,2\cosh^{-1}(ax)\right)}{a^2}$$

```
[Out] (2^(-3 - n)*ArcCosh[a*x]^n*Gamma[1 + n, -2*ArcCosh[a*x]])/(a^2*(-ArcCosh[a*
x])^n) + (2^(-3 - n)*Gamma[1 + n, 2*ArcCosh[a*x]])/a^2
```

Rubi [A] time = 0.087337, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5670, 5448, 12, 3308, 2181}

$$\frac{2^{-n-3}\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-2\cosh^{-1}(ax)\right)}{a^2}+\frac{2^{-n-3}\text{Gamma}\left(n+1,2\cosh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCosh[a*x]^n,x]
```

```
[Out] (2^(-3 - n)*ArcCosh[a*x]^n*Gamma[1 + n, -2*ArcCosh[a*x]])/(a^2*(-ArcCosh[a*
x])^n) + (2^(-3 - n)*Gamma[1 + n, 2*ArcCosh[a*x]])/a^2
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x\cosh^{-1}(ax)^n\,dx &= \frac{\text{Subst}\left(\int x^n\cosh(x)\sinh(x)\,dx, x, \cosh^{-1}(ax)\right)}{a^2}\\
&= \frac{\text{Subst}\left(\int \frac{1}{2}x^n\sinh(2x)\,dx, x, \cosh^{-1}(ax)\right)}{a^2}\\
&= \frac{\text{Subst}\left(\int x^n\sinh(2x)\,dx, x, \cosh^{-1}(ax)\right)}{2a^2}\\
&= -\frac{\text{Subst}\left(\int e^{-2x}x^n\,dx, x, \cosh^{-1}(ax)\right)}{4a^2} + \frac{\text{Subst}\left(\int e^{2x}x^n\,dx, x, \cosh^{-1}(ax)\right)}{4a^2}\\
&= \frac{2^{-3-n}\left(-\cosh^{-1}(ax)\right)^{-n}\cosh^{-1}(ax)^n\Gamma\left(1+n, -2\cosh^{-1}(ax)\right)}{a^2} + \frac{2^{-3-n}\Gamma\left(1+n, 2\cosh^{-1}(ax)\right)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.0447461, size = 58, normalized size = 0.98

$$\frac{2^{-n-3}\left(-\cosh^{-1}(ax)\right)^{-n}\left(\left(-\cosh^{-1}(ax)\right)^n\text{Gamma}\left(n+1, 2\cosh^{-1}(ax)\right) + \cosh^{-1}(ax)^n\text{Gamma}\left(n+1, -2\cosh^{-1}(ax)\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCosh[a*x]^n,x]
```

```
[Out] (2^(-3 - n)*(ArcCosh[a*x]^n*Gamma[1 + n, -2*ArcCosh[a*x]] + (-ArcCosh[a*x])
^n*Gamma[1 + n, 2*ArcCosh[a*x]]))/(a^2*(-ArcCosh[a*x])^n)
```

Maple [C] time = 0.031, size = 38, normalized size = 0.6

$$\frac{(\operatorname{arccosh}(ax))^{2+n}}{a^2(2+n)} {}_1F_2(1+\frac{n}{2}; \frac{3}{2}, 2+\frac{n}{2}; (\operatorname{arccosh}(ax))^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^n,x)
```

```
[Out] 1/a^2/(2+n)*arccosh(a*x)^(2+n)*hypergeom([1+1/2*n],[3/2,2+1/2*n],arccosh(a*
x)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcosh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(x*arccosh(a*x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x \operatorname{arcosh}(ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x*arccosh(a*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{acosh}^n(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**n,x)
```

```
[Out] Integral(x*acosh(a*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^n,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.131 $\int \cosh^{-1}(ax)^n\,dx$

Optimal. Leaf size=49

$$\frac{\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\operatorname{Gamma}\left(n+1,-\cosh^{-1}(ax)\right)}{2a}+\frac{\operatorname{Gamma}\left(n+1,\cosh^{-1}(ax)\right)}{2a}$$

```
[Out] (ArcCosh[a*x]^n*Gamma[1 + n, -ArcCosh[a*x]])/(2*a*(-ArcCosh[a*x])^n) + Gamm
a[1 + n, ArcCosh[a*x]]/(2*a)
```

Rubi [A] time = 0.0492336, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5658, 3308, 2181}

$$\frac{\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\operatorname{Gamma}\left(n+1,-\cosh^{-1}(ax)\right)}{2a}+\frac{\operatorname{Gamma}\left(n+1,\cosh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^n,x]
```

```
[Out] (ArcCosh[a*x]^n*Gamma[1 + n, -ArcCosh[a*x]])/(2*a*(-ArcCosh[a*x])^n) + Gamm
a[1 + n, ArcCosh[a*x]]/(2*a)
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
```

```
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \cosh^{-1}(ax)^n\,dx = \frac{\text{Subst}\left(\int x^n\sinh(x)\,dx, x, \cosh^{-1}(ax)\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int e^{-x}x^n\,dx, x, \cosh^{-1}(ax)\right)}{2a} + \frac{\text{Subst}\left(\int e^{x}x^n\,dx, x, \cosh^{-1}(ax)\right)}{2a}$$
$$= \frac{\left(-\cosh^{-1}(ax)\right)^{-n}\cosh^{-1}(ax)^n\Gamma\left(1+n, -\cosh^{-1}(ax)\right)}{2a} + \frac{\Gamma\left(1+n, \cosh^{-1}(ax)\right)}{2a}$$

Mathematica [A] time = 0.0241498, size = 43, normalized size = 0.88

$$\frac{\cosh^{-1}(ax)^n\left(-\cosh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1, -\cosh^{-1}(ax)\right)+\text{Gamma}\left(n+1, \cosh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^n,x]
```

```
[Out] ((ArcCosh[a*x]^n*Gamma[1 + n, -ArcCosh[a*x]])/(-ArcCosh[a*x])^n + Gamma[1 +
 n, ArcCosh[a*x]])/(2*a)
```

Maple [C] time = 0.041, size = 40, normalized size = 0.8

$$\frac{(\text{arccosh}\,(ax))^{2+n}}{a\,(2+n)}\,{}_1F_2(1+\frac{n}{2};\frac{3}{2},2+\frac{n}{2};\frac{(\text{arccosh}\,(ax))^2}{4})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^n,x)
```

```
[Out] 1/a/(2+n)*arccosh(a*x)^(2+n)*hypergeom([1+1/2*n],[3/2,2+1/2*n],1/4*arccosh(
a*x)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcosh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\operatorname{arcosh}(ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acosh}^n(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**n,x)
```

```
[Out] Integral(acosh(a*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^n,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.132 $\int \frac{\cosh^{-1}(ax)^n}{x} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\cosh^{-1}(ax)^n}{x}, x\right)$$

```
[Out] Unintegrable[ArcCosh[a*x]^n/x, x]
```

Rubi [A] time = 0.017206, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^{-1}(ax)^n}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCosh[a*x]^n/x,x]
```

```
[Out] Defer[Int][ArcCosh[a*x]^n/x, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^n}{x} dx = \int \frac{\cosh^{-1}(ax)^n}{x} dx$$

Mathematica [A] time = 0.32685, size = 0, normalized size = 0.

$$\int \frac{\cosh^{-1}(ax)^n}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCosh[a*x]^n/x,x]
```

```
[Out] Integrate[ArcCosh[a*x]^n/x, x]
```

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{arccosh}\,(ax))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^n/x,x)
```

```
[Out] int(arccosh(a*x)^n/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arcosh}\,(ax)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^n/x,x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^n/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{arcosh}\,(ax)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^n/x,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^n/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{acosh}^n\,(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**n/x,x)
```

```
[Out] Integral(acosh(a*x)**n/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^n/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.133 $\int x^3 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=84

$$\frac{1}{4}x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{3bx\sqrt{cx-1}\sqrt{cx+1}}{32c^3}-\frac{3b\cosh^{-1}(cx)}{32c^4}-\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{16c}$$

```
[Out] (-3*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(32*c^3) - (b*x^3*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])/(16*c) - (3*b*ArcCosh[c*x])/(32*c^4) + (x^4*(a + b*ArcCosh[c*x])
)/4
```

Rubi [A] time = 0.0390041, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5662, 100, 12, 90, 52}

$$\frac{1}{4}x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{3bx\sqrt{cx-1}\sqrt{cx+1}}{32c^3}-\frac{3b\cosh^{-1}(cx)}{32c^4}-\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(32*c^3) - (b*x^3*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])/(16*c) - (3*b*ArcCosh[c*x])/(32*c^4) + (x^4*(a + b*ArcCosh[c*x])
)/4
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\cosh^{-1}(cx)\right)dx &= \frac{1}{4}x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{4}(bc)\int\frac{x^4}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\
&= -\frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}+\frac{1}{4}x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{b\int\frac{3x^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{16c}\\
&= -\frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}+\frac{1}{4}x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{(3b)\int\frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{16c}\\
&= -\frac{3bx\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}+\frac{1}{4}x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{(3b)\int\frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{32c^3}\\
&= -\frac{3bx\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}-\frac{3b\cosh^{-1}(cx)}{32c^4}+\frac{1}{4}x^4\left(a+b\cosh^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] time = 0.037223, size = 105, normalized size = 1.25

$$\frac{ax^4}{4}-\frac{3bx\sqrt{cx-1}\sqrt{cx+1}}{32c^3}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{cx-1}}{\sqrt{cx+1}}\right)}{16c^4}-\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{16c}+\frac{1}{4}bx^4\cosh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (a*x^4)/4 - (3*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(32*c^3) - (b*x^3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])/(16*c) + (b*x^4*ArcCosh[c*x])/4 - (3*b*ArcTanh[Sqrt[-
1 + c*x]/Sqrt[1 + c*x]])/(16*c^4)
```

Maple [A] time = 0.012, size = 109, normalized size = 1.3

$$\frac{x^4a}{4}+\frac{bx^4\mathrm{arccosh}\,(cx)}{4}-\frac{bx^3}{16\,c}\sqrt{cx-1}\sqrt{cx+1}-\frac{3\,bx}{32\,c^3}\sqrt{cx-1}\sqrt{cx+1}-\frac{3\,b}{32\,c^4}\sqrt{cx-1}\sqrt{cx+1}\ln\left(cx+\sqrt{c^2x^2-1}\right)\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arccosh(c*x)-1/16*b*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-3
/32*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-3/32/c^4*b*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.1697, size = 130, normalized size = 1.55

$$\frac{1}{4}\,ax^4+\frac{1}{32}\left(8\,x^4\,\mathrm{arcosh}\,(cx)-\left(\frac{2\,\sqrt{c^2x^2-1}x^3}{c^2}+\frac{3\,\sqrt{c^2x^2-1}x}{c^4}+\frac{3\,\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqr
t(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt
(c^2)*c^4))*c)*b
```

Fricas [A] time = 2.45928, size = 161, normalized size = 1.92

$$\frac{8\,ac^4x^4 + \left(8\,bc^4x^4 - 3\,b\right)\log\left(cx + \sqrt{c^2x^2 - 1}\right) - \left(2\,bc^3x^3 + 3\,bcx\right)\sqrt{c^2x^2 - 1}}{32\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a*c^4*x^4 + (8*b*c^4*x^4 - 3*b)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b
*c^3*x^3 + 3*b*c*x)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [A] time = 1.45855, size = 87, normalized size = 1.04

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{acosh}(cx)}{4} - \frac{bx^3\sqrt{c^2x^2-1}}{16c} - \frac{3bx\sqrt{c^2x^2-1}}{32c^3} - \frac{3b \operatorname{acosh}(cx)}{32c^4} & \text{for}\ c \neq 0 \\ \frac{x^4\left(a+\frac{i\pi b}{2}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*acosh(c*x)/4 - b*x**3*sqrt(c**2*x**2 - 1)/(16*
c) - 3*b*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 3*b*acosh(c*x)/(32*c**4), Ne(c,
0)), (x**4*(a + I*pi*b/2)/4, True))
```

Giac [A] time = 1.46898, size = 123, normalized size = 1.46

$$\frac{1}{4}\,ax^4 + \frac{1}{32}\left(8\,x^4\log\left(cx + \sqrt{c^2x^2 - 1}\right) - \left(\sqrt{c^2x^2 - 1}x\left(\frac{2\,x^2}{c^2} + \frac{3}{c^4}\right) - \frac{3\,\log\left(\left|-x|c| + \sqrt{c^2x^2 - 1}\right|\right)}{c^4|c|}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/4*a*x^4 + 1/32*(8*x^4*log(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)*x
*(2*x^2/c^2 + 3/c^4) - 3*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^4*abs(c
)))*c)*b
```

3.134 $\int x^2 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=71

$$\frac{1}{3}x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2b\sqrt{cx-1}\sqrt{cx+1}}{9c^3}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{9c}$$

```
[Out] (-2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^3) - (b*x^2*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])/(9*c) + (x^3*(a + b*ArcCosh[c*x]))/3
```

Rubi [A] time = 0.0295427, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5662, 100, 12, 74}

$$\frac{1}{3}x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2b\sqrt{cx-1}\sqrt{cx+1}}{9c^3}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^3) - (b*x^2*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])/(9*c) + (x^3*(a + b*ArcCosh[c*x]))/3
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+b\cosh^{-1}(cx)\right)dx &= \frac{1}{3}x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}(bc)\int\frac{x^3}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\
&= -\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9c}+\frac{1}{3}x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{b\int\frac{2x}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{9c}\\
&= -\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9c}+\frac{1}{3}x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{(2b)\int\frac{x}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{9c}\\
&= -\frac{2b\sqrt{-1+cx}\sqrt{1+cx}}{9c^3}-\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9c}+\frac{1}{3}x^3\left(a+b\cosh^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] time = 0.043911, size = 54, normalized size = 0.76

$$\frac{1}{9}\left(3ax^3-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^2x^2+2\right)}{c^3}+3bx^3\cosh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*a*x^3 - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2 + c^2*x^2))/c^3 + 3*b*x^3*Arc
Cosh[c*x])/9
```

Maple [A] time = 0.004, size = 55, normalized size = 0.8

$$\frac{1}{c^3}\left(\frac{c^3x^3a}{3}+b\left(\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-\frac{c^2x^2+2}{9}\sqrt{cx-1}\sqrt{cx+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^3*(1/3*c^3*x^3*a+b*(1/3*c^3*x^3*arccosh(c*x)-1/9*(c*x-1)^(1/2)*(c*x+1)^
(1/2)*(c^2*x^2+2)))
```

Maxima [A] time = 1.17177, size = 78, normalized size = 1.1

$$\frac{1}{3}\,ax^3+\frac{1}{9}\left(3\,x^3\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt
(c^2*x^2 - 1)/c^4))*b
```

Fricas [A] time = 2.42938, size = 140, normalized size = 1.97

$$\frac{3\,bc^3x^3\log\left(cx+\sqrt{c^2x^2-1}\right)+3\,ac^3x^3-\left(bc^2x^2+2\,b\right)\sqrt{c^2x^2-1}}{9\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*b*c^3*x^3*log(c*x + sqrt(c^2*x^2 - 1)) + 3*a*c^3*x^3 - (b*c^2*x^2 +
2*b)*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [A] time = 0.67728, size = 71, normalized size = 1.

$$\begin{cases}\frac{ax^3}{3}+\frac{bx^3\,\mathrm{acosh}\,(cx)}{3}-\frac{bx^2\sqrt{c^2x^2-1}}{9c}-\frac{2b\sqrt{c^2x^2-1}}{9c^3} & \text{for } c\neq 0\\ \frac{x^3\left(a+\frac{i\pi b}{2}\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*x**3/3 + b*x**3*acosh(c*x)/3 - b*x**2*sqrt(c**2*x**2 - 1)/(9*c
) - 2*b*sqrt(c**2*x**2 - 1)/(9*c**3), Ne(c, 0)), (x**3*(a + I*pi*b/2)/3, Tr
ue))
```

Giac [A] time = 1.45294, size = 84, normalized size = 1.18

$$\frac{1}{3}\,ax^3 + \frac{1}{9}\left(3\,x^3\log\left(cx + \sqrt{c^2x^2 - 1}\right) - \frac{\left(c^2x^2 - 1\right)^{\frac{3}{2}} + 3\,\sqrt{c^2x^2 - 1}}{c^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*a*x^3 + 1/9*(3*x^3*log(c*x + sqrt(c^2*x^2 - 1)) - ((c^2*x^2 - 1)^(3/2)
+ 3*sqrt(c^2*x^2 - 1))/c^3)*b
```

3.135 $\int x\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}x^2\left(a+b\cosh^{-1}(cx)\right)-\frac{b\cosh^{-1}(cx)}{4c^2}-\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{4c}$$

```
[Out] -(b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (b*ArcCosh[c*x])/(4*c^2) + (x^2
*(a + b*ArcCosh[c*x]))/2
```

Rubi [A] time = 0.0197901, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5662, 90, 52}

$$\frac{1}{2}x^2\left(a+b\cosh^{-1}(cx)\right)-\frac{b\cosh^{-1}(cx)}{4c^2}-\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (b*ArcCosh[c*x])/(4*c^2) + (x^2
*(a + b*ArcCosh[c*x]))/2
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+b\cosh^{-1}(cx)\right)dx &= \frac{1}{2}x^2\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{2}(bc)\int\frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\
&= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c}+\frac{1}{2}x^2\left(a+b\cosh^{-1}(cx)\right)-\frac{b\int\frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{4c}\\
&= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{b\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}x^2\left(a+b\cosh^{-1}(cx)\right)
\end{aligned}
$$

Mathematica [A] time = 0.035222, size = 76, normalized size = 1.38

$$
\frac{ax^2}{2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{cx-1}}{\sqrt{cx+1}}\right)}{2c^2}+\frac{1}{2}bx^2\cosh^{-1}(cx)-\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{4c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (a*x^2)/2 - (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) + (b*x^2*ArcCosh[c*x])
/2 - (b*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]])/(2*c^2)
```

Maple [A] time = 0.004, size = 86, normalized size = 1.6

$$
\frac{ax^2}{2}+\frac{bx^2\operatorname{arccosh}(cx)}{2}-\frac{bx}{4c}\sqrt{cx-1}\sqrt{cx+1}-\frac{b}{4c^2}\sqrt{cx-1}\sqrt{cx+1}\ln\left(cx+\sqrt{c^2x^2-1}\right)\frac{1}{\sqrt{c^2x^2-1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arccosh(c*x)-1/4*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4/
c^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2
```

```
))
```

Maxima [A] time = 1.15047, size = 101, normalized size = 1.84

$$\frac{1}{2}\,ax^2+\frac{1}{4}\left(2\,x^2\,\mathrm{arcosh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2-1}x}{c^2}+\frac{\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^2}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^
2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^2)))*b
```

Fricas [A] time = 2.50037, size = 132, normalized size = 2.4

$$\frac{2\,ac^2x^2-\sqrt{c^2x^2-1}bcx+\left(2\,bc^2x^2-b\right)\log\left(cx+\sqrt{c^2x^2-1}\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*x^2 - sqrt(c^2*x^2 - 1)*b*c*x + (2*b*c^2*x^2 - b)*log(c*x + sq
rt(c^2*x^2 - 1)))/c^2
```

Sympy [A] time = 0.372885, size = 61, normalized size = 1.11

$$\begin{cases}\frac{ax^2}{2}+\frac{bx^2\,\mathrm{acosh}\left(cx\right)}{2}-\frac{bx\sqrt{c^2x^2-1}}{4c}-\frac{b\,\mathrm{acosh}\left(cx\right)}{4c^2} & \text{for } c\neq 0\\ \frac{x^2\left(a+\frac{i\pi b}{2}\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*acosh(c*x)/2 - b*x*sqrt(c**2*x**2 - 1)/(4*c) -
 b*acosh(c*x)/(4*c**2), Ne(c, 0)), (x**2*(a + I*pi*b/2)/2, True))
```

Giac [A] time = 1.43285, size = 108, normalized size = 1.96

$$\frac{1}{2}\,ax^2+\frac{1}{4}\left(2\,x^2\log\left(cx+\sqrt{c^2x^2-1}\right)-c\left(\frac{\sqrt{c^2x^2-1}x}{c^2}-\frac{\log\left(\left|-x|c|+\sqrt{c^2x^2-1}\right|\right)}{c^2|c|}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*x^2*log(c*x + sqrt(c^2*x^2 - 1)) - c*(sqrt(c^2*x^2 - 1)*
x/c^2 - log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^2*abs(c))))*b
```

3.136 $\int \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=35

$$ax-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}+bx\cosh^{-1}(cx)$$

```
[Out] a*x - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c + b*x*ArcCosh[c*x]
```

Rubi [A] time = 0.0143607, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5654, 74}

$$ax-\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c}+bx\cosh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCosh[c*x],x]
```

```
[Out] a*x - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c + b*x*ArcCosh[c*x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(cx)\right)dx = ax + b\int \cosh^{-1}(cx)\,dx$$
$$= ax + bx\cosh^{-1}(cx) - (bc)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$
$$= ax - \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c} + bx\cosh^{-1}(cx)$$

Mathematica [A] time = 0.0207335, size = 35, normalized size = 1.

$$ax - \frac{b\sqrt{cx-1}\sqrt{cx+1}}{c} + bx\cosh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCosh[c*x],x]
```

```
[Out] a*x - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c + b*x*ArcCosh[c*x]
```

Maple [A] time = 0.003, size = 34, normalized size = 1.

$$ax + \frac{b}{c}\left(cx\mathrm{arccosh}\,(cx) - \sqrt{cx-1}\sqrt{cx+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccosh(c*x),x)
```

```
[Out] a*x+b/c*(c*x*arccosh(c*x)-(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [A] time = 1.10648, size = 41, normalized size = 1.17

$$ax + \frac{\left(cx\,\mathrm{arcosh}\,(cx) - \sqrt{c^2x^2-1}\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(c*x),x, algorithm="maxima")
```

```
[Out] a*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b/c
```

Fricas [A] time = 2.49282, size = 95, normalized size = 2.71

$$\frac{bcx\log\left(cx+\sqrt{c^2x^2-1}\right)+acx-\sqrt{c^2x^2-1}b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(c*x),x, algorithm="fricas")
```

```
[Out] (b*c*x*log(c*x + sqrt(c^2*x^2 - 1)) + a*c*x - sqrt(c^2*x^2 - 1)*b)/c
```

Sympy [A] time = 0.166587, size = 31, normalized size = 0.89

$$ax+b\left(\begin{cases}x\operatorname{acosh}(cx)-\frac{\sqrt{c^2x^2-1}}{c} & \text{for } c\neq 0\\ \frac{i\pi x}{2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acosh(c*x),x)
```

```
[Out] a*x + b*Piecewise((x*acosh(c*x) - sqrt(c**2*x**2 - 1)/c, Ne(c, 0)), (I*pi*x
/2, True))
```

Giac [A] time = 1.29573, size = 55, normalized size = 1.57

$$\left(x\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{\sqrt{c^2x^2-1}}{c}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(c*x),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*b + a*x
```

3.137 $\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx$

Optimal. Leaf size=55

$$-\frac{1}{2}b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2b}+\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)$$

```
[Out] (a + b*ArcCosh[c*x])^2/(2*b) + (a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c
*x])] - (b*PolyLog[2, -E^(-2*ArcCosh[c*x])])/2
```

Rubi [A] time = 0.0766328, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5660, 3718, 2190, 2279, 2391}

$$\frac{1}{2}b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2b}+\log\left(e^{2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])/x,x]
```

```
[Out] -(a + b*ArcCosh[c*x])^2/(2*b) + (a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c
*x])] + (b*PolyLog[2, -E^(2*ArcCosh[c*x])])/2
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx &= \text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \cosh^{-1}(cx)\right)\\
&= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2b}+2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \cosh^{-1}(cx)\right)\\
&= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2b}+\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}(cx)}\right)-b\,\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,d\right.\\
&= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2b}+\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}(cx)}\right)-\frac{1}{2}b\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx\right.\\
&= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2b}+\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}(cx)}\right)+\frac{1}{2}b\,\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)
\end{aligned}$$

Mathematica [A] time = 0.0401445, size = 48, normalized size = 0.87

$$\frac{1}{2}b\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+2\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\right)-\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)\right)+a\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/x,x]
```

```
[Out] a*Log[x] + (b*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])])
 - PolyLog[2, -E^(-2*ArcCosh[c*x])]))/2
```

Maple [A] time = 0.03, size = 75, normalized size = 1.4

$$a\ln\left(cx\right)-\frac{b\left(\mathrm{arccosh}\left(cx\right)\right)^{2}}{2}+b\,\mathrm{arccosh}\left(cx\right)\ln\left(\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)^{2}+1\right)+\frac{b}{2}\mathrm{polylog}\left(2,-\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x,x)
```

```
[Out] a*ln(c*x)-1/2*b*arccosh(c*x)^2+b*arccosh(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))^2+1)+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{x}\,dx+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x) + a*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arcosh}\left(cx\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acosh}(cx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x,x)
```

```
[Out] Integral((a + b*acosh(c*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/x, x)
```

3.138 $\int \frac{a+b\cosh^{-1}(cx)}{x^2}\,dx$

Optimal. Leaf size=37

$$bc\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\cosh^{-1}(cx)}{x}$$

```
[Out] -((a + b*ArcCosh[c*x])/x) + b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rubi [A] time = 0.0239983, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5662, 92, 205}

$$bc\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{a+b\cosh^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/x^2,x]
```

```
[Out] -((a + b*ArcCosh[c*x])/x) + b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2}\,dx = -\frac{a+b\cosh^{-1}(cx)}{x}+(bc)\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$
$$= -\frac{a+b\cosh^{-1}(cx)}{x}+\left(bc^2\right)\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx, x, \sqrt{-1+cx}\sqrt{1+cx}\right)$$
$$= -\frac{a+b\cosh^{-1}(cx)}{x}+bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

Mathematica [A] time = 0.0716771, size = 65, normalized size = 1.76

$$-\frac{a}{x}+\frac{bc\sqrt{c^2x^2-1}\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{b\cosh^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcCosh[c*x])/x + (b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*
x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [A] time = 0.004, size = 59, normalized size = 1.6

$$-\frac{a}{x}-\frac{b\operatorname{arccosh}(cx)}{x}-bc\sqrt{cx-1}\sqrt{cx+1}\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2,x)
```

```
[Out] -a/x-b/x*arccosh(c*x)-c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arc
tan(1/(c^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.6757, size = 43, normalized size = 1.16

$$-\left(c\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)+\frac{\operatorname{arcosh}(cx)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -(c*arcsin(1/(sqrt(c^2)*abs(x))) + arccosh(c*x)/x)*b - a/x
```

Fricas [B] time = 2.5959, size = 171, normalized size = 4.62

$$\frac{2\,bcx\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+bx\log\left(-cx+\sqrt{c^2x^2-1}\right)+(bx-b)\log\left(cx+\sqrt{c^2x^2-1}\right)-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] (2*b*c*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) + b*x*log(-c*x + sqrt(c^2*x^2 - 1
)) + (b*x - b)*log(c*x + sqrt(c^2*x^2 - 1)) - a)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\operatorname{acosh}(cx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\operatorname{arcosh}(cx)+a}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/x^2, x)
```

3.139 $\int \frac{a+b\cosh^{-1}(cx)}{x^3}\,dx$

Optimal. Leaf size=43

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x}-\frac{a+b\cosh^{-1}(cx)}{2x^2}$$

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (a + b*ArcCosh[c*x])/(2*x^2)
```

Rubi [A] time = 0.0200365, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5662, 95}

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x}-\frac{a+b\cosh^{-1}(cx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/x^3,x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (a + b*ArcCosh[c*x])/(2*x^2)
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3}\,dx = -\frac{a+b\cosh^{-1}(cx)}{2x^2} + \frac{1}{2}(bc)\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$
$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{a+b\cosh^{-1}(cx)}{2x^2}$$

Mathematica [A] time = 0.0175018, size = 48, normalized size = 1.12

$$-\frac{a}{2x^2} - \frac{b\cosh^{-1}(cx)}{2x^2} + \frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/x^3,x]
```

```
[Out] -a/(2*x^2) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (b*ArcCosh[c*x])/(2
*x^2)
```

Maple [A] time = 0.004, size = 52, normalized size = 1.2

$$c^2\left(-\frac{a}{2\,c^2x^2} + b\left(-\frac{\operatorname{arccosh}(cx)}{2\,c^2x^2} + \frac{1}{2\,cx}\sqrt{cx-1}\sqrt{cx+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3,x)
```

```
[Out] c^2*(-1/2*a/c^2/x^2+b*(-1/2/c^2/x^2*arccosh(c*x)+1/2*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/c/x))
```

Maxima [A] time = 1.74016, size = 49, normalized size = 1.14

$$\frac{1}{2}b\left(\frac{\sqrt{c^2x^2-1}c}{x} - \frac{\operatorname{arcosh}(cx)}{x^2}\right) - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*(sqrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) - 1/2*a/x^2
```

Fricas [A] time = 2.31892, size = 108, normalized size = 2.51

$$\frac{\sqrt{c^2x^2-1}bcx+ax^2-b\log\left(cx+\sqrt{c^2x^2-1}\right)-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(c^2*x^2 - 1)*b*c*x + a*x^2 - b*log(c*x + sqrt(c^2*x^2 - 1)) - a)/
x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{acosh}(cx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcosh}(cx)+a}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/x^3, x)
```

3.140 $\int \frac{a+b\cosh^{-1}(cx)}{x^4}\,dx$

Optimal. Leaf size=71

$$-\frac{a+b\cosh^{-1}(cx)}{3x^3}+\frac{1}{6}bc^3\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6x^2}$$

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2) - (a + b*ArcCosh[c*x])/(3*x^3) +
 (b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/6
```

Rubi [A] time = 0.0334076, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5662, 103, 12, 92, 205}

$$-\frac{a+b\cosh^{-1}(cx)}{3x^3}+\frac{1}{6}bc^3\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/x^4,x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2) - (a + b*ArcCosh[c*x])/(3*x^3) +
 (b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/6
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{x^4}\,dx &= -\frac{a+b\cosh^{-1}(cx)}{3x^3}+\frac{1}{3}(bc)\int \frac{1}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\
&= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6x^2}-\frac{a+b\cosh^{-1}(cx)}{3x^3}+\frac{1}{6}(bc)\int \frac{c^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\
&= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6x^2}-\frac{a+b\cosh^{-1}(cx)}{3x^3}+\frac{1}{6}\left(bc^3\right)\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\
&= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6x^2}-\frac{a+b\cosh^{-1}(cx)}{3x^3}+\frac{1}{6}\left(bc^4\right)\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx,x,\sqrt{-1+cx}\sqrt{1+cx}\right)\\
&= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6x^2}-\frac{a+b\cosh^{-1}(cx)}{3x^3}+\frac{1}{6}bc^3\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)
\end{aligned}$$

Mathematica [A] time = 0.100809, size = 101, normalized size = 1.42

$$-\frac{a}{3x^3}+\frac{bc^3\sqrt{c^2x^2-1}\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6x^2}-\frac{b\cosh^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/x^4,x]
```

```
[Out] -a/(3*x^3) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2) - (b*ArcCosh[c*x])/
(3*x^3) + (b*c^3*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])
```

Maple [A] time = 0.006, size = 82, normalized size = 1.2

$$-\frac{a}{3\,x^3}-\frac{b\mathrm{arccosh}\,(cx)}{3\,x^3}-\frac{c^3b}{6}\sqrt{cx-1}\sqrt{cx+1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\frac{1}{\sqrt{c^2x^2-1}}+\frac{bc}{6\,x^2}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arccosh(c*x)-1/6*c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^
2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))+1/6*b*c*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/x^2
```

Maxima [A] time = 1.69955, size = 73, normalized size = 1.03

$$-\frac{1}{6}\left(\left(c^2\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)-\frac{\sqrt{c^2x^2-1}}{x^2}\right)c+\frac{2\,\mathrm{arcosh}\,(cx)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*arcsin(1/(sqrt(c^2)*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arcc
osh(c*x)/x^3)*b - 1/3*a/x^3
```

Fricas [A] time = 2.64239, size = 234, normalized size = 3.3

$$\frac{2\,bc^3x^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+2\,bx^3\log\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}bcx+2\left(bx^3-b\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*c^3*x^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 2*b*x^3*log(-c*x + sqrt
(c^2*x^2 - 1)) + sqrt(c^2*x^2 - 1)*b*c*x + 2*(b*x^3 - b)*log(c*x + sqrt(c^2
*x^2 - 1)) - 2*a)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acosh}(cx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acosh(c*x))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/x^4, x)
```

3.141 $\int \frac{a+b\cosh^{-1}(cx)}{x^5} dx$

Optimal. Leaf size=72

$$-\frac{a+b\cosh^{-1}(cx)}{4x^4}+\frac{bc^3\sqrt{cx-1}\sqrt{cx+1}}{6x}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{12x^3}$$

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(12*x^3) + (b*c^3*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])/(6*x) - (a + b*ArcCosh[c*x])/(4*x^4)
```

Rubi [A] time = 0.0322867, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5662, 103, 12, 95}

$$-\frac{a+b\cosh^{-1}(cx)}{4x^4}+\frac{bc^3\sqrt{cx-1}\sqrt{cx+1}}{6x}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/x^5,x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(12*x^3) + (b*c^3*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])/(6*x) - (a + b*ArcCosh[c*x])/(4*x^4)
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{x^5}\,dx &= -\frac{a+b\cosh^{-1}(cx)}{4x^4}+\frac{1}{4}(bc)\int \frac{1}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\
&= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{12x^3}-\frac{a+b\cosh^{-1}(cx)}{4x^4}+\frac{1}{12}(bc)\int \frac{2c^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\
&= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{12x^3}-\frac{a+b\cosh^{-1}(cx)}{4x^4}+\frac{1}{6}\left(bc^3\right)\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\
&= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{12x^3}+\frac{bc^3\sqrt{-1+cx}\sqrt{1+cx}}{6x}-\frac{a+b\cosh^{-1}(cx)}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.0289037, size = 50, normalized size = 0.69

$$\frac{-3a+bcx\sqrt{cx-1}\sqrt{cx+1}\left(2c^2x^2+1\right)-3b\cosh^{-1}(cx)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/x^5,x]
```

```
[Out] (-3*a + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 + 2*c^2*x^2) - 3*b*ArcCosh[c*
x])/(12*x^4)
```

Maple [A] time = 0.004, size = 62, normalized size = 0.9

$$c^4\left(-\frac{a}{4\,c^4x^4}+b\left(-\frac{\operatorname{arccosh}(cx)}{4\,c^4x^4}+\frac{2\,c^2x^2+1}{12\,c^3x^3}\sqrt{cx-1}\sqrt{cx+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^5,x)
```

```
[Out] c^4*(-1/4*a/c^4/x^4+b*(-1/4/c^4/x^4*arccosh(c*x)+1/12*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*(2*c^2*x^2+1)/c^3/x^3))
```

Maxima [A] time = 1.64523, size = 77, normalized size = 1.07

$$\frac{1}{12}\left(\left(\frac{2\sqrt{c^2x^2-1}c^2}{x}+\frac{\sqrt{c^2x^2-1}}{x^3}\right)c-\frac{3\,\operatorname{arcosh}(cx)}{x^4}\right)b-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*((2*sqrt(c^2*x^2 - 1)*c^2/x + sqrt(c^2*x^2 - 1)/x^3)*c - 3*arccosh(c*x
)/x^4)*b - 1/4*a/x^4
```

Fricas [A] time = 2.5122, size = 139, normalized size = 1.93

$$\frac{3\,ax^4-3\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(2\,bc^3x^3+bcx\right)\sqrt{c^2x^2-1}-3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*x^4 - 3*b*log(c*x + sqrt(c^2*x^2 - 1)) + (2*b*c^3*x^3 + b*c*x)*sq
rt(c^2*x^2 - 1) - 3*a)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\operatorname{acosh}(cx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**5,x)
```

```
[Out] Integral((a + b*acosh(c*x))/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/x^5, x)
```

3.142 $\int x^2\sqrt{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=213

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{-\frac{3a}{b}}\text{Erfi}\left(\right.}{4}$$

```
[Out] (x^3*Sqrt[a + b*ArcCosh[c*x]])/3 - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b
*ArcCosh[c*x]]/Sqrt[b]])/(16*c^3) - (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sq
rt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(48*c^3) - (Sqrt[b]*Sqrt[Pi]*Erfi
[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*c^3*E^(a/b)) - (Sqrt[b]*Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```

Rubi [A] time = 0.78825, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{-\frac{3a}{b}}\text{Erfi}\left(\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*ArcCosh[c*x]],x]
```


```
[Out] (x^3*Sqrt[a + b*ArcCosh[c*x]])/3 - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b
*ArcCosh[c*x]]/Sqrt[b]])/(16*c^3) - (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sq
rt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(48*c^3) - (Sqrt[b]*Sqrt[Pi]*Erfi
[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*c^3*E^(a/b)) - (Sqrt[b]*Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```


Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^2\sqrt{a+b\cosh^{-1}(cx)}\,dx = \frac{1}{3}x^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{1}{6}(bc)\int \frac{x^3}{\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

$$= \frac{1}{3}x^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{\cosh^3(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{6c^3}$$

$$= \frac{1}{3}x^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4\sqrt{a+bx}} + \frac{\cosh(3x)}{4\sqrt{a+bx}}\right) dx, x, \cosh^{-1}(cx)\right)}{6c^3}$$

$$= \frac{1}{3}x^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{24c^3} - \frac{b\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x\right.}{8c^3}$$

$$= \frac{1}{3}x^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{48c^3} - \frac{b\,\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{a+bx}}\,dx, x, \text{c}\right.}{48c^3}$$

$$= \frac{1}{3}x^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{24c^3} - \frac{\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\right.}{}$$

$$= \frac{1}{3}x^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3} - \frac{\sqrt{b}e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right.}{48c^3}$$

<u>Mathematica</u> [A] time = 0.480813, size = 214, normalized size = 1.

$$\frac{e^{-\frac{3a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\text{Gamma}\left(\frac{3}{2},-\frac{3(a+b\,\text{co}}{}\right.\right.}{72c^3\sqrt{-\frac{(a+b\,\text{c}}{}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcCosh[c*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Ga
mma[3/2, a/b + ArcCosh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2,
(-3*(a + b*ArcCosh[c*x]))/b] + 9*E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma
[3/2, -((a + b*ArcCosh[c*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCos
h[c*x])/b)]*Gamma[3/2, (3*(a + b*ArcCosh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sq
rt[-((a + b*ArcCosh[c*x])^2/b^2)])
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int x^2\sqrt{a+b\,\mathrm{arccosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{arcosh}\,(cx)+a}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a+b\,\mathrm{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.143 $\int x\sqrt{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=145

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2}-\frac{\sqrt{a+b\cosh^{-1}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\cosh^{-1}(cx)}$$

```
[Out] -Sqrt[a + b*ArcCosh[c*x]]/(4*c^2) + (x^2*Sqrt[a + b*ArcCosh[c*x]])/2 - (Sqr
t[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]
)/(16*c^2) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]])/(16*c^2*E^((2*a)/b))
```

Rubi [A] time = 0.649158, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2}-\frac{\sqrt{a+b\cosh^{-1}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\cosh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] -Sqrt[a + b*ArcCosh[c*x]]/(4*c^2) + (x^2*Sqrt[a + b*ArcCosh[c*x]])/2 - (Sqr
t[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]
)/(16*c^2) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]])/(16*c^2*E^((2*a)/b))
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
```

```
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x\sqrt{a+b\cosh^{-1}(cx)}\,dx = \frac{1}{2}x^2\sqrt{a+b\cosh^{-1}(cx)} - \frac{1}{4}(bc)\int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

$$= \frac{1}{2}x^2\sqrt{a+b\cosh^{-1}(cx)} - \frac{b\,\mathrm{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{4c^2}$$

$$= \frac{1}{2}x^2\sqrt{a+b\cosh^{-1}(cx)} - \frac{b\,\mathrm{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}} + \frac{\cosh(2x)}{2\sqrt{a+bx}}\right)dx, x, \cosh^{-1}(cx)\right)}{4c^2}$$

$$= -\frac{\sqrt{a+b\cosh^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\cosh^{-1}(cx)} - \frac{b\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{8c^2}$$

$$= -\frac{\sqrt{a+b\cosh^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\cosh^{-1}(cx)} - \frac{b\,\mathrm{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{16c^2} - \frac{b\,\mathrm{Su}}{}$$

$$= -\frac{\sqrt{a+b\cosh^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\cosh^{-1}(cx)} - \frac{\mathrm{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{8c^2}$$

$$= -\frac{\sqrt{a+b\cosh^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\cosh^{-1}(cx)} - \frac{\sqrt{b}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2} - \frac{\sqrt{b}e^{-\frac{2a}{b}}}{}$$

Mathematica [A] time = 0.380288, size = 136, normalized size = 0.94

$$\frac{-\sqrt{2\pi}\sqrt{b}\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)+\sqrt{2\pi}\sqrt{b}\left(\sinh\left(\frac{2a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)+8\,c}{32c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (8*Sqrt[a + b*ArcCosh[c*x]]*Cosh[2*ArcCosh[c*x]] + Sqrt[b]*Sqrt[2*Pi]*Erfi[
(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(-Cosh[(2*a)/b] + Sinh[(2*a)/b]
) - Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cos
h[(2*a)/b] + Sinh[(2*a)/b]))/(32*c^2)
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int x\sqrt{a+b\mathrm{arccosh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int(x*(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{arcosh}(cx) + a} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \sqrt{a + b \operatorname{acosh}(cx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.144 $\int \sqrt{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=102

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\cosh^{-1}(cx)}$$

```
[Out] x*Sqrt[a + b*ArcCosh[c*x]] - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCo
sh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/
Sqrt[b]])/(4*c*E^(a/b))
```

Rubi [A] time = 0.412052, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5654, 5781, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\cosh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] x*Sqrt[a + b*ArcCosh[c*x]] - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCo
sh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/
Sqrt[b]])/(4*c*E^(a/b))
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
```

```
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a+b\cosh^{-1}(cx)}\,dx = x\sqrt{a+b\cosh^{-1}(cx)} - \frac{1}{2}(bc)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$
$$= x\sqrt{a+b\cosh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{2c}$$
$$= x\sqrt{a+b\cosh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{4c} - \frac{b\,\text{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{4c}$$
$$= x\sqrt{a+b\cosh^{-1}(cx)} - \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{2c} - \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{2c}$$
$$= x\sqrt{a+b\cosh^{-1}(cx)} - \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}$$

Mathematica [A] time = 0.222177, size = 100, normalized size = 0.98

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(\frac{e^{\frac{2a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}} + \frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}}\right)}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcCosh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])/Sqr
t[a/b + ArcCosh[c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a + b
*ArcCosh[c*x])/b)]))/(2*c*E^(a/b))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \sqrt{a+b\text{arccosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{arcosh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \operatorname{acosh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.145 $\int x^2 \left(a + b \cosh^{-1}(cx)\right)^{3/2} \, dx$

Optimal. Leaf size=292

$$-\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}-\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^{-\frac{3a}{b}}\text{E}\cdots}{}$$

```
[Out] -(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(3*c^3) - (b*x^2
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(6*c) + (x^3*(a + b
*ArcCosh[c*x])^(3/2))/3 - (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCos
h[c*x]]/Sqrt[b]])/(32*c^3) - (b^(3/2)*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*S
qrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(96*c^3) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt
[a + b*ArcCosh[c*x]]/Sqrt[b]])/(32*c^3*E^(a/b)) + (b^(3/2)*Sqrt[Pi/3]*Erfi[
(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(96*c^3*E^((3*a)/b))
```

Rubi [A] time = 1.25055, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5664, 5759, 5718, 5658, 3308, 2180, 2205, 2204, 5670, 5448}

$$-\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}-\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^{-\frac{3a}{b}}\text{E}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] -(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(3*c^3) - (b*x^2
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(6*c) + (x^3*(a + b
*ArcCosh[c*x])^(3/2))/3 - (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCos
h[c*x]]/Sqrt[b]])/(32*c^3) - (b^(3/2)*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*S
qrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(96*c^3) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt
[a + b*ArcCosh[c*x]]/Sqrt[b]])/(32*c^3*E^(a/b)) + (b^(3/2)*Sqrt[Pi/3]*Erfi[
(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(96*c^3*E^((3*a)/b))
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
```

```
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^2 \left(a+b\cosh^{-1}(cx)\right)^{3/2} dx = \frac{1}{3}x^3\left(a+b\cosh^{-1}(cx)\right)^{3/2} - \frac{1}{2}(bc)\int \frac{x^3\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$

$$= -\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\cosh^{-1}(cx)\right)^{3/2} + \frac{1}{12}b^2\int \frac{}{\sqrt{a+}}$$

$$= -\frac{b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{6c} + \frac{1}{3}$$

$$= -\frac{b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{6c} + \frac{1}{3}$$

$$= -\frac{b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{6c} + \frac{1}{3}$$

$$= -\frac{b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{6c} + \frac{1}{3}$$

$$= -\frac{b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{6c} + \frac{1}{3}$$

$$= -\frac{b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{6c} + \frac{1}{3}$$

Mathematica [A] time = 2.25499, size = 540, normalized size = 1.85

$$\frac{ae^{-\frac{3a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\text{Gamma}\left(\frac{3}{2},-\frac{3(a+b\,c}{}\right.\right.}{72c^3\sqrt{-\frac{(a+b}{}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcCosh[c*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*
Gamma[3/2, a/b + ArcCosh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2
```

```
, (-3*(a + b*ArcCosh[c*x]))/b] + 9*E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gam
ma[3/2, -((a + b*ArcCosh[c*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcC
osh[c*x])/b)]*Gamma[3/2, (3*(a + b*ArcCosh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*
Sqrt[-((a + b*ArcCosh[c*x])^2/b^2)]) + (Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[a + b*ArcCosh[c*x]] + 8*Sqrt[b]*c*x*ArcCosh[
c*x]*Sqrt[a + b*ArcCosh[c*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCos
h[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[3*Pi]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a
)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]
]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*ArcCosh[c*x]]*(2*
ArcCosh[c*x]*Cosh[3*ArcCosh[c*x]] - Sinh[3*ArcCosh[c*x]])))/(288*c^3)
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int x^{2} \left(a + b \operatorname{arccosh}\left(c x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{\frac{3}{2}} x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{acosh}(cx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.146 $\int x\left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=184

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^2}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^2}-\frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\cosh^{-1}(\right.$$

```
[Out] (-3*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(8*c) - (a +
 b*ArcCosh[c*x])^(3/2)/(4*c^2) + (x^2*(a + b*ArcCosh[c*x])^(3/2))/2 - (3*b^
(3/2)*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]
])/(64*c^2) + (3*b^(3/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])
/Sqrt[b]])/(64*c^2*E^((2*a)/b))
```

Rubi [A] time = 0.816092, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5664, 5759, 5676, 5670, 5448, 12, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^2}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^2}-\frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\cosh^{-1}(\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-3*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(8*c) - (a +
 b*ArcCosh[c*x])^(3/2)/(4*c^2) + (x^2*(a + b*ArcCosh[c*x])^(3/2))/2 - (3*b^
(3/2)*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]
])/(64*c^2) + (3*b^(3/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])
/Sqrt[b]])/(64*c^2*E^((2*a)/b))
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x\left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx=\frac{1}{2}x^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{1}{4}(3bc)\int\frac{x^2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$

$$=-\frac{3bx\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{8c}+\frac{1}{2}x^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}+\frac{1}{16}\left(3b^2\right)\int\frac{}{\sqrt{a}}$$

$$=-\frac{3bx\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{8c}-\frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\cosh^{-1}(c\right.$$

$$=-\frac{3bx\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{8c}-\frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\cosh^{-1}(c\right.$$

$$=-\frac{3bx\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{8c}-\frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\cosh^{-1}(c\right.$$

$$=-\frac{3bx\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{8c}-\frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\cosh^{-1}(c\right.$$

$$=-\frac{3bx\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{8c}-\frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\cosh^{-1}(c\right.$$

$$=-\frac{3bx\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{8c}-\frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\cosh^{-1}(c\right.$$

Mathematica [A] time = 1.00846, size = 165, normalized size = 0.9

$$\frac{-3\sqrt{2\pi}b^{3/2}\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)+3\sqrt{2\pi}b^{3/2}\left(\cosh\left(\frac{2a}{b}\right)-\sinh\left(\frac{2a}{b}\right)\right)\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{128c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (3*b^(3/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cos
h[(2*a)/b] - Sinh[(2*a)/b]) - 3*b^(3/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*
ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[a + b*ArcC
osh[c*x]]*(4*a*Cosh[2*ArcCosh[c*x]] + 4*b*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]]
 - 3*b*Sinh[2*ArcCosh[c*x]]))/(128*c^2)
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{arccosh}(cx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x*(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\operatorname{arcosh}(cx)+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\,\mathrm{acosh}\,(cx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.147 $\int \left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=140

$$-\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\right.$$

```
[Out] (-3*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(2*c) + x*(a +
 b*ArcCosh[c*x])^(3/2) - (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh
[c*x]]/Sqrt[b]])/(8*c) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/
Sqrt[b]])/(8*c*E^(a/b))
```

Rubi [A] time = 0.427901, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5654, 5718, 5658, 3308, 2180, 2205, 2204}

$$-\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(3/2),x]
```


```
[Out] (-3*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(2*c) + x*(a +
 b*ArcCosh[c*x])^(3/2) - (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh
[c*x]]/Sqrt[b]])/(8*c) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/
Sqrt[b]])/(8*c*E^(a/b))
```


Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
```

```
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(cx)\right)^{3/2} dx = x\left(a+b\cosh^{-1}(cx)\right)^{3/2} - \frac{1}{2}(3bc)\int \frac{x\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$

$$= -\frac{3b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c} + x\left(a+b\cosh^{-1}(cx)\right)^{3/2} + \frac{1}{4}\left(3b^2\right)\int \frac{}{\sqrt{a+b\,\mathrm{c}}}$$

$$= -\frac{3b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c} + x\left(a+b\cosh^{-1}(cx)\right)^{3/2} - \frac{(3b)\,\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}\right.}{\sqrt{x}}\right.}{}$$

$$= -\frac{3b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c} + x\left(a+b\cosh^{-1}(cx)\right)^{3/2} - \frac{(3b)\,\mathrm{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\right.}}{\sqrt{x}}\right.}{}$$

$$= -\frac{3b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c} + x\left(a+b\cosh^{-1}(cx)\right)^{3/2} - \frac{(3b)\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\right.}{}$$

$$= -\frac{3b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c} + x\left(a+b\cosh^{-1}(cx)\right)^{3/2} - \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+}}{}\right.}{8c}$$

Mathematica [A] time = 0.6246, size = 269, normalized size = 1.92

$$\frac{ae^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(\frac{e^{\frac{2a}{b}}\,\mathrm{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}} + \frac{\mathrm{Gamma}\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}}\right)}{2c} + \frac{b\left(\frac{\sqrt{\pi}(2a-3b)\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c}}{\sqrt{b}}\right.}{\sqrt{b}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcCosh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])/S
qrt[a/b + ArcCosh[c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a +
 b*ArcCosh[c*x])/b)]))/(2*c*E^(a/b)) + (b*(-12*Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x)*Sqrt[a + b*ArcCosh[c*x]] + 8*c*x*ArcCosh[c*x]*Sqrt[a + b*ArcCosh[c
*x]] + ((2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a
/b] - Sinh[a/b]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*
x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(8*c)
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int (a+b\operatorname{arccosh}(cx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arcosh}(cx)+a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\operatorname{acosh}(cx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.148 $\int x^2 \left(a+b\cosh^{-1}(cx)\right)^{5/2}\,dx$

Optimal. Leaf size=337

$$-\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^3}-\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{576c^3}-\frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^3}-\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e \cdots}{}$$

```
[Out] (5*b^2*x*Sqrt[a + b*ArcCosh[c*x]])/(6*c^2) + (5*b^2*x^3*Sqrt[a + b*ArcCosh[
c*x]])/36 - (5*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(3/2))/(
9*c^3) - (5*b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(3/2))/
(18*c) + (x^3*(a + b*ArcCosh[c*x])^(5/2))/3 - (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*
Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(64*c^3) - (5*b^(5/2)*E^((3*a)/b)*Sq
rt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(576*c^3) - (15*b
^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(64*c^3*E^(a/b)) -
(5*b^(5/2)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(57
6*c^3*E^((3*a)/b))
```

Rubi [A] time = 2.08993, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5664, 5759, 5718, 5654, 5781, 3307, 2180, 2204, 2205, 3312}

$$-\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^3}-\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{576c^3}-\frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^3}-\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e \cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCosh[c*x])^(5/2),x]
```

```
[Out] (5*b^2*x*Sqrt[a + b*ArcCosh[c*x]])/(6*c^2) + (5*b^2*x^3*Sqrt[a + b*ArcCosh[
c*x]])/36 - (5*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(3/2))/(
9*c^3) - (5*b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(3/2))/
(18*c) + (x^3*(a + b*ArcCosh[c*x])^(5/2))/3 - (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*
Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(64*c^3) - (5*b^(5/2)*E^((3*a)/b)*Sq
rt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(576*c^3) - (15*b
^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(64*c^3*E^(a/b)) -
(5*b^(5/2)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(57
6*c^3*E^((3*a)/b))
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^2 \left(a+b\cosh^{-1}(cx)\right)^{5/2}\,dx = \frac{1}{3}x^3\left(a+b\cosh^{-1}(cx)\right)^{5/2} - \frac{1}{6}(5bc)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$

$$= -\frac{5bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{18c} + \frac{1}{3}x^3\left(a+b\cosh^{-1}(cx)\right)^{5/2} + \frac{1}{12}\left(5b^2\right)$$

$$= \frac{5}{36}b^2x^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{9c^3} - \frac{5bx^2\sqrt{-1+c}}{}$$

$$= \frac{5b^2x\sqrt{a+b\cosh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cos\right.}{9c^3}$$

$$= \frac{5b^2x\sqrt{a+b\cosh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cos\right.}{9c^3}$$

$$= \frac{5b^2x\sqrt{a+b\cosh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cos\right.}{9c^3}$$

$$= \frac{5b^2x\sqrt{a+b\cosh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cos\right.}{9c^3}$$

$$= \frac{5b^2x\sqrt{a+b\cosh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cos\right.}{9c^3}$$

$$= \frac{5b^2x\sqrt{a+b\cosh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cos\right.}{9c^3}$$

Mathematica [B] time = 10.694, size = 924, normalized size = 2.74

$$\frac{e^{-\frac{3a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\text{Gamma}\left(\frac{3}{2},-\frac{3\left(a+b\cos\right.}{}\right.\right.}{72c^3\sqrt{-\frac{\left(a+\right.}{}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcCosh[c*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a + b*ArcCosh[c*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)
]*Gamma[3/2, a/b + ArcCosh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3
```

```
/2, (-3*(a + b*ArcCosh[c*x]))/b] + 9*E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*G
amma[3/2, -((a + b*ArcCosh[c*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*Ar
cCosh[c*x])/b)]*Gamma[3/2, (3*(a + b*ArcCosh[c*x]))/b]))/(72*c^3*E^((3*a)/b
)*Sqrt[-((a + b*ArcCosh[c*x])^2/b^2)]) + (a*Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[a + b*ArcCosh[c*x]] + 8*Sqrt[b]*c*x*ArcC
osh[c*x]*Sqrt[a + b*ArcCosh[c*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*Ar
cCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[3*
Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[
(3*a)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqr
t[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*ArcCosh[c*x]]
*(2*ArcCosh[c*x]*Cosh[3*ArcCosh[c*x]] - Sinh[3*ArcCosh[c*x]])))/(144*c^3) -
 (27*(-4*b*Sqrt[a + b*ArcCosh[c*x]]*(2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
*(a - 5*b*ArcCosh[c*x]) + b*c*x*(15 + 4*ArcCosh[c*x]^2)) + Sqrt[b]*(4*a^2 +
 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b
] - Sinh[a/b]) + Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*
ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(12*a^2 + 12*a*b
+ 5*b^2)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[
(3*a)/b] - Sinh[(3*a)/b]) + Sqrt[b]*(12*a^2 - 12*a*b + 5*b^2)*Sqrt[3*Pi]*Er
f[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b
]) - 12*b*Sqrt[a + b*ArcCosh[c*x]]*(b*(5 + 12*ArcCosh[c*x]^2)*Cosh[3*ArcCos
h[c*x]] + 2*(a - 5*b*ArcCosh[c*x])*Sinh[3*ArcCosh[c*x]]))/(1728*c^3)
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int x^{2} \left(a + b \operatorname{arccosh}\left(c x\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^(5/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{arcosh}\left(c x\right) + a\right)^{\frac{5}{2}} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(5/2)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.149 $\int x\left(a+b\cosh^{-1}(cx)\right)^{5/2}\,dx$

Optimal. Leaf size=228

$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{256c^2}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{256c^2}-\frac{15b^2\sqrt{a+b\cosh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b}$$

```
[Out] (-15*b^2*Sqrt[a + b*ArcCosh[c*x]])/(64*c^2) + (15*b^2*x^2*Sqrt[a + b*ArcCos
h[c*x]])/32 - (5*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(3/2
))/(8*c) - (a + b*ArcCosh[c*x])^(5/2)/(4*c^2) + (x^2*(a + b*ArcCosh[c*x])^(
5/2))/2 - (15*b^(5/2)*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCos
h[c*x]])/Sqrt[b]])/(256*c^2) - (15*b^(5/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a
+ b*ArcCosh[c*x]])/Sqrt[b]])/(256*c^2*E^((2*a)/b))
```

Rubi [A] time = 1.3278, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5664, 5759, 5676, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{256c^2}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{256c^2}-\frac{15b^2\sqrt{a+b\cosh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCosh[c*x])^(5/2),x]
```



```
[Out] (-15*b^2*Sqrt[a + b*ArcCosh[c*x]])/(64*c^2) + (15*b^2*x^2*Sqrt[a + b*ArcCos
h[c*x]])/32 - (5*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(3/2
))/(8*c) - (a + b*ArcCosh[c*x])^(5/2)/(4*c^2) + (x^2*(a + b*ArcCosh[c*x])^(
5/2))/2 - (15*b^(5/2)*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCos
h[c*x]])/Sqrt[b]])/(256*c^2) - (15*b^(5/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a
+ b*ArcCosh[c*x]])/Sqrt[b]])/(256*c^2*E^((2*a)/b))
```


Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
```

```
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x\left(a+b\cosh^{-1}(cx)\right)^{5/2}\,dx=\frac{1}{2}x^2\left(a+b\cosh^{-1}(cx)\right)^{5/2}-\frac{1}{4}(5bc)\int\frac{x^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$

$$=-\frac{5bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{8c}+\frac{1}{2}x^2\left(a+b\cosh^{-1}(cx)\right)^{5/2}+\frac{1}{16}\left(15b^2\right)\int$$

$$=\frac{15}{32}b^2x^2\sqrt{a+b\cosh^{-1}(cx)}-\frac{5bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{8c}-\frac{\left(a+b\cosh^{-}\right.}{4c^2}$$

$$=\frac{15}{32}b^2x^2\sqrt{a+b\cosh^{-1}(cx)}-\frac{5bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{8c}-\frac{\left(a+b\cosh^{-}\right.}{4c^2}$$

$$=\frac{15}{32}b^2x^2\sqrt{a+b\cosh^{-1}(cx)}-\frac{5bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{8c}-\frac{\left(a+b\cosh^{-}\right.}{4c^2}$$

$$=-\frac{15b^2\sqrt{a+b\cosh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\cosh^{-1}(cx)}-\frac{5bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\,\mathrm{co}\right.}{8c}$$

$$=-\frac{15b^2\sqrt{a+b\cosh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\cosh^{-1}(cx)}-\frac{5bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\,\mathrm{co}\right.}{8c}$$

$$=-\frac{15b^2\sqrt{a+b\cosh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\cosh^{-1}(cx)}-\frac{5bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\,\mathrm{co}\right.}{8c}$$

$$=-\frac{15b^2\sqrt{a+b\cosh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\cosh^{-1}(cx)}-\frac{5bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\,\mathrm{co}\right.}{8c}$$

Mathematica [A] time = 1.87998, size = 207, normalized size = 0.91

$$\frac{8\sqrt{a+b\cosh^{-1}(cx)}\left(\left(16a^2+15b^2\right)\cosh\left(2\cosh^{-1}(cx)\right)+4b\cosh^{-1}(cx)\left(8a\cosh\left(2\cosh^{-1}(cx)\right)-5b\sinh\left(2\cosh^{-1}(c\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcCosh[c*x])^(5/2),x]
```

```
[Out] (-15*b^(5/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(C
osh[(2*a)/b] - Sinh[(2*a)/b]) - 15*b^(5/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a +
 b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[a + b*A
```

```
rcCosh[c*x]]*((16*a^2 + 15*b^2)*Cosh[2*ArcCosh[c*x]] + 16*b^2*ArcCosh[c*x]^
2*Cosh[2*ArcCosh[c*x]] - 20*a*b*Sinh[2*ArcCosh[c*x]] + 4*b*ArcCosh[c*x]*(8*
a*Cosh[2*ArcCosh[c*x]] - 5*b*Sinh[2*ArcCosh[c*x]])))/(512*c^2)
```

Maple [F] time = 0.124, size = 0, normalized size = 0.

$$\int x\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^(5/2),x)
```

```
[Out] int(x*(a+b*arccosh(c*x))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{\frac{5}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(5/2)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.150 $\int \left(a+b\cosh^{-1}(cx)\right)^{5/2}\,dx$

Optimal. Leaf size=160

$$-\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c}-\frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c}+\frac{15}{4}b^2x\sqrt{a+b\cosh^{-1}(cx)}-\frac{5b\sqrt{cx-1}\sqrt{cx+1}\,(}{2}$$

```
[Out] (15*b^2*x*Sqrt[a + b*ArcCosh[c*x]])/4 - (5*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(
a + b*ArcCosh[c*x])^(3/2))/(2*c) + x*(a + b*ArcCosh[c*x])^(5/2) - (15*b^(5/
2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*c) - (15*b^(
5/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*c*E^(a/b))
```

Rubi [A] time = 0.750641, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5654, 5718, 5781, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c}-\frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c}+\frac{15}{4}b^2x\sqrt{a+b\cosh^{-1}(cx)}-\frac{5b\sqrt{cx-1}\sqrt{cx+1}\,(}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(5/2),x]
```

```
[Out] (15*b^2*x*Sqrt[a + b*ArcCosh[c*x]])/4 - (5*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(
a + b*ArcCosh[c*x])^(3/2))/(2*c) + x*(a + b*ArcCosh[c*x])^(5/2) - (15*b^(5/
2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*c) - (15*b^(
5/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*c*E^(a/b))
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
```

```
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(cx)\right)^{5/2}\,dx = x\left(a+b\cosh^{-1}(cx)\right)^{5/2} - \frac{1}{2}(5bc)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$
$$= -\frac{5b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{2c} + x\left(a+b\cosh^{-1}(cx)\right)^{5/2} + \frac{1}{4}\left(15b^2\right)\int \sqrt{a+b}$$
$$= \frac{15}{4}b^2x\sqrt{a+b\cosh^{-1}(cx)} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{2c} + x\left(a+b\cosh^{-1}(cx\right.$$
$$= \frac{15}{4}b^2x\sqrt{a+b\cosh^{-1}(cx)} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{2c} + x\left(a+b\cosh^{-1}(cx\right.$$
$$= \frac{15}{4}b^2x\sqrt{a+b\cosh^{-1}(cx)} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{2c} + x\left(a+b\cosh^{-1}(cx\right.$$
$$= \frac{15}{4}b^2x\sqrt{a+b\cosh^{-1}(cx)} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{2c} + x\left(a+b\cosh^{-1}(cx\right.$$
$$= \frac{15}{4}b^2x\sqrt{a+b\cosh^{-1}(cx)} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{2c} + x\left(a+b\cosh^{-1}(cx\right.$$

Mathematica [B] time = 2.3507, size = 452, normalized size = 2.82

$$8a^2e^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(\frac{e^{\frac{2a}{b}}\operatorname{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}} + \frac{\operatorname{Gamma}\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}}\right) - \sqrt{\pi}\sqrt{b}\left(4a^2-12ab+15b^2\right)\left(\sinh\left(\frac{a}{b}\right)+c\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(5/2),x]
```

```
[Out] (4*b*Sqrt[a + b*ArcCosh[c*x]]*(2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a -
5*b*ArcCosh[c*x]) + b*c*x*(15 + 4*ArcCosh[c*x]^2)) + (8*a^2*Sqrt[a + b*ArcC
osh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])/Sqrt[a/b + ArcCosh[
c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a + b*ArcCosh[c*x])/b
)]))/E^(a/b) - Sqrt[b]*(4*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*A
rcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) - Sqrt[b]*(4*a^2 - 12*a*b + 1
5*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b
]) + 4*a*b*(-12*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[a + b*ArcCosh[c*x
```

```
]] + 8*c*x*ArcCosh[c*x]*Sqrt[a + b*ArcCosh[c*x]] + ((2*a + 3*b)*Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]))/Sqrt[b] + ((2
*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[
a/b]))/Sqrt[b]))/(16*c)
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int (a+b\operatorname{arccosh}(cx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(5/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arcosh}(cx)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.151 $\int \frac{x^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=194

$$-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}-\frac{\sqrt{\frac{\pi}{3}}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}$$

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^3) -
 (E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(
8*Sqrt[b]*c^3) + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*Sqrt[
b]*c^3*E^(a/b)) + (Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[
b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

Rubi [A] time = 0.360347, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}-\frac{\sqrt{\frac{\pi}{3}}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^3) -
 (E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(
8*Sqrt[b]*c^3) + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*Sqrt[
b]*c^3*E^(a/b)) + (Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[
b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{4\sqrt{a+bx}} + \frac{\sinh(3x)}{4\sqrt{a+bx}}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{4c^3} + \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{4c^3}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{8c^3} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{8c^3} + \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{a+b}}\right.}{}$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{4bc^3} - \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{4bc^3} -$$

$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} - \frac{e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} +$$

Mathematica [A] time = 0.346328, size = 195, normalized size = 1.01

$$\frac{e^{-\frac{3a}{b}}\left(3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\text{Gamma}\left(\frac{1}{2}, \frac{a}{b}+\cosh^{-1}(cx)\right) + \sqrt{3}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{1}{2}, -\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right) + 3e^{\frac{2a}{b}}\sqrt{-}\right.}{24c^3\sqrt{a+b\cosh^{-1}(cx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (3*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] + Sq
rt[3]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/
b] + 3*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcC
osh[c*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*
(a + b*ArcCosh[c*x]))/b])/(24*c^3*E^((3*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{a + b\text{arccosh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int(x^2/(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{b\,\mathrm{arcosh}\,(cx) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*arccosh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + b\,\mathrm{acosh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.152 $\int \frac{x}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=107

$$\frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2}-\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2}$$

```
[Out] -(E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(
4*Sqrt[b]*c^2) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b
]])/(4*Sqrt[b]*c^2*E^((2*a)/b))
```

Rubi [A] time = 0.184953, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5670, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2}-\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] -(E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(
4*Sqrt[b]*c^2) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b
]])/(4*Sqrt[b]*c^2*E^((2*a)/b))
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{c^2}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{c^2}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{2c^2}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{4c^2} + \frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{4c^2}$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{2bc^2} + \frac{\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{2bc^2}$$

$$= -\frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2}$$

Mathematica [A] time = 0.219912, size = 104, normalized size = 0.97

$$-\frac{\sqrt{\frac{\pi}{2}}\left(\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)+\left(\sinh\left(\frac{2a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\right)\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)\right)}{4\sqrt{b}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] -(Sqrt[Pi/2]*(Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(-Cosh[(2*a)
/b] + Sinh[(2*a)/b]) + Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cos
h[(2*a)/b] + Sinh[(2*a)/b])))/(4*Sqrt[b]*c^2)
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{a+b\text{arccosh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int(x/(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{b\,\mathrm{arcosh}\,(cx) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*arccosh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + b\,\mathrm{acosh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.153 $\int \frac{1}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{\pi}e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}-\frac{\sqrt{\pi}e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) + (
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
```

Rubi [A] time = 0.104876, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5658, 3308, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}-\frac{\sqrt{\pi}e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) + (
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc}\\
&= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{2bc}+\frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{2bc}\\
&= -\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{bc}+\frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{bc}\\
&= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}
\end{aligned}$$

Mathematica [A] time = 0.100201, size = 100, normalized size = 1.14

$$\frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)\right)}{2c\sqrt{a+b\cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] + Sqrt
[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)])/(2*c*E^(
a/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\mathrm{arccosh}\left(cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\mathrm{arcosh}\left(cx\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccosh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\operatorname{acosh}(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.154 $\int \frac{x^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=231

$$\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}e^{-\frac{3a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh}}{\sqrt{b}}\right.}{4b^{3/2}c^3}$$

```
[Out] (-2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (E^(
a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) + (E^(
(3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b^(
3/2)*c^3) + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^
3*E^(a/b)) + (Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/
(4*b^(3/2)*c^3*E^((3*a)/b))
```

Rubi [A] time = 0.310078, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}e^{-\frac{3a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh}}{\sqrt{b}}\right.}{4b^{3/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (E^(
a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) + (E^(
(3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b^(
3/2)*c^3) + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^
3*E^(a/b)) + (Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/
(4*b^(3/2)*c^3*E^((3*a)/b))
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
```

```
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2\,\text{Subst}\left(\int\left(-\frac{\cosh(x)}{4\sqrt{a+bx}}-\frac{3\cosh(3x)}{4\sqrt{a+bx}}\right)dx, x, \cosh^{-1}(cx)\right)}{bc^3}$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\text{Subst}\left(\int\frac{\cosh(x)}{\sqrt{a+bx}}dx, x, \cosh^{-1}(cx)\right)}{2bc^3} + \frac{3\,\text{Subst}\left(\int\frac{\cosh(3x)}{\sqrt{a+bx}}dx, x,\right.}{2bc^3}$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\text{Subst}\left(\int\frac{e^{-x}}{\sqrt{a+bx}}dx, x, \cosh^{-1}(cx)\right)}{4bc^3} + \frac{\text{Subst}\left(\int\frac{e^{x}}{\sqrt{a+bx}}dx, x, \cos\right.}{4bc^3}$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{2b^2c^3} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\right.}{}$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} + \frac{e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} + \cdots$$

Mathematica [A] time = 0.658086, size = 247, normalized size = 1.07

$$\frac{e^{-\frac{3a}{b}}\left(-e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+\sqrt{3}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)+e^{\frac{2a}{b}}\sqrt{-\cdots}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*E^((3*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - E^((4*a)/b)*Sqrt[a/b
 + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] + Sqrt[3]*Sqrt[-((a + b*Arc
Cosh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] + E^((2*a)/b)*Sqrt[-
((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] - Sqrt[3]*E
^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b]
- 2*E^((3*a)/b)*Sinh[3*ArcCosh[c*x]])/(4*b*c^3*E^((3*a)/b)*Sqrt[a + b*ArcCo
sh[c*x]])
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int x^2 (a + b\mathrm{arccosh}\,(cx))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x^2/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(b\,\mathrm{arcosh}\,(cx) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a + b\,\mathrm{acosh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*acosh(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.155 $\int \frac{x}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}-\frac{2x\sqrt{cx-1}\sqrt{cx+1}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

```
[Out] (-2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (E^((2
*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(b^(3/2)
*c^2) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(b^(3
/2)*c^2*E^((2*a)/b))
```

Rubi [A] time = 0.154964, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}-\frac{2x\sqrt{cx-1}\sqrt{cx+1}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (E^((2
*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(b^(3/2)
*c^2) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(b^(3
/2)*c^2*E^((2*a)/b))
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{2\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc^2}$$

$$= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\mathrm{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc^2} + \frac{\mathrm{Subst}\left(\int \frac{e^{2x}}{\sqrt{a+bx}}\,dx, x, \cos\right.}{bc^2}$$

$$= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{2\,\mathrm{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{b^2c^2} + \frac{2\,\mathrm{Subst}\left(\int e^{-\frac{2}{}}\right.}{}$$

$$= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}$$

Mathematica [A] time = 1.20243, size = 135, normalized size = 0.96

$$\frac{\sqrt{2\pi}\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)+\sqrt{2\pi}\left(\cosh\left(\frac{2a}{b}\right)-\sinh\left(\frac{2a}{b}\right)\right)\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)-\frac{2\sqrt{b}\sinh\left(2\cosh^{-1}(cx)\right)}{\sqrt{a+b\cosh^{-1}(cx)}}}{2b^{3/2}c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(2*a)/b]
 - Sinh[(2*a)/b]) + Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[
b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - (2*Sqrt[b]*Sinh[2*ArcCosh[c*x]])/Sqrt
[a + b*ArcCosh[c*x]])/(2*b^(3/2)*c^2)
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{arccosh}(cx)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(b\operatorname{arcosh}(cx)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a + b\operatorname{acosh}(cx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral(x/(a + b*acosh(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.156 $\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{cx-1}\sqrt{cx+1}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

```
[Out] (-2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))
```

Rubi [A] time = 0.410395, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5656, 5781, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{cx-1}\sqrt{cx+1}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2c)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{2\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc} + \frac{\mathrm{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}\right.}{bc}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{2\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{b^2c} + \frac{2\,\mathrm{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\right.}{}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}$$

Mathematica [A] time = 0.237044, size = 132, normalized size = 1.1

$$\frac{e^{-\frac{a}{b}}\left(-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\mathrm{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)-2e^{a/b}\sqrt{\frac{cx-1}{cx+1}}(cx\right.}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(-3/2),x]
```

```
[Out] (-2*E^(a/b)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - E^((2*a)/b)*Sqrt[a/b + A
rcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] + Sqrt[-((a + b*ArcCosh[c*x])/b
)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)])/(b*c*E^(a/b)*Sqrt[a + b*ArcCosh[c
*x]])
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int (a+b\mathrm{arccosh}\,(cx))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(1/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arcosh}(cx) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \operatorname{acosh}(cx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.157 $\int \frac{x^2}{\left(a+b\cosh^{-1}(cx)\right)^{5/2}}\,dx$

Optimal. Leaf size=276

$$-\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3}-\frac{\sqrt{3\pi}e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3}+\frac{\sqrt{3\pi}e^{-\frac{3a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3}$$

```
[Out] (-2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*b*c*(a + b*ArcCosh[c*x])^(3/2)) +
(8*x)/(3*b^2*c^2*Sqrt[a + b*ArcCosh[c*x]]) - (4*x^3)/(b^2*Sqrt[a + b*ArcCos
h[c*x]]) - (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(6*b^(5
/2)*c^3) - (E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/S
qrt[b]])/(2*b^(5/2)*c^3) + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]
)/(6*b^(5/2)*c^3*E^(a/b)) + (Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*
x]])/Sqrt[b]])/(2*b^(5/2)*c^3*E^((3*a)/b))
```

Rubi [A] time = 1.33041, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5668, 5775, 5670, 5448, 3308, 2180, 2204, 2205, 5658}

$$-\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3}-\frac{\sqrt{3\pi}e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3}+\frac{\sqrt{3\pi}e^{-\frac{3a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*ArcCosh[c*x])^(5/2),x]
```

```
[Out] (-2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*b*c*(a + b*ArcCosh[c*x])^(3/2)) +
(8*x)/(3*b^2*c^2*Sqrt[a + b*ArcCosh[c*x]]) - (4*x^3)/(b^2*Sqrt[a + b*ArcCos
h[c*x]]) - (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(6*b^(5
/2)*c^3) - (E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/S
qrt[b]])/(2*b^(5/2)*c^3) + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]
)/(6*b^(5/2)*c^3*E^(a/b)) + (Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*
x]])/Sqrt[b]])/(2*b^(5/2)*c^3*E^((3*a)/b))
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\cosh^{-1}(cx)\right)^{5/2}}\,dx = -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{4\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx}{3bc} + \frac{(2c)\int \frac{x^3}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}}{b}$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{8x}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\cosh^{-1}(cx)}} + \frac{12\int \frac{}{\sqrt{a+b}}}{}$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{8x}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\cosh^{-1}(cx)}} + \frac{8\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{8x}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\cosh^{-1}(cx)}} + \frac{4\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{8x}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\cosh^{-1}(cx)}} + \frac{8\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{8x}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\cosh^{-1}(cx)}} + \frac{4e^{a/b}\sqrt{\pi}\mathrm{e}}{}$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{8x}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\cosh^{-1}(cx)}} + \frac{4e^{a/b}\sqrt{\pi}\mathrm{e}}{}$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{8x}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{e^{a/b}\sqrt{\pi}\mathrm{er}}{}$$

Mathematica [A] time = 2.04002, size = 340, normalized size = 1.23

$$\frac{e^{-3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}\left(-6\sqrt{3}be^{3\cosh^{-1}(cx)}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{3/2}\mathrm{Gamma}\left(\frac{1}{2},-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)-2be^{\frac{2a}{b}+3\cosh^{-1}(cx)}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{3/}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + b*ArcCosh[c*x])^(5/2),x]
```

```
[Out] (2*E^((4*a)/b + 3*ArcCosh[c*x])*Sqrt[a/b + ArcCosh[c*x]]*(a + b*ArcCosh[c*x
])*Gamma[1/2, a/b + ArcCosh[c*x]] - 6*Sqrt[3]*b*E^(3*ArcCosh[c*x])*(-((a +
b*ArcCosh[c*x])/b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] - 2*b*E^(
(2*a)/b + 3*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])/b))^(3/2)*Gamma[1/2, -((a
 + b*ArcCosh[c*x])/b)] + E^((3*a)/b)*(-((1 + E^(2*ArcCosh[c*x]))*(a*(6 - 4*
E^(2*ArcCosh[c*x]) + 6*E^(4*ArcCosh[c*x])) + b*(-1 + 6*ArcCosh[c*x] - 4*E^(
2*ArcCosh[c*x])*ArcCosh[c*x] + E^(4*ArcCosh[c*x])*(1 + 6*ArcCosh[c*x])))) +
 6*Sqrt[3]*E^(3*(a/b + ArcCosh[c*x]))*Sqrt[a/b + ArcCosh[c*x]]*(a + b*ArcCo
sh[c*x])*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b]))/(12*b^2*c^3*E^(3*(a/b + A
rcCosh[c*x]))*(a + b*ArcCosh[c*x])^(3/2))
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int x^{2} \left(a + b \operatorname{arccosh}\left(c x\right)\right)^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccosh(c*x))^(5/2),x)
```

```
[Out] int(x^2/(a+b*arccosh(c*x))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(b \operatorname{arcosh}\left(c x\right) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arccosh(c*x) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a + b \operatorname{acosh}(cx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acosh(c*x))**(5/2),x)
```

```
[Out] Integral(x**2/(a + b*acosh(c*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.158 $\int \frac{x}{\left(a+b\cosh^{-1}(cx)\right)^{5/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{2\sqrt{2\pi}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}+\frac{2\sqrt{2\pi}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}+\frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}}-\frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(c}}$$

```
[Out] (-2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*b*c*(a + b*ArcCosh[c*x])^(3/2)) + 4/
(3*b^2*c^2*Sqrt[a + b*ArcCosh[c*x]]) - (8*x^2)/(3*b^2*Sqrt[a + b*ArcCosh[c*
x]]) - (2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqr
t[b]])/(3*b^(5/2)*c^2) + (2*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x
]])/Sqrt[b]])/(3*b^(5/2)*c^2*E^((2*a)/b))
```

Rubi [A] time = 0.89348, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5668, 5775, 5670, 5448, 12, 3308, 2180, 2204, 2205, 5676}

$$-\frac{2\sqrt{2\pi}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}+\frac{2\sqrt{2\pi}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}+\frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}}-\frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcCosh[c*x])^(5/2),x]
```

```
[Out] (-2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*b*c*(a + b*ArcCosh[c*x])^(3/2)) + 4/
(3*b^2*c^2*Sqrt[a + b*ArcCosh[c*x]]) - (8*x^2)/(3*b^2*Sqrt[a + b*ArcCosh[c*
x]]) - (2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqr
t[b]])/(3*b^(5/2)*c^2) + (2*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x
]])/Sqrt[b]])/(3*b^(5/2)*c^2*E^((2*a)/b))
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
```

```
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{x}{\left(a+b\cosh^{-1}(cx)\right)^{5/2}}\,dx = -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{2\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx}{3bc} + \frac{(4c)\int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}(a+b\cosh}}{3b}$$

$$= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} + \frac{16\int}{\sqrt{a}}$$

$$= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} + \frac{16\,\mathrm{Subs}}{}$$

$$= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} + \frac{16\,\mathrm{Subs}}{}$$

$$= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} + \frac{8\,\mathrm{Subst}}{}$$

$$= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{4\,\mathrm{Subst}}{}$$

$$= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8\,\mathrm{Subst}}{}$$

$$= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2e^{\frac{2a}{b}}\sqrt{2\pi}}{}$$

Mathematica [A] time = 1.51707, size = 157, normalized size = 0.84

$$\frac{-2\sqrt{2\pi}\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)+2\sqrt{2\pi}\left(\cosh\left(\frac{2a}{b}\right)-\sinh\left(\frac{2a}{b}\right)\right)\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)-\frac{\sqrt{b}(4\cos}{}}{3b^{5/2}c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*ArcCosh[c*x])^(5/2),x]
```

```
[Out] (2*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(2*a)/
b] - Sinh[(2*a)/b]) - 2*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/S
qrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - (Sqrt[b]*(4*(a + b*ArcCosh[c*x])*
Cosh[2*ArcCosh[c*x]] + b*Sinh[2*ArcCosh[c*x]]))/(a + b*ArcCosh[c*x])^(3/2))
/(3*b^(5/2)*c^2)
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{arccosh}(cx)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccosh(c*x))^(5/2),x)
```

```
[Out] int(x/(a+b*arccosh(c*x))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(b\operatorname{arcosh}(cx)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arccosh(c*x) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a+b\operatorname{acosh}(cx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acosh(c*x))**(5/2),x)
```

```
[Out] Integral(x/(a + b*acosh(c*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.159 $\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{5/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{2\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}+\frac{2\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}-\frac{4x}{3b^2\sqrt{a+b\cosh^{-1}(cx)}}-\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}}$$

```
[Out] (-2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*b*c*(a + b*ArcCosh[c*x])^(3/2)) - (4*x
)/(3*b^2*Sqrt[a + b*ArcCosh[c*x]]) - (2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Arc
Cosh[c*x]]/Sqrt[b]])/(3*b^(5/2)*c) + (2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*
x]]/Sqrt[b]])/(3*b^(5/2)*c*E^(a/b))
```

Rubi [A] time = 0.455902, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5656, 5775, 5658, 3308, 2180, 2205, 2204}

$$-\frac{2\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}+\frac{2\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}-\frac{4x}{3b^2\sqrt{a+b\cosh^{-1}(cx)}}-\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(-5/2),x]
```

```
[Out] (-2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*b*c*(a + b*ArcCosh[c*x])^(3/2)) - (4*x
)/(3*b^2*Sqrt[a + b*ArcCosh[c*x]]) - (2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Arc
Cosh[c*x]]/Sqrt[b]])/(3*b^(5/2)*c) + (2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*
x]]/Sqrt[b]])/(3*b^(5/2)*c*E^(a/b))
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{5/2}}\,dx = -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{(2c)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx}{3b}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{4x}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} + \frac{4\int \frac{1}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{3b^2}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{4x}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{4\,\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\,\mathrm{c}\right.}{3b^3c}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{4x}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2\,\mathrm{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\,\mathrm{co}\right.}{3b^3c}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{4x}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{4\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\,\mathrm{co}}\right.}{3b^3c}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{3bc\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{4x}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} + \frac{2e^{-}}{}$$

Mathematica [A] time = 1.01488, size = 192, normalized size = 1.3

$$\frac{e^{-\frac{a+b\cosh^{-1}(cx)}{b}}\left(2e^{\frac{2a}{b}+\cosh^{-1}(cx)}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\left(a+b\cosh^{-1}(cx)\right)\mathrm{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)-2\left(be^{\cosh^{-1}(cx)}\left(-\frac{a+b\cos}{}\right.\right.\right.}{3b^2c\left(a+b\cosh\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(-5/2),x]
```

```
[Out] (2*E^((2*a)/b + ArcCosh[c*x])*Sqrt[a/b + ArcCosh[c*x]]*(a + b*ArcCosh[c*x])
*Gamma[1/2, a/b + ArcCosh[c*x]] - 2*(E^(a/b)*(b*E^ArcCosh[c*x]*Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x) + (1 + E^(2*ArcCosh[c*x]))*(a + b*ArcCosh[c*x])) +
 b*E^ArcCosh[c*x]*(-((a + b*ArcCosh[c*x])/b))^(3/2)*Gamma[1/2, -((a + b*Arc
Cosh[c*x])/b)]))/(3*b^2*c*E^((a + b*ArcCosh[c*x])/b)*(a + b*ArcCosh[c*x])^(
```

3/2))

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(cx))^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^(5/2),x)
```

```
[Out] int(1/(a+b*arccosh(c*x))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{arcosh}(cx) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\operatorname{acosh}(cx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.160 $\int \frac{x^2}{\left(a+b\cosh^{-1}(cx)\right)^{7/2}}\,dx$

Optimal. Leaf size=361

$$\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3}+\frac{3\sqrt{3\pi}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3}+\frac{3\sqrt{3\pi}e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cos}}{\sqrt{b}}\right.}{5b^{7/2}c^3}$$

```
[Out] (-2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(5*b*c*(a + b*ArcCosh[c*x])^(5/2)) +
(8*x)/(15*b^2*c^2*(a + b*ArcCosh[c*x])^(3/2)) - (4*x^3)/(5*b^2*(a + b*ArcCo
sh[c*x])^(3/2)) + (16*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*b^3*c^3*Sqrt[a + b*
ArcCosh[c*x]]) - (24*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(5*b^3*c*Sqrt[a + b*
ArcCosh[c*x]]) + (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(
15*b^(7/2)*c^3) + (3*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh
[c*x]])/Sqrt[b]])/(5*b^(7/2)*c^3) + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]
/Sqrt[b]])/(15*b^(7/2)*c^3*E^(a/b)) + (3*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a +
b*ArcCosh[c*x]])/Sqrt[b]])/(5*b^(7/2)*c^3*E^((3*a)/b))
```

Rubi [A] time = 1.62671, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5668, 5775, 5666, 3307, 2180, 2204, 2205, 5656, 5781}

$$\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3}+\frac{3\sqrt{3\pi}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3}+\frac{3\sqrt{3\pi}e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cos}}{\sqrt{b}}\right.}{5b^{7/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*ArcCosh[c*x])^(7/2),x]
```


```
[Out] (-2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(5*b*c*(a + b*ArcCosh[c*x])^(5/2)) +
(8*x)/(15*b^2*c^2*(a + b*ArcCosh[c*x])^(3/2)) - (4*x^3)/(5*b^2*(a + b*ArcCo
sh[c*x])^(3/2)) + (16*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*b^3*c^3*Sqrt[a + b*
ArcCosh[c*x]]) - (24*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(5*b^3*c*Sqrt[a + b*
ArcCosh[c*x]]) + (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(
15*b^(7/2)*c^3) + (3*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh
[c*x]])/Sqrt[b]])/(5*b^(7/2)*c^3) + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]
/Sqrt[b]])/(15*b^(7/2)*c^3*E^(a/b)) + (3*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a +
```

b*ArcCosh[c*x]])/Sqrt[b]])/(5*b^(7/2)*c^3*E^((3*a)/b))

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\cosh^{-1}(cx)\right)^{7/2}}\,dx = -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} - \frac{4\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{5/2}}\,dx}{5bc} + \frac{(6c)\int \frac{x^3}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}}{5b}$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} + \frac{8x}{15b^2c^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} +$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} + \frac{8x}{15b^2c^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} +$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} + \frac{8x}{15b^2c^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} +$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} + \frac{8x}{15b^2c^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} +$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} + \frac{8x}{15b^2c^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} +$$

$$= -\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} + \frac{8x}{15b^2c^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} +$$

Mathematica [A] time = 2.46149, size = 394, normalized size = 1.09

$$\frac{-2e^{-\cosh^{-1}(cx)}\left(a+b\cosh^{-1}(cx)\right)\left(2e^{\frac{a}{b}+\cosh^{-1}(cx)}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\left(a+b\cosh^{-1}(cx)\right)\mathrm{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)-2a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + b*ArcCosh[c*x])^(7/2),x]
```

```
[Out] (-6*b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - (2*(a + b*ArcCosh[c*x])*(-2*
a + b - 2*b*ArcCosh[c*x] + 2*E^(a/b + ArcCosh[c*x])*Sqrt[a/b + ArcCosh[c*x]
]*(a + b*ArcCosh[c*x])*Gamma[1/2, a/b + ArcCosh[c*x]]))/E^ArcCosh[c*x] - (2
```

```
*(a + b*ArcCosh[c*x])*(E^(a/b + ArcCosh[c*x])*(2*a + b + 2*b*ArcCosh[c*x])
+ 2*b*(-((a + b*ArcCosh[c*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c*x])/b
)]))/E^(a/b) - 3*(a + b*ArcCosh[c*x])*((12*Sqrt[3]*b*(-((a + b*ArcCosh[c*x]
)/b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b])/E^((3*a)/b) + (2*(b +
6*a*(-1 + E^(6*ArcCosh[c*x])) - 6*b*ArcCosh[c*x] + b*E^(6*ArcCosh[c*x])*(1
+ 6*ArcCosh[c*x]) + 6*Sqrt[3]*E^(3*(a/b + ArcCosh[c*x]))*Sqrt[a/b + ArcCosh
[c*x]]*(a + b*ArcCosh[c*x])*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b]))/E^(3*A
rcCosh[c*x])) - 6*b^2*Sinh[3*ArcCosh[c*x]])/(60*b^3*c^3*(a + b*ArcCosh[c*x]
)^(5/2))
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int x^{2} \left(a+b \operatorname{arccosh}\left(c x\right)\right)^{-\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccosh(c*x))^(7/2),x)
```

```
[Out] int(x^2/(a+b*arccosh(c*x))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arccosh(c*x) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acosh(c*x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.161 $\int \frac{x}{\left(a+b\cosh^{-1}(cx)\right)^{7/2}}\,dx$

Optimal. Leaf size=229

$$\frac{8\sqrt{2\pi}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2}+\frac{8\sqrt{2\pi}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2}+\frac{4}{15b^2c^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}}-\frac{8x^2}{15b^2\left(a+b\cosh^{-1}(\right.}$$

```
[Out] (-2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(5*b*c*(a + b*ArcCosh[c*x])^(5/2)) + 4/
(15*b^2*c^2*(a + b*ArcCosh[c*x])^(3/2)) - (8*x^2)/(15*b^2*(a + b*ArcCosh[c*
x])^(3/2)) - (32*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*b^3*c*Sqrt[a + b*ArcCo
sh[c*x]]) + (8*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]]
)/Sqrt[b]])/(15*b^(7/2)*c^2) + (8*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCo
sh[c*x]])/Sqrt[b]])/(15*b^(7/2)*c^2*E^((2*a)/b))
```

Rubi [A] time = 0.870413, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5668, 5775, 5666, 3307, 2180, 2204, 2205, 5676}

$$\frac{8\sqrt{2\pi}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2}+\frac{8\sqrt{2\pi}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2}+\frac{4}{15b^2c^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}}-\frac{8x^2}{15b^2\left(a+b\cosh^{-1}(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcCosh[c*x])^(7/2),x]
```


```
[Out] (-2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(5*b*c*(a + b*ArcCosh[c*x])^(5/2)) + 4/
(15*b^2*c^2*(a + b*ArcCosh[c*x])^(3/2)) - (8*x^2)/(15*b^2*(a + b*ArcCosh[c*
x])^(3/2)) - (32*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*b^3*c*Sqrt[a + b*ArcCo
sh[c*x]]) + (8*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]]
)/Sqrt[b]])/(15*b^(7/2)*c^2) + (8*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCo
sh[c*x]])/Sqrt[b]])/(15*b^(7/2)*c^2*E^((2*a)/b))
```


Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
```

```
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{x}{\left(a+b\cosh^{-1}(cx)\right)^{7/2}}\,dx = -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} - \frac{2\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{5/2}}\,dx}{5bc} + \frac{(4c)\int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{5/2}}}{5b}$$

$$= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} + \frac{4}{15b^2c^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{8x^2}{15b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \cdots$$

$$= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} + \frac{4}{15b^2c^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{8x^2}{15b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \cdots$$

$$= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} + \frac{4}{15b^2c^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{8x^2}{15b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \cdots$$

$$= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} + \frac{4}{15b^2c^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{8x^2}{15b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \cdots$$

$$= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} + \frac{4}{15b^2c^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{8x^2}{15b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \cdots$$

Mathematica [A] time = 1.60954, size = 175, normalized size = 0.76

$$\frac{\frac{\sqrt{b}\left(-\sinh\left(2\cosh^{-1}(cx)\right)\left(16\left(a+b\cosh^{-1}(cx)\right)^2+3b^2\right)-4b\cosh\left(2\cosh^{-1}(cx)\right)\left(a+b\cosh^{-1}(cx)\right)\right)}{\left(a+b\cosh^{-1}(cx)\right)^{5/2}} + 8\sqrt{2\pi}\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)\cdots}{15b^{7/2}c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*ArcCosh[c*x])^(7/2),x]
```

```
[Out] (8*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(2*a)/
b] - Sinh[(2*a)/b]) + 8*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/S
qrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + (Sqrt[b]*(-4*b*(a + b*ArcCosh[c*x
])*Cosh[2*ArcCosh[c*x]] - (3*b^2 + 16*(a + b*ArcCosh[c*x])^2)*Sinh[2*ArcCos
h[c*x]]))/(a + b*ArcCosh[c*x])^(5/2))/(15*b^(7/2)*c^2)
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{arccosh}\left(cx\right)\right)^{-\frac{7}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccosh(c*x))^(7/2),x)
```

```
[Out] int(x/(a+b*arccosh(c*x))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arccosh(c*x) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acosh(c*x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.162 $\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{7/2}}\,dx$

Optimal. Leaf size=188

$$\frac{4\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c}-\frac{4x}{15b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}}-\frac{8\sqrt{cx-1}\sqrt{cx+1}}{15b^3c\sqrt{a+b\cosh^{-1}(cx)}}-$$

```
[Out] (-2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(5*b*c*(a + b*ArcCosh[c*x])^(5/2)) - (4*x
)/(15*b^2*(a + b*ArcCosh[c*x])^(3/2)) - (8*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(1
5*b^3*c*Sqrt[a + b*ArcCosh[c*x]]) + (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcC
osh[c*x]]/Sqrt[b]])/(15*b^(7/2)*c) + (4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*
x]]/Sqrt[b]])/(15*b^(7/2)*c*E^(a/b))
```

Rubi [A] time = 0.771664, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5656, 5775, 5781, 3307, 2180, 2204, 2205}

$$\frac{4\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c}-\frac{4x}{15b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}}-\frac{8\sqrt{cx-1}\sqrt{cx+1}}{15b^3c\sqrt{a+b\cosh^{-1}(cx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(-7/2),x]
```


```
[Out] (-2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(5*b*c*(a + b*ArcCosh[c*x])^(5/2)) - (4*x
)/(15*b^2*(a + b*ArcCosh[c*x])^(3/2)) - (8*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(1
5*b^3*c*Sqrt[a + b*ArcCosh[c*x]]) + (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcC
osh[c*x]]/Sqrt[b]])/(15*b^(7/2)*c) + (4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*
x]]/Sqrt[b]])/(15*b^(7/2)*c*E^(a/b))
```


Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
```

```
c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{7/2}}\,dx = -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} + \frac{(2c)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{5/2}}\,dx}{5b}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} - \frac{4x}{15b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{4\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx}{15b^2}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} - \frac{4x}{15b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{8\sqrt{-1+cx}\sqrt{1+cx}}{15b^3c\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(8c}{}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} - \frac{4x}{15b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{8\sqrt{-1+cx}\sqrt{1+cx}}{15b^3c\sqrt{a+b\cosh^{-1}(cx)}} + \frac{8\,\mathrm{S}}{}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} - \frac{4x}{15b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{8\sqrt{-1+cx}\sqrt{1+cx}}{15b^3c\sqrt{a+b\cosh^{-1}(cx)}} + \frac{4\,\mathrm{S}}{}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} - \frac{4x}{15b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{8\sqrt{-1+cx}\sqrt{1+cx}}{15b^3c\sqrt{a+b\cosh^{-1}(cx)}} + \frac{8\,\mathrm{S}}{}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{5bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}} - \frac{4x}{15b^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}} - \frac{8\sqrt{-1+cx}\sqrt{1+cx}}{15b^3c\sqrt{a+b\cosh^{-1}(cx)}} + \frac{4e^{}}{}$$

Mathematica [A] time = 0.698332, size = 214, normalized size = 1.14

$$\frac{-\frac{2e^{-\cosh^{-1}(cx)}\left(a+b\cosh^{-1}(cx)\right)\left(2e^{\frac{a}{b}+\cosh^{-1}(cx)}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\left(a+b\cosh^{-1}(cx)\right)\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)-2a-2b\cosh^{-1}(cx)+b\right)}{b^2} - \frac{2e^{-\frac{a}{b}}\left(a+b\cosh^{-1}(c\right.}{}}{15bc\left(a+b\cosh^{-1}(cx)\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(-7/2),x]
```

```
[Out] (-6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - (2*(a + b*ArcCosh[c*x])*(-2*a +
b - 2*b*ArcCosh[c*x] + 2*E^(a/b + ArcCosh[c*x])*Sqrt[a/b + ArcCosh[c*x]]*(a
```

```
 + b*ArcCosh[c*x])*Gamma[1/2, a/b + ArcCosh[c*x]]))/(b^2*E^ArcCosh[c*x]) -
(2*(a + b*ArcCosh[c*x])*(E^(a/b + ArcCosh[c*x])*(2*a + b + 2*b*ArcCosh[c*x]
) + 2*b*(-((a + b*ArcCosh[c*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c*x])
/b)]))/(b^2*E^(a/b)))/(15*b*c*(a + b*ArcCosh[c*x])^(5/2))
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(cx))^{-\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^(7/2),x)
```

```
[Out] int(1/(a+b*arccosh(c*x))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arcosh}(cx) + a)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(-7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.163 $\int \sqrt{fx}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=128

$$-\frac{16b^2c^2(fx)^{7/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2x^2\right)}{105f^3}-\frac{8bc\sqrt{1-cx}(fx)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right.}{15f^2\sqrt{cx-1}}$$

```
[Out] (2*(f*x)^(3/2)*(a + b*ArcCosh[c*x])^2)/(3*f) - (8*b*c*(f*x)^(5/2)*Sqrt[1 -
c*x]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x^2])/(15*f^
2*Sqrt[-1 + c*x]) - (16*b^2*c^2*(f*x)^(7/2)*HypergeometricPFQ[{1, 7/4, 7/4}
, {9/4, 11/4}, c^2*x^2])/(105*f^3)
```

Rubi [A] time = 0.285291, antiderivative size = 141, normalized size of antiderivative = 1.1, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5662, 5763}

$$-\frac{16b^2c^2(fx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{105f^3}-\frac{8bc\sqrt{1-c^2x^2}(fx)^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{15f^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{2(fx)^{3/2}\left(a+b\cos\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f*x]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (2*(f*x)^(3/2)*(a + b*ArcCosh[c*x])^2)/(3*f) - (8*b*c*(f*x)^(5/2)*Sqrt[1 -
c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x^2])/(1
5*f^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (16*b^2*c^2*(f*x)^(7/2)*Hypergeometri
cPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^2])/(105*f^3)
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
```

```
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \sqrt{fx}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{2(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{3f} - \frac{(4bc)\int \frac{(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{3f}$$

$$= \frac{2(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{3f} - \frac{8bc(fx)^{5/2}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2\right.}{15f^2\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.397293, size = 118, normalized size = 0.92

$$\frac{2}{105}x\sqrt{fx}\left(35\left(a+b\cosh^{-1}(cx)\right)^2-4bcx\left(2bcx\text{HypergeometricPFQ}\left(\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2x^2\right)+\frac{7\sqrt{1-c^2x^2}\text{Hype}}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f*x]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (2*x*Sqrt[f*x]*(35*(a + b*ArcCosh[c*x])^2 - 4*b*c*x*((7*Sqrt[1 - c^2*x^2]*(
a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x^2])/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + 2*b*c*x*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4},
c^2*x^2])))/105
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int (a+b\mathrm{arccosh}\,(cx))^2\sqrt{fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(f*x)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2*(f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(f*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\operatorname{arcosh}(cx)^2 + 2\,ab\operatorname{arcosh}(cx) + a^2\right)\sqrt{fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(f*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*sqrt(f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{fx}\,(a + b\operatorname{acosh}(cx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(f*x)**(1/2),x)
```

```
[Out] Integral(sqrt(f*x)*(a + b*acosh(c*x))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(f*x)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.164 $\int (dx)^m \left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=181

$$-\frac{2b^2c^2(dx)^{m+3}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2}\right\},\left\{\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2}\right\},c^2x^2\right)}{d^3(m+1)(m+2)(m+3)}-\frac{2bc\sqrt{1-cx}(dx)^{m+2}\text{Hypergeometri}}{d^2(m+1}$$

```
[Out] ((d*x)^(1 + m)*(a + b*ArcCosh[c*x])^2)/(d*(1 + m)) - (2*b*c*(d*x)^(2 + m)*S
qrt[1 - c*x]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)
/2, c^2*x^2])/(d^2*(1 + m)*(2 + m)*Sqrt[-1 + c*x]) - (2*b^2*c^2*(d*x)^(3 +
m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, c^2*x
^2])/(d^3*(1 + m)*(2 + m)*(3 + m))
```

Rubi [A] time = 0.309833, antiderivative size = 194, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5662, 5763}

$$-\frac{2b^2c^2(dx)^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};c^2x^2\right)}{d^3(m+1)(m+2)(m+3)}-\frac{2bc\sqrt{1-c^2x^2}(dx)^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)\left(a+b\cosh^{-1}(cx\right.}{d^2(m+1)(m+2)\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcCosh[c*x])^2)/(d*(1 + m)) - (2*b*c*(d*x)^(2 + m)*S
qrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (2 + m)/2, (4
+ m)/2, c^2*x^2])/(d^2*(1 + m)*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b
^2*c^2*(d*x)^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2,
 5/2 + m/2}, c^2*x^2])/(d^3*(1 + m)*(2 + m)*(3 + m))
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (dx)^m \left(a+b\cosh^{-1}(cx)\right)^2 dx = \frac{(dx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)^2}{d(1+m)} - \frac{(2bc)\int \frac{(dx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{d(1+m)}$$

$$= \frac{(dx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)^2}{d(1+m)} - \frac{2bc(dx)^{2+m}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\right.}{d^2(1+m)(2+m)\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.246303, size = 164, normalized size = 0.91

$$\frac{x(dx)^m\left(-\frac{2b^2c^2x^2\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2}\right\},\left\{\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2}\right\},c^2x^2\right)}{m^2+5m+6}-\frac{2bcx\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}\right.}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (x*(d*x)^m*((a + b*ArcCosh[c*x])^2 - (2*b*c*x*Sqrt[1 - c^2*x^2]*(a + b*ArcC
osh[c*x])*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/((2 + m)*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b^2*c^2*x^2*HypergeometricPFQ[{1, 3/2 + m
/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, c^2*x^2])/(6 + 5*m + m^2)))/(1 + m)
```

Maple [F] time = 2.246, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\,\mathrm{arccosh}\,(cx)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arccosh(c*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2 \operatorname{arcosh}(cx)^2 + 2\,ab \operatorname{arcosh}(cx) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{acosh}(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*acosh(c*x))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.165 $\int (dx)^m \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=106

$$\frac{(dx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{d(m+1)}-\frac{bc\sqrt{1-c^2x^2}(dx)^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{d^2(m+1)(m+2)\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(d*(1 + m)) - (b*c*(d*x)^(2 + m)*Sqrt[
1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(d^2*(1
 + m)*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.053476, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5662, 126, 365, 364}

$$\frac{(dx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{d(m+1)}-\frac{bc\sqrt{1-c^2x^2}(dx)^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{d^2(m+1)(m+2)\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcCosh[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(d*(1 + m)) - (b*c*(d*x)^(2 + m)*Sqrt[
1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(d^2*(1
 + m)*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+b\cosh^{-1}(cx)\right)dx &= \frac{(dx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{d(1+m)}-\frac{(bc)\int\frac{(dx)^{1+m}}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{d(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{d(1+m)}-\frac{\left(bc\sqrt{-1+c^2x^2}\right)\int\frac{(dx)^{1+m}}{\sqrt{-1+c^2x^2}}\,dx}{d(1+m)\sqrt{-1+cx}\sqrt{1+cx}}\\ &= \frac{(dx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{d(1+m)}-\frac{\left(bc\sqrt{1-c^2x^2}\right)\int\frac{(dx)^{1+m}}{\sqrt{1-c^2x^2}}\,dx}{d(1+m)\sqrt{-1+cx}\sqrt{1+cx}}\\ &= \frac{(dx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{d(1+m)}-\frac{bc(dx)^{2+m}\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};c^2x^2\right)}{d^2(1+m)(2+m)\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

Mathematica [A] time = 0.13697, size = 87, normalized size = 0.82

$$\frac{x(dx)^m\left(-\frac{bcx\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{(m+2)\sqrt{cx-1}\sqrt{cx+1}}+a+b\cosh^{-1}(cx)\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (x*(d*x)^m*(a + b*ArcCosh[c*x] - (b*c*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1
[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/((2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
)))/(1 + m)
```

Maple [F] time = 2.023, size = 0, normalized size = 0.

$$\int (dx)^m (a + b\operatorname{arccosh}(cx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arccosh(c*x)),x)
```

```
[Out] int((d*x)^m*(a+b*arccosh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \operatorname{arcosh}(cx) + a)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b\operatorname{acosh}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d*x)**m*(a + b*acosh(c*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.166 $\int \frac{(dx)^m}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b\cosh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcCosh[c*x]), x]
```

Rubi [A] time = 0.0281057, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcCosh[c*x]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcCosh[c*x]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\cosh^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\cosh^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.457408, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcCosh[c*x]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcCosh[c*x]), x]
```

Maple [A] time = 0.811, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b\mathrm{arccosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arccosh(c*x)),x)
```

```
[Out] int((d*x)^m/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b\,\mathrm{arcosh}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b*arccosh(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx)^m}{b\,\mathrm{arcosh}\,(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arccosh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b\,\mathrm{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d*x)**m/(a + b*acosh(c*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{arcosh}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arccosh(c*x) + a), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=", ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```